Analysis of 2007 Data from the Numeracy Development Projects: What Does the Picture Show?

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This paper presents an analysis of data gathered from teachers' assessments of their students using the Numeracy Project Assessment (NumPA) during 2007. The patterns of performance and progress were similar to those found in previous years. Slight fluctuations in the proportion of students at particular levels seem to coincide with changes in the proportion of students from low-decile schools involved with the Numeracy Development Projects (NDP). Comparison with the Ministry of Education curriculum expectations raises concerns about the gaps between what students attained after one year of NDP participation and the goals of the mathematics and statistics learning area of the new curriculum, particularly for students at years 6–9. An analysis of the relationships between strategy and knowledge domains shows the importance of place value and basic facts, particularly for the additive domain.

The Numeracy Development Projects (NDP) were first implemented in New Zealand schools in 2000. Since then, approximately 800,000 students and 29,000 teachers have participated in the NDP (Hyland, 2008). Like other educational reform initiatives worldwide, the NDP were set up to improve mathematics teaching and learning at primary and, later, at secondary levels (Bobis, Clarke, & Clarke, et al., 2005; Ministry of Education, 2001; Young-Loveridge, 2008). Key features of the NDP include the Number Framework, which consists of a sequence of global stages that describe the mental processes that students use to solve problems with numbers (strategies) as well as the key pieces of knowledge that students need to learn in order to be able to use strategies effectively (knowledge). The diagnostic interview (Numeracy Project Assessment [NumPA]), designed to provide teachers with valuable information about their students' knowledge and mental strategies, is aligned to the Framework. The initial professional development programme for teachers requires the participation of the school over a minimum of two years.

At the beginning and end of the first year of the professional development programme, teachers use the individual diagnostic interview to gather data on their students' mathematics achievement, which is sent to a secure website for later analysis. This data has been a valuable source of information for shaping the project over recent years (see Young-Loveridge, 2005, 2006, 2007). By 2009, virtually every primary and intermediate school in New Zealand (thus catering for students in years 0–8) will have had the opportunity to participate in this professional development.

The present study focuses on the analysis of data, gathered in 2007 by teachers of year 5–9 students, from schools in their first year of the NDP. The analysis also explores the inter-relationships among the strategy domains and selected knowledge domains. Comparisons are made between this 2007 data and the Ministry of Education (n.d.) curriculum expectations that are aligned to the mathematics and statistics learning area of *The New Zealand Curriculum* (Ministry of Education, 2007). The curriculum expectations are an indication of the expected levels of achievement once schools have everything in place for numeracy teaching and learning (Education Review Office, 2007). Some schools reach these levels within the two years of professional development, while others take longer.

Method

Participants

The cohort of students whose teachers participated in their school's first year of NDP consisted of 16 651 students in years 5–9 (see Appendix A, p. 212). The proportion of students from different decile bands varied according to year level, with years 5–6 including approximately 25% of low-decile students and close to 50% of high-decile students, whereas the corresponding figures for years 7 and 8 were 15% and just over 33% respectively. The year 9 cohort (including students in Secondary Numeracy Project [SNP] schools as well as students in area schools) had even fewer low-decile students (13%) and about 33% high-decile students.

Procedure

The teachers conducted initial assessments using the NumPA with their students in or about March 2007 and participated in a programme of workshops and in-class visits over the course of the year. The programme concluded with final assessments of the students in or about November. Data was sent to a secure website and made available for analysis at the beginning of 2008.

Results

Several key questions are addressed in this section. These include:

1. How do the 2007 data compare with data from previous years?
2. How do the 2007 data compare with the Ministry of Education curriculum expectations?
3. What is the relationship between performance on strategy domains and key knowledge domains?

Comparison of Data with Previous Years

Appendix B (p. 192) shows the percentages of students at each stage on the Number Framework at the beginning and end of 2007 for students in years 5–9. The percentage of students who were counters (stages 0–4) halved from the beginning of the year to the end, with a particularly marked effect at earlier year levels. There was a corresponding increase in the percentage of advanced part–whole thinkers at stages 6 and 7 (see figures 1–3). It is interesting to note that, at the end of the year, year 9 students were at very similar stages to year 8 students on the additive and multiplicative domains, whereas there was noticeable improvement for year 9 on the proportional domain. For years 5–9, there were noticeable improvements at each year level across all three domains. However, it is interesting to note that year 8 students performed almost as well as year 9 students on the additive and multiplicative domains.

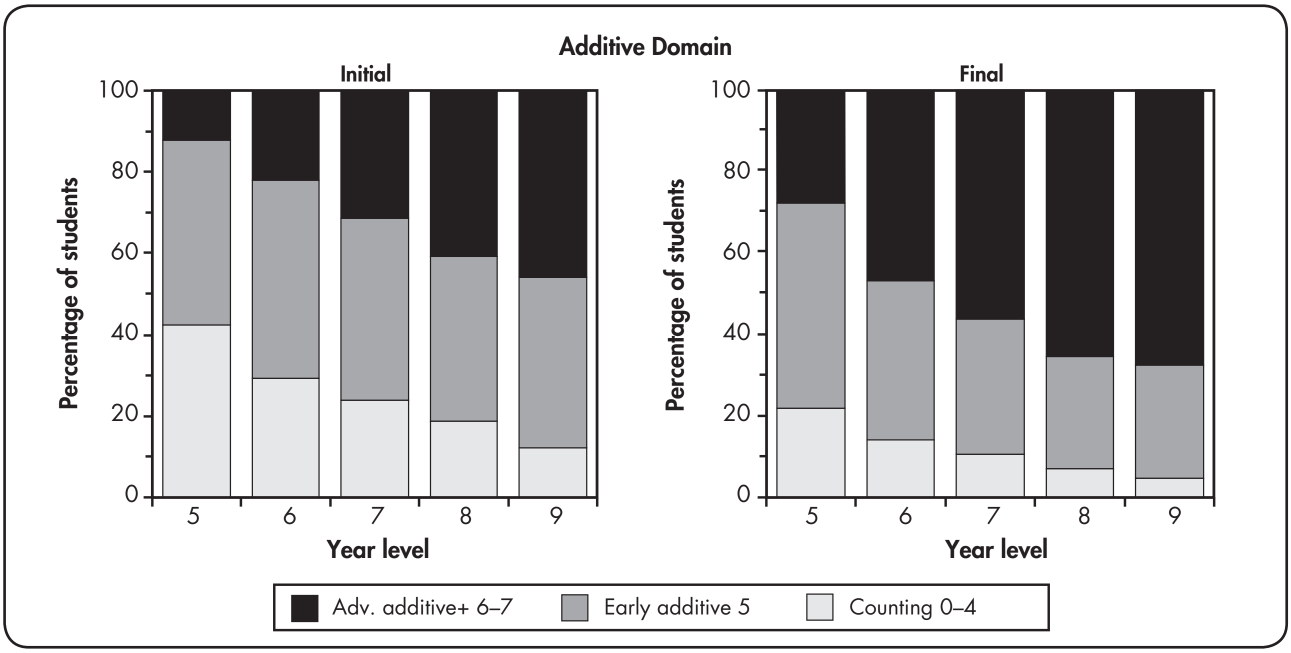

Figure 1. Percentage of students at stages on the Framework for the additive domain

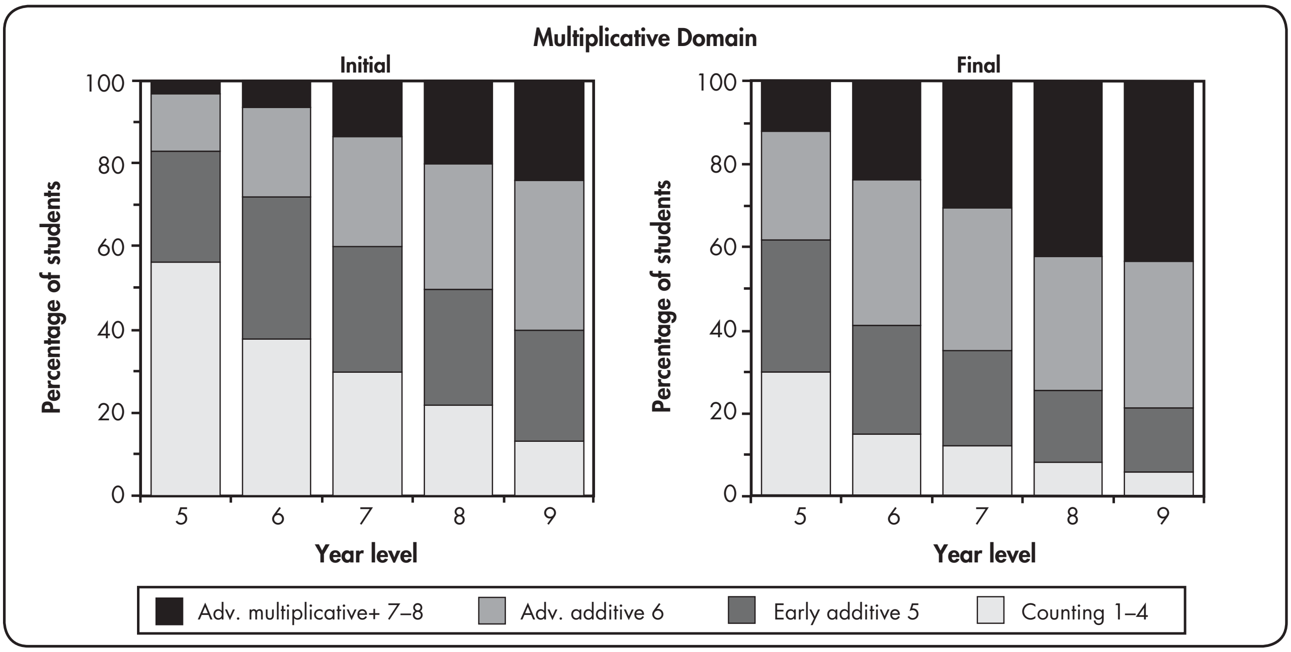

Figure 2. Percentage of students at stages on the Framework for the multiplicative domain

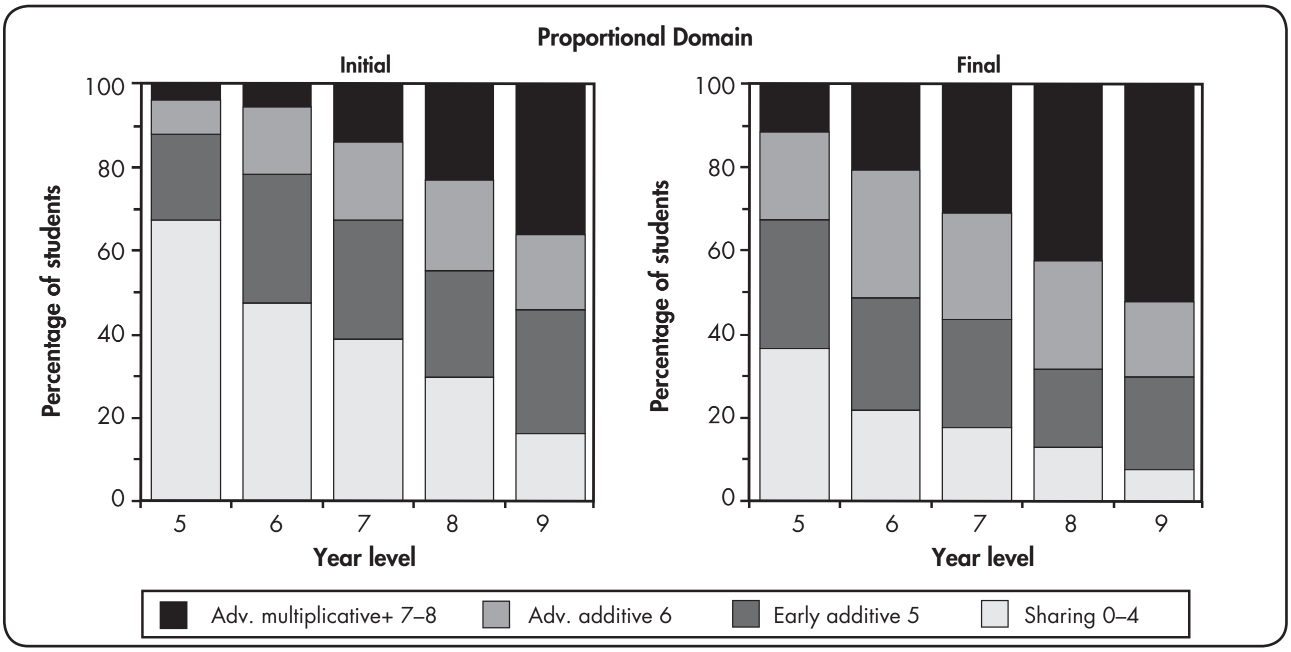

Figure 3. Percentage of students at stages on the Framework for the proportional domain

In lieu of a control group, a matched comparison analysis was carried out between adjacent years, comparing ***younger*** students ***after*** almost a year of the NDP with ***older*** students ***before*** the NDP had got underway (for example, year 5 students after a year of the NDP compared with year 6 students before the NDP had got underway). At this point in time (when the younger students were given their final assessment and the older students were given their initial assessment), the older students were only approximately one-third of a year older than the younger students. Hence differences favouring the younger students were strongly indicative of a "treatment" effect. Figures 4–6 show the difference in numbers of counters (students at stages 0–4) and advanced part–whole thinkers (students at stages 6–8). In every case, there were fewer younger students than older students still at a counting stage by the end of a year on the NDP, and more younger students than older students at an advanced part–whole stage.

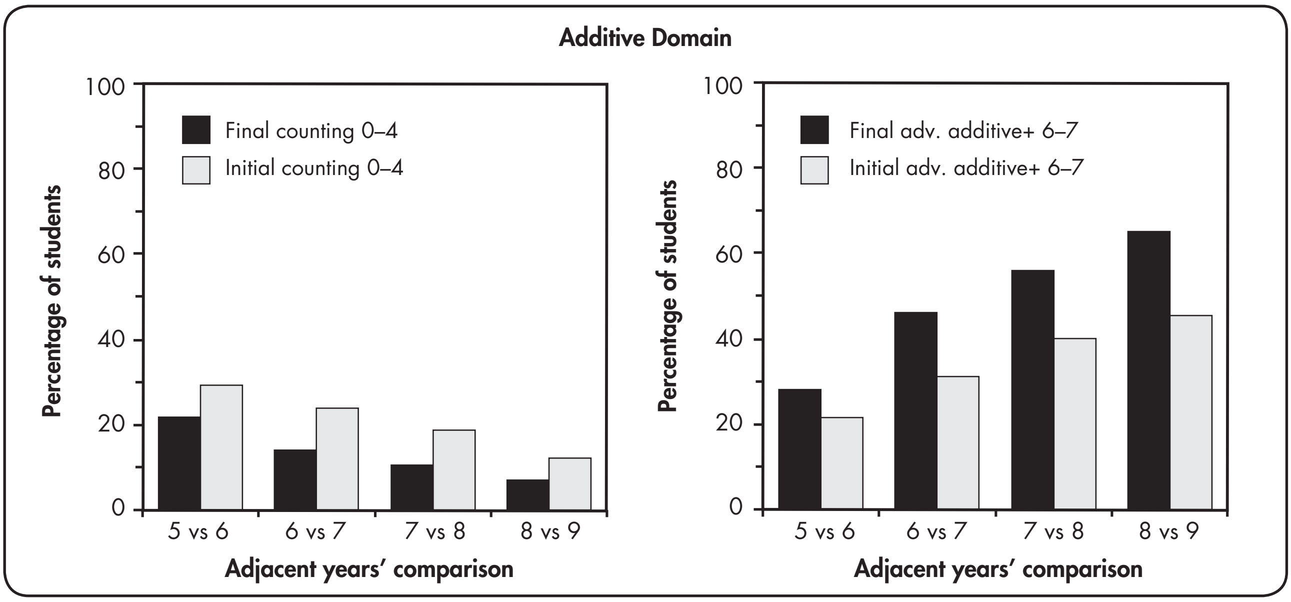

Figure 4. Comparison of ***younger*** students ***after*** NDP with ***older*** students ***before*** NDP on the additive domain

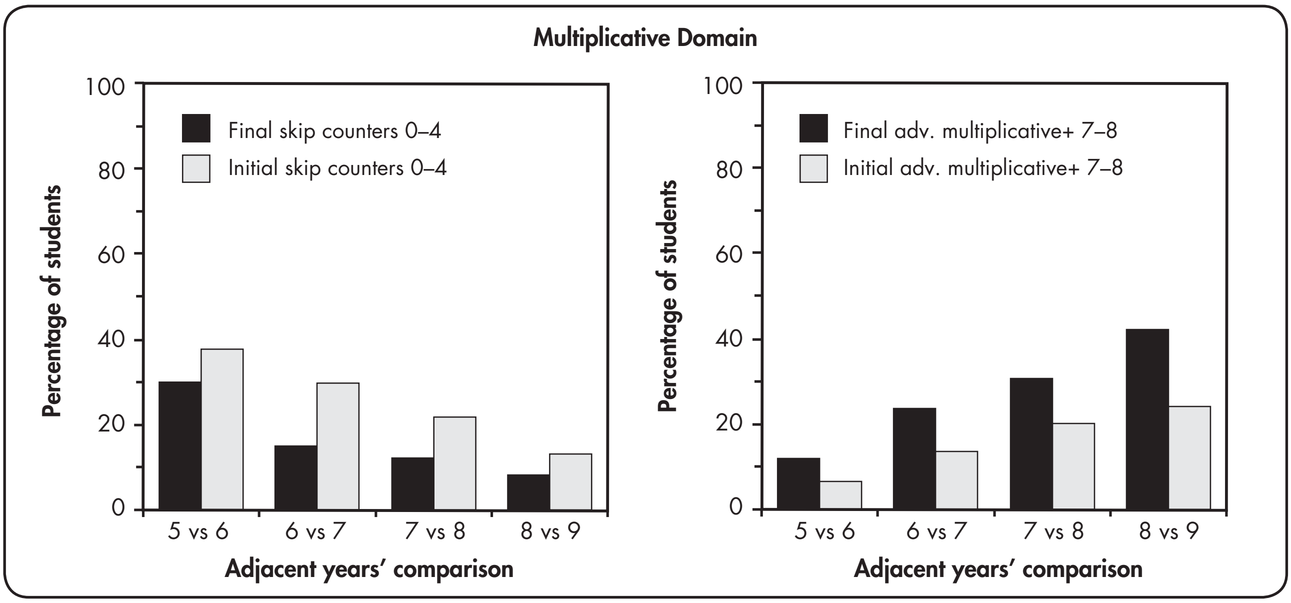

Figure 5. Comparison of ***younger*** students ***after*** NDP with ***older*** students ***before*** NDP on the multiplicative domain

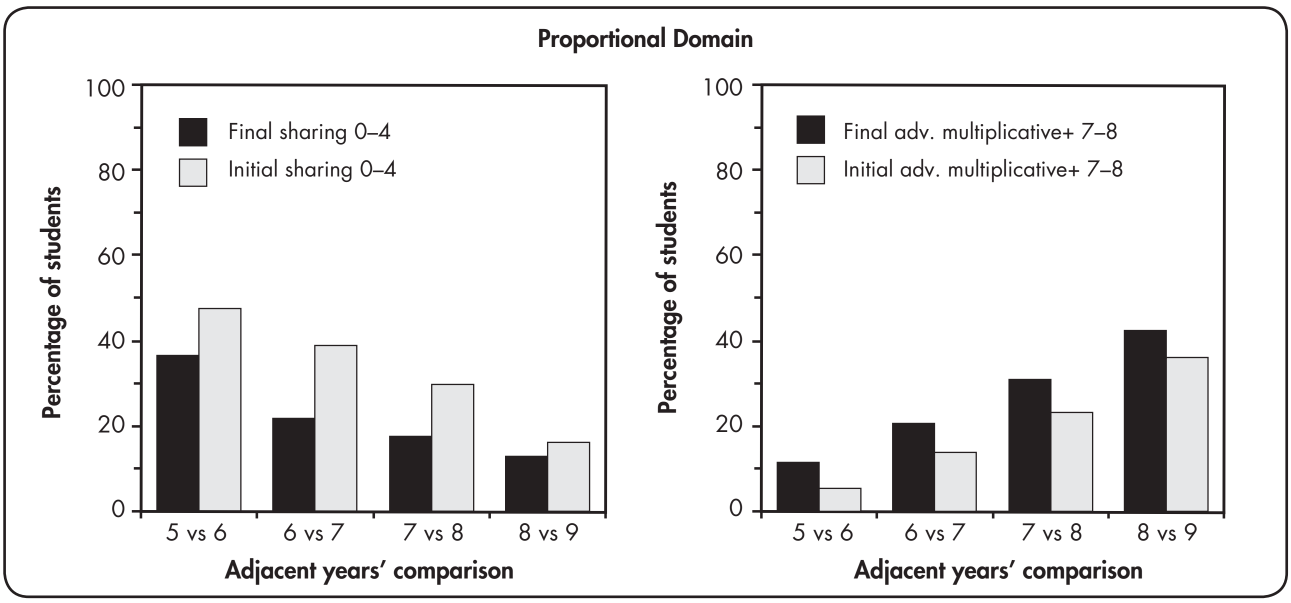

Figure 6. Comparison of ***younger*** students ***after*** NDP with ***older*** students ***before*** NDP on the proportional domain

Appendix C (p. 195) shows the percentages of students at each stage on the Framework at the end of a year on the NDP (2002–2007) for students in years 5–8. The results shown in Appendix C need to be interpreted alongside Appendix A, which reports the composition of the cohort of year 5–9 students at each year level over the period 2002–2007. Fluctuations from year to year in the proportion of students at particular stages on the Framework coincide roughly with similar changes in the composition of the cohort. For example, in 2007, the proportion of year 6 students in first-year NDP schools at stage 6 or higher on the additive domain increased from that in 2002 (from 36% to 46%). This could be explained by improvements in the support offered by numeracy facilitators, who by 2007 were more experienced, or it could be due to an increase in the proportion of students from high-decile schools taking part in the NDP (from 27% to 46%) and/or to the decrease in proportion of students from low-decile schools taking part in the NDP (from 34% to 28%). There was an even more pronounced improvement at year 7 (from 40% to 56%) and year 8 (from 51% to 65%) over the same period. This improvement coincides with increases in the proportion of high-decile students (from 20% to 37% at year 7 and from 19% to 40% at year 8) and a corresponding decrease in low-decile students (from 33% to 15% at year 7 and from 33% to 16% at year 8) over the same period. Similar improvements were found on the multiplicative and proportional domains, though the differences were smaller (see figures 7 and 8).

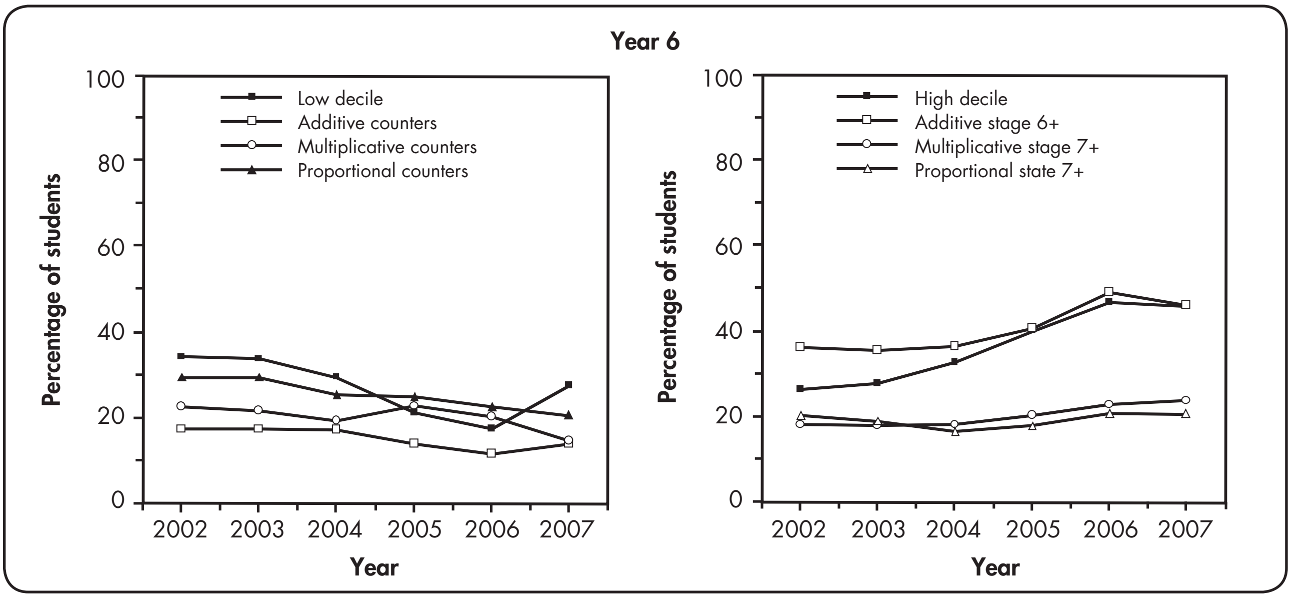

Figure 7. Percentage of year 6 students who used particular strategies to solve problems in each of the domains and attended schools in a particular decile band between 2002 and 2007

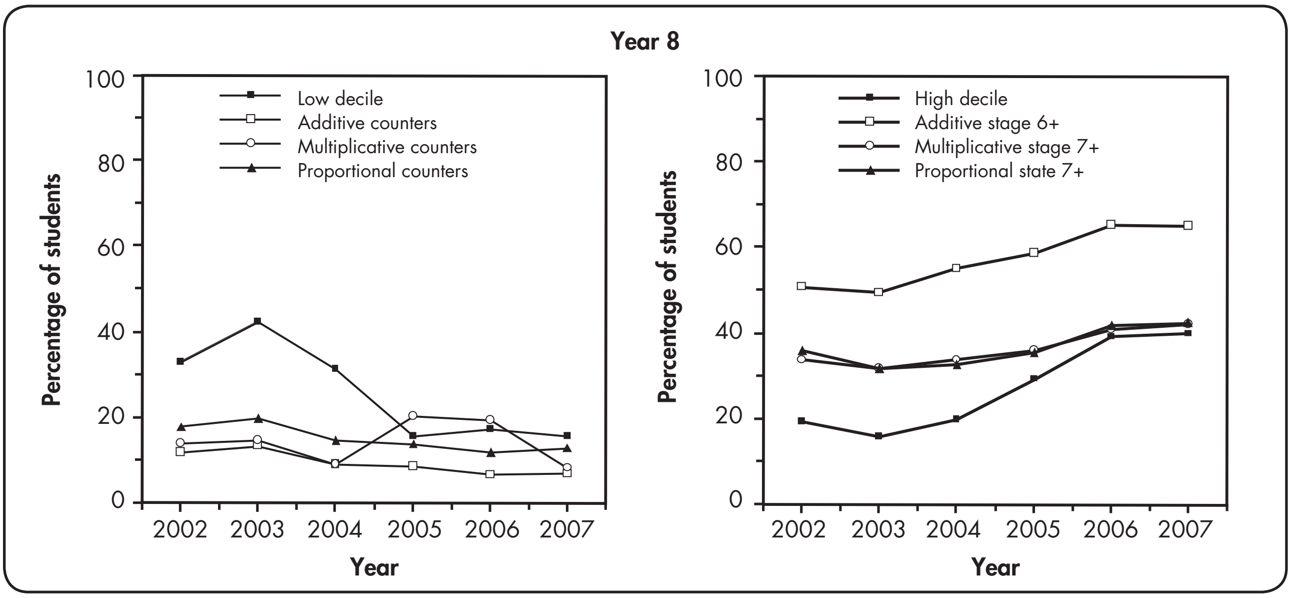

Figure 8. Percentage of year 8 students who used particular strategies to solve problems in each of the domains and attended schools in a particular decile band between 2002 and 2007

Appendices D and E (p. 197 and p. 200) show the percentages of students at each stage on the Framework as a function of socio-economic status (as reflected in school decile) and ethnicity respectively. In general, there were more high-decile students at higher stages on the Framework than middle-decile or low-decile students. However, an analysis of change from the beginning of the year until the end shows that, in many instances, the drop in percentage of students who were still using a counting strategy was greater for low-decile students than for high-decile students, partly because more low-decile students were at lower stages on the Framework initially. However, it must be remembered that school decile is a very blunt indicator of the socio-economic status of families whose children attend a school, particularly from year 7 onwards, when intermediate and secondary schools often take in students from families of widely different socio-economic status.

Comparison of Data with Curriculum Expectations

Appendix F (p. 203) presents the percentages of year 5–9 students at the end of 2007, the end of the first year of the school's involvement in the NDP, on each of the strategy domains aligned with the appropriate Ministry of Education (n.d.) curriculum expectations for those year levels.

Comparing end of first year results with expectations shows, for example, that more than three-quarters (78%) of year 5 students were at stage 5 (early additive part–whole) or higher on the Framework. This fits reasonably well with the curriculum expectations for this year level in that the majority of students were at stages 5 and 6 by the end of year 5. Only 20% of students fell into the category of "cause for concern" because of being at stage 4 (advanced counting), and just 2% were deemed to be "at risk" by virtue of being at stages 2 or 3 (counting from one either on materials or by imaging).

At the end of year 6, just under 50% of the students were at stage 6 (advanced additive–early multiplicative part–whole) or higher on the Framework (additive domain), and the 58% from high-decile schools was still well short of the majority expected to have reached this stage by the end of their primary school years. It is clear from the findings that it is a huge step to go from just beginning to partition single-digit and teen numbers in solving addition and subtraction problems to having a range of different mental strategies available for solving multi-digit addition and subtraction problems. More than one-third (39%) of year 6 students fell into the "cause for concern" category because of being at stage 5 (early additive), while 14% of this cohort was judged to be "at risk" because they were still using some kind of counting strategy.

A little over half (56%) of year 7 students were at stage 6 (advanced additive–early multiplicative part–whole) or higher on the Framework. A third (33%) of the students were "cause for concern", and 11% were considered to be "at risk". Although this appears to be an improvement on the year 6 results, the magnitude of the difference is not great enough to ensure that adequate progress was being made by the end of the following year (year 8).

According to the Ministry of Education (n.d.) curriculum expectations, the majority of students at year 8 should be at stage 7 (advanced multiplicative–early proportional part–whole) by the end of the year. However, only 42% of students had reached stage 7 by the end of year 8. Thirty-three percent of students were a "cause for concern", and 25% of students were now considered to be "at risk".

There was little improvement at year 9 on the multiplicative domain, with just 43% of students being at stage 7 by the end of the year. However, performance was slightly better on the proportional domain, with 53% of year 9 students reaching stage 7 by the end of the year. The proportion of year 9 students at stage 8 was virtually identical on both the multiplicative and proportional domains (13% and 12% respectively).

Appendix F (p. 203) also shows the percentages of year 5–9 students from low- and high-decile schools on each stage on the Framework for each strategy domain, aligned with the curriculum expectations. A comparison of the percentages of students who are “not OK” (that is, they have been categorised as either “cause for concern” or “at risk”) shows considerably greater risk status for students from low-decile schools than for those from high-decile schools. The level of “risk” varies somewhat as a function of the domain, partly because it is more difficult to meet the requirements for stage 7 (advanced multiplicative) on the additive domain than it is on either the multiplicative or proportional domains. When this is combined with school decile, it appears that as much as 85% of year 8 students from low-decile schools were “below expectations”. The picture was slightly better for year 8 students from high-decile schools, but even so, 67% of these students were “below expectations” on the additive domain. The year 9 figures are no better for the additive domain. The multiplicative and proportional domains are better, but even then, just under 50% of year 8 and 9 students from high-decile schools were “below expectations”. Regardless of the differences as a function of domain and school decile, the message is very clear that schools need to do more to help their students become advanced multiplicative thinkers if we are to provide students with mathematics teaching and learning in line with international standards.

These results are consistent with those of international research literature, which shows that it takes at least two years, sometimes longer, for educational change to have an impact on students’ learning (for example, Lamon, 2007). If we want to see long-term benefits from the NDP, it is vital that the professional development continues beyond the initial two-year period. Many schools may need support for considerably longer if they are to meet the Ministry of Education (n.d.) curriculum expectations.

Relationships between Strategy and Knowledge Domains

Correlations were calculated among the strategy and knowledge domains (see Table 1).

Table 1
Correlations among Strategy and Knowledge Domains

Domain	Multiplicative	Proportional	Fractions	Place Value	Basic Facts	FNWS	BNWS
Additive	0.80	0.75	0.70	0.81	0.83	0.79	0.80
Multiplicative		0.80	0.74	0.76	0.75	0.64	0.66
Proportional			0.73	0.73	0.72	0.62	0.64
Fractions				0.76	0.71	0.61	0.62
Place Value					0.82	0.76	0.76
Basic Facts						0.78	0.80
Forward NWS							0.92

The highest correlation was between forward and backward number word sequence (0.92). Other correlations that matched or exceeded 0.80 included those between the additive and multiplicative domains (0.80) and those between the multiplicative and proportional domains (0.80). Basic facts were strongly related to the additive domain (0.83), as was place value (0.81). Basic facts were strongly related to place value (0.82) and to backward number word sequence (0.80). It is interesting to note that the relationship between fractions and the proportional domain was somewhat smaller (0.73).

Appendix G (p. 206) shows the percentages of year 5–9 students at each stage on the strategy domains (stages 4 and above) that were at various stages on strategy and selected knowledge domains at the end of 2007. It is interesting to note that among the students who were at stage 4 (advanced counting) on the additive domain, just over 60% had reasonable basic facts (that is, stage 5 or higher), whereas only 42% were able to identify the number of tens in numbers to 1000, and only 10% were able to identify the number of tens, hundreds, or thousands in any number. This contrasts with 32% of students at stage 5 (early additive) and 64% of those at stage 6 (advanced additive–early multiplicative). It seems that it is not until students reach stage 7 (advanced multiplicative–early proportional) that they completely master place value for whole numbers, and by then, the majority (76%) also understand how place value works with decimal numbers.

An examination of the percentages of students at various stages on the multiplicative domain shows a high level of consistency with those found on the additive domain. For example, at stage 4 on both domains, about 33% of students could identify unit fractions, 40% could order unit fractions, approximately 8% could co-ordinate numerators and denominators, and few if any could recognise equivalent fractions or order a collection of mixed fractions (with different numerators and denominators).

As in 2006, stage 7 on the additive domain was more difficult to reach than stage 7 on either the multiplicative or proportional domains (see Young-Loveridge, 2007). A greater proportion of the stage 7 additive students reached the top of the Framework (for example, stage 8) on other domains than did stages 7 and 8 multiplicative or stages 7 and 8 proportional students. There may be a case to be made for re-evaluating items to align them in difficulty. While there is bound to be some variation in difficulty level from task to task, Appendix G shows remarkable levels of consistency across domains. For example, for those at stage 4 on the additive, multiplicative, and proportional domains, the proportion at stage 4 on fractions was 34%, 37%, and 32% respectively.

Place Value and Basic Facts Knowledge: Relationships with Other Domains

Appendices H and I (p. 208 and p. 210) present information about the percentages of year 5–9 students at stages on the place value and the basic facts domains respectively after one year of the NDP and the stages they had reached on other domains on the Framework at that time. It is clear that students need to be able to use ten as a unit (stage 4 place value) if they are to be able to use even the simplest part–whole strategies for solving addition, subtraction, multiplication, division, or fractions problems. Almost all who were at stage 5 on the place value domain (who were able to give the number of tens in 230 and could identify 6.8 on a number line) were able to use some kind of part–whole strategy to solve problems in the various domains. For those at stage 4 on basic facts (knowing addition facts to 20), this was valuable for using part–whole strategies to solve problems involving addition and subtraction but not as important for problems in the multiplicative and proportional domains. Students needed to be at stage 5 or even stage 6 in order to be able to use part–whole strategies for solving problems in all three domains.

Discussion

The consistency of the comparisons between younger students after a year on the NDP with older students before the NDP got underway, looking just at students at extremes of the Number Framework (those still counting and those using advanced part–whole strategies), are consistent with similar findings in an earlier study (see Young-Loveridge, 2007). Larger numbers of students in the earlier study enabled an effect size analysis to be done separately for each major ethnic group. In that study, Pasifika students showed the largest average effect sizes (0.40), followed by students attending low-decile schools (0.38), and Māori students (0.35). When just the proportional domain was considered, the average effects sizes were even greater, with Pasifika students having an average effect size of close to half a standard deviation (0.48), followed by students from low-decile schools (0.45), and then Māori students (0.42). The average effect size for European students across all domains was 0.33, compared with that for just the proportional domain at 0.37. These effect sizes are a healthy magnitude, particularly in view of the fact that they are deflated by the "control" group being on average 4 months older than the "treatment" group at the time of the assessments.

The advantage of effect sizes is that they allow us to look at the impact of the NDP on students' learning in relation to those of other projects. (According to Fan [2001], effect sizes of 0.20, 0.50, 0.80, are considered small, medium, and large respectively). For example, the average effect sizes found for the United Kingdom National Numeracy Strategy were 0.17 or 0.18 (see Brown, Askew, Millet, & Rhodes, 2003), an initiative described by Fullan and Earl (2002, p. 3) as "an impressive success". Average effect sizes on the NDP for Pasifika, Māori, and low-decile students were more than double that and provide robust evidence that the project has had a substantial impact on the mathematics learning of those students.

Curriculum Expectations

As Appendix F shows, most students in year 5 whose teachers had recently completed the first year of the NDP professional development programme seemed to meet the curriculum expectations outlined by the Ministry of Education (n.d.). However, from about year 6 onwards, the proportion of students below the expected levels was close to half, and this increased steadily up to years 8 and 9. According to the Ministry of Education, "These expectations are for schools with everything in place for numeracy teaching and learning" (Ministry of Education, n.d.). Therefore, it would be unreasonable to expect all students in senior primary/intermediate to be "at or above expectations" at the end of their school's first year of NDP. Many schools may need two or three more years to become ready to meet these curriculum expectations. According to the Ministry of Education, the curriculum expectations:

> ... are intended to assist principals and teachers in setting high yet attainable expectations, and develop teaching and learning programmes for all students at each year level in their school. (Ministry of Education, n.d.)

These expectations consist of the Global Strategy Stage (GloSS) sequence on the Number Framework and indications about which stage/s the majority of students at each year level should have reached or exceeded by the end of that year (at or above expected achievement). Also included are the categories of "cause for concern" (immediately below expected levels) and "at risk" (substantially below expected levels). The Ministry of Education points out that "These expectations, and the indications of when to consider students to be 'at risk', 'cause for concern', or 'high achievers', are a guide only" (Ministry of Education, n.d.).

The curriculum expectations have a strong foundation in international literature, including research evidence showing that students need to be advanced multiplicative thinkers if they are to engage with the formal algebra being introduced at secondary schools (Brown & Quinn, 2006; Lamon, 2007; Wu, 2002). An analysis of the curricula in other education systems reveals that the expectations for students at the end of year 8 in New Zealand are assigned somewhat later than in other jurisdictions. For example, in the United States, students at grade 5 (year 6) are expected to be "developing an understanding of and fluency with addition and subtraction of fractions and decimals" and the following year (grade 6 [year 7]) "developing an understanding of and fluency with multiplication and division of fractions and decimals" as well as "connecting ratio and rate to multiplication and division" (NCTM, n.d.). These expectations are at least stage 7 advanced multiplicative part–whole thinking on the New Zealand Number Framework – expectations for the end of year 8. At grade 7 (year 8), the United States students are "developing an understanding of and applying proportionality", which matches stage 8 advanced proportional part–whole, an expectation for the end of year 9 in New Zealand. In New South Wales, the expectation at grade 6 (year 7) is that a student "operates with fractions, decimals, percentages, ratios and rates" (Board of Studies NSW, 2006).

Data from the Longitudinal Study provides evidence of the stages on the Number Framework reached by students in schools that participated in that study and is reported with the Ministry of Education (n.d.) curriculum expectations (see also Tagg & Thomas, 2007). The analysis shows that 70% of these students met the year 6 curriculum expectations by reaching stage 6 on the Framework, or higher. This contrasts with between 47% and 59% of students (depending on the domain) whose teachers were in their school's first year of the NDP. The longitudinal cohort for 2006 included students from each of the decile bands (15% low, 40% middle, and 46% high), but the sample was skewed in favour of students attending high- and middle-decile schools, although Tagg and Thomas had tried to ensure that the sample of schools in the longitudinal study was representative of schools overall.

It is not surprising that most students whose teachers have completed just one year of the NDP have not yet reached the levels hoped for in the curriculum expectations. Research shows that bringing about major reform in mathematics pedagogy takes considerable time, at least two years or more, according to Lamon (2007). The lack of teacher content knowledge in mathematics, particularly in the area of fractions, has already been documented (see Ward & Thomas, 2007, this volume). There is clearly much more work needed to help teachers develop a deeper understanding of the proportional structure of multiplication, division, and fractions (Sophian, 2007).

The curriculum expectations provide an important set of goalposts for New Zealand teachers and schools. This is a considerable challenge, and after only one year of NDP, it is doubtful that teachers at upper primary and intermediate school levels have had sufficient opportunity to fully understand all the ideas. It would be very useful to analyse the data from students whose teachers were in their second year of the NDP to see whether there is evidence of higher achievement by these students relative to those whose teachers have had only one year on the project.

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