Does Teacher Knowledge Make a Difference?

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This study reports on the development of a teacher assessment to assist in targeting professional development support. In order to be effective, the assessment must identify those teachers whose knowledge gaps are likely to impact on their students' achievement. Measures of teacher content knowledge (CK) and teacher pedagogical content knowledge (PCK) were obtained using a teaching of fractions assessment. The relationship between teachers' CK and PCK scores was examined, and a strong correlation was found. In general, those teachers who received low scores on the CK questions also received low scores on the PCK questions, while those teachers with high CK scores received a wide range of PCK scores. The relationship between teacher scores on the assessment and student achievement information was also investigated. The gains made by students whose teachers received the top two-thirds of scores in the assessment were significantly greater than those made by students whose teachers received the lower third of scores, where students were initially rated at stages 0 and 6 of the Number Framework.

Background

Teachers of mathematics require a broad range of knowledge in order to be effective. Knowledge of mathematics content, knowledge of students as learners, and knowledge of effective teaching strategies have all been identified as important components of this knowledge (National Council of Teachers of Mathematics, 2000). Of particular significance among the research on teacher knowledge is Shulman's (1986) definition of teacher pedagogical content knowledge (PCK) as knowledge of a subject "for teaching". Shulman distinguished this from pure subject knowledge by describing PCK as knowing the best ways to explain the subject to learners; using the most constructive examples to illustrate specific concepts; and having an idea of the misconceptions and pre-conceptions that learners may bring with them to the learning.

Studies have sought to describe and measure teacher PCK in the field of mathematics. Questions based on teaching and learning scenarios have been used successfully to provide a reliable measure of teacher PCK (Hill, Schilling, & Ball, 2004). In a further study, these measures were mapped against student achievement data. Teacher knowledge was shown to be a significant predictor of student gains, with a similar size of impact as student background characteristics such as ethnicity and gender (Hill, Rowan, & Ball, 2005). In particular, it was found that teachers within the lower third of the sample had significantly lower student achievement. This finding was used as a rationale to recommend the provision of targeted professional development programmes for these teachers.

> Our results suggest that those who may benefit most [from content-focused professional development programmes] are the teachers in the lowest third of the distribution of knowledge and that efforts to recruit teachers into professional development and pre-service coursework might focus heavily on those with weak subject matter knowledge for teaching. (Hill, Rowan, & Ball, 2005, p. 41)

In order to address this need in the Numeracy Development Projects (NDP), an assessment was developed to evaluate teachers' knowledge for teaching fractions (Ward & Thomas, 2007). Fractions were selected as an appropriate area in which to assess teacher knowledge because they are recognised as one of the most important areas in the primary school mathematics curriculum (Behr, Lesh, Post, & Silver, 1983; Lamon, 2007) and also as one of the most challenging to teach and learn effectively (Smith, 2002; Lamon, 2007). The assessment consists of seven teaching and learning scenarios related to fractions, with questions, based on these scenarios, that investigate teachers' knowledge of the mathematical content involved and the most effective teaching methods. Trials found that the assessment was effective in producing a range of teachers' scores and that it was efficient because it was quick to administer and teachers did not find the process unduly stressful (Ward & Thomas, 2007).

In addition to being an important area within the mathematics and statistics learning area of the new curriculum, fractions represent the start of students' development of proportional reasoning skills. The fracturing or breaking of whole numbers provides a basis for understanding the rational number system, which is a prerequisite for being able to reason proportionally (Lamon, 2005). This is particularly significant in the context of the NDP because multiplicative and proportional thinking have been identified as areas of concern (Ward & Thomas, 2007; Young-Loveridge, 2006, 2007; Young-Loveridge, Taylor, Hāwera, & Sharma, 2007). NDP results indicate that approximately two-thirds of students are unable to think either multiplicatively or proportionally by the end of their primary school years (Young-Loveridge, 2007). More detailed analysis has shown that both students and teachers have difficulty performing operations with fractional numbers, with 34% of teachers and 87% of students unable to correctly answer an addition of unlike fractions task such as three-quarters plus seven-eighths (Young-Loveridge et al., 2007; Ward & Thomas, 2007). In addition, approximately 25% of students and 14% of teachers incorrectly identified the appropriate method for solving such problems as being to "add across", adding together both numerators and denominators.

> It is clear from the results that teachers need a great deal more help in coming to understand multiplicative thinking and proportional reasoning. (Young-Loveridge, 2007, p. 30)

The present study continues work on the teaching of fractions assessment. If the assessment is to prove useful in identifying teachers for targeted professional development support, which ultimately improves teacher knowledge and increases student achievement, it is important to ensure that it is able to identify those teachers whose students make less progress than expected. This paper reports on work to identify these teachers and includes an initial investigation of the relationship between teacher scores on the assessment and student achievement data.

Method

Materials

Information on teacher knowledge was collected using the teaching of fractions assessment. The assessment is designed to collect information on teachers' content knowledge (CK) and PCK in the area of fractions. It was developed with the support of a reference group of numeracy facilitators and practising classroom teachers in 2006 (Ward & Thomas, 2007), with further modifications made in 2007. The seven teaching and learning scenarios used in the assessment each focus on different aspects of fractions and operations with fractions: equivalent fractions; addition of fractions; proportions; multiplication of fractions; ordering fractions; part to whole reasoning; and fractional number lines. In general, the items described a scenario in which a student was finding it difficult to answer a problem involving fractions. Teachers were asked to identify the correct answer to the problem and then describe either the key understandings involved or the teacher actions required to help the student develop these understandings. All items included an option for the teachers to indicate if they were unsure how to respond.

The teacher responses to the scenarios were evaluated using a marking criteria developed alongside the assessment. Questions involving pure CK were marked as either correct or incorrect. Responses based on the teachers' PCK were categorised as conceptually focused or other. Conceptually-focused responses were those that were most likely to support students in developing a sound understanding of fractions and operating with fractions. Other responses included those that were mathematically incorrect, unclear, irrelevant, or overly broad, as well as those based solely on learning and applying rules or procedures. A second researcher marked a random sample of 20% of the teacher assessments. There was agreement between the two markers on 97% of the ratings made. The assessment was marked out of a total of 17 points; the CK component had a total possible score of seven points, and the PCK component was worth ten points.

Student achievement data was collected by teachers using the Numeracy Project Assessment (NumPA). The NumPA is an individual interview that provides information on students' knowledge and strategy stages aligned to the Number Framework.

Process

Teacher assessments were distributed to schools early in October 2007; the participants were asked to complete them and return them within a fortnight. Detailed instructions for the completion of the assessments were provided, with the teachers asked to complete the assessments individually. To enable teacher assessment results to be matched with student achievement data, the teachers recorded on their assessments the class name they used in the online Numeracy Database. The completed teacher assessments were marked by the researcher, using the marking criteria.

Two sets of student achievement information were collected by the teachers: initial data was collected before instruction began, and final data was collected near the end of the 2007 school year. Student data was entered into the online Numeracy Database and was accessed by the researcher from the database on 16 January, 2008.

Participants

Forty-five schools in the first year of their participation in the NDP were invited to participate, and 18 schools accepted the invitation. The sample included data from all teachers who had taught a full year in one classroom, completed the assessment independently, and provided enough information in their responses to indicate their understandings. All the students from the teachers' classes for whom initial and final results in all three strategy domains were available were included in the sample.

The final sample consisted of 17 schools: one low-decile (1–3) school, ten medium-decile (4–7) schools, and six high-decile (8–10) schools. Eighty-eight teacher assessments were included in the sample. Table 1 provides a summary of the year levels taught by these teachers. Participating teachers had up to 30 years' experience teaching, with the majority of teachers' experience being at the year level of the students they were currently teaching.

Table 1
Participating Teachers

Year levels	Number of teachers	Percentage of teachers
1–3	27	31
4–6	18	20
7–8	32	36
9	11	13
Total	88	100

On average, each participating teacher provided complete results for 22 students. Table 2 shows the distribution of participating students across year levels.

Table 2
Participating Students

Year level	Number of students	Percentage of students
1	168	9
2	188	10
3	172	9
4	119	6
5	140	7
6	122	6
7	371	19
8	445	23
9	221	11
Total	1946	100

Analysis

Teachers' scores in the teaching of fractions assessment were analysed, with the CK and PCK components isolated for consideration. Patterns and trends in responses were identified to enable the teachers to be split into two groups. Group one consisted of approximately one-third of the teachers and included those teachers with the lowest scores in the assessment. Group two was made up of the remaining teachers, those with the highest scores in the assessment.

Demographic factors such as year levels taught and years of teaching experience were compared for the two groups.

Student achievement information from each of the teacher groups was compared and the student data was aligned to the Number Framework. The Framework is known to have non-linear student gains across stages (Irwin, 2003, Irwin & Niederer, 2002, Ward & Thomas, 2002), so analysis was carried out by comparing the number of stages students gained from each starting stage. Gains were measured using a generated Global Strategy Stage (GloSS) score, which combines information from all three strategy domains of the Framework (Thomas & Tagg, 2006). A student's GloSS is usually determined by using the GloSS assessment forms, but for the purposes of this study, initial and final GloSS results for each student were generated by taking the highest strategy stage reported for that student across the three strategy domains.

Findings

A wide range of teacher scores were obtained in the assessment. As shown by Figure 1, the majority of teachers scored in the middle of the range, with 76 of the 88 teachers scoring between five and 14 points, from a maximum score of 17. Smaller groups of teachers scored at the extremities of the range, with seven teachers scoring less than five points and five teachers scoring more than 14 points.

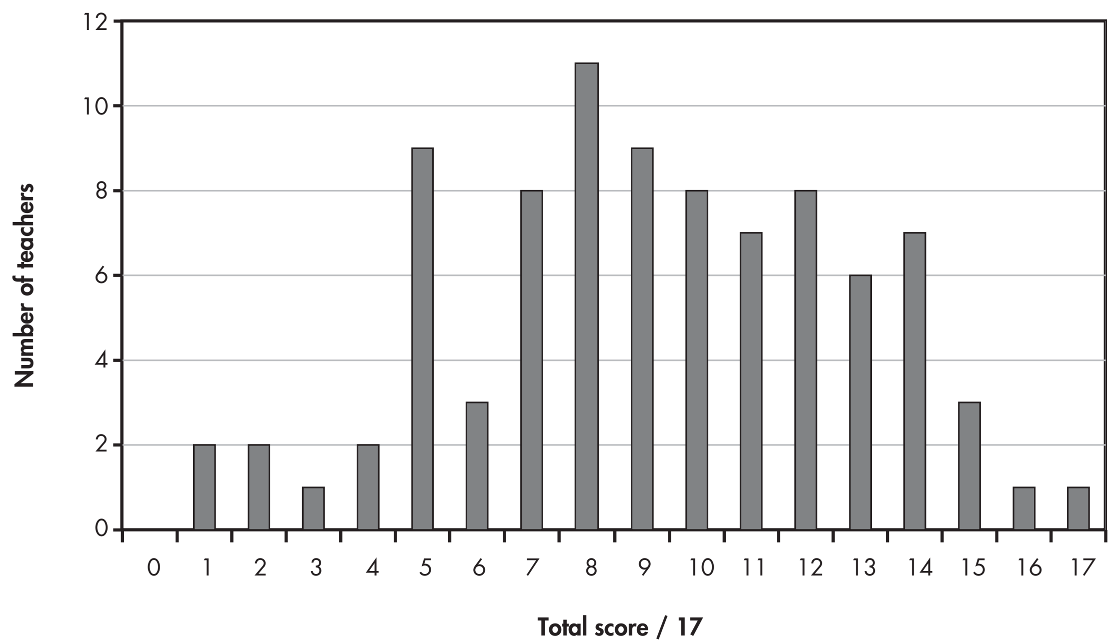

Figure 1. Distribution of teacher scores on the teaching of fractions assessment

Clearly, those teachers with scores of less than five points received low scores on both the CK and PCK components of the assessment. In contrast, teachers with scores of more than 14 points scored highly on both components of the assessment. Figure 2 plots the teachers' PCK against CK scores, with the line of best fit included. Note that the numerals in brackets give the number of teachers at each of the data points.

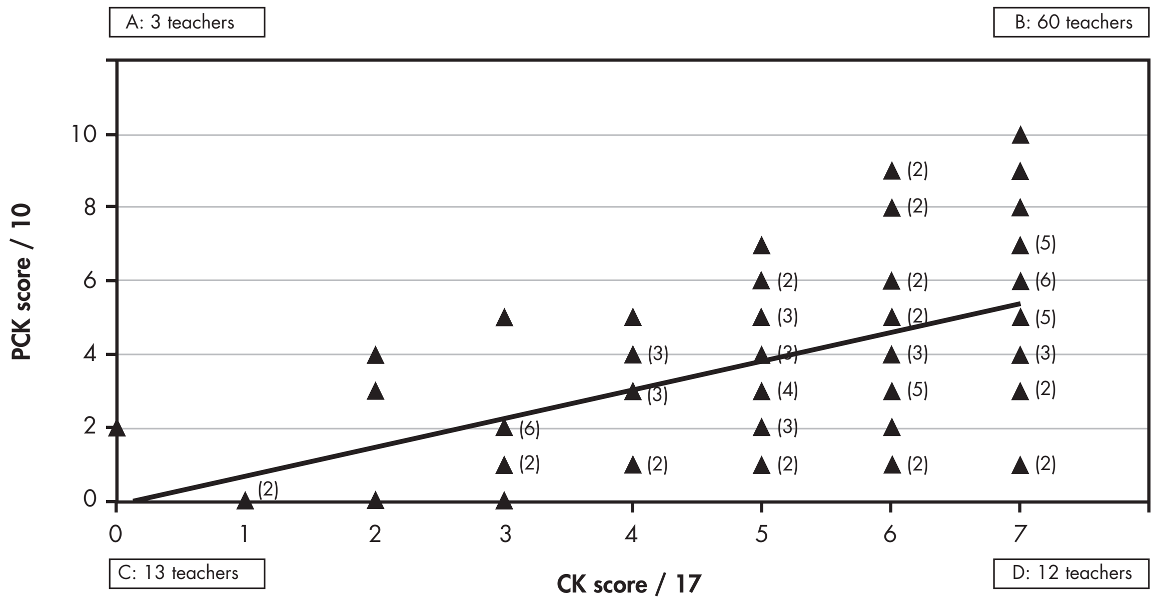

Figure 2. Teacher scores, CK vs PCK

Teacher PCK is strongly linked to teacher CK with a correlation of 0.57 and a p-value of less than 0.01. In general, those teachers with low CK are also likely to have low PCK. This finding is not surprising because those teachers who are unfamiliar with specific content are unlikely to know effective ways to teach that content. In contrast, the teachers with high CK scores received a wide range of PCK scores. For example, 26 teachers answered all seven content questions correctly, but the PCK scores of these teachers ranged from one out of ten to ten out of ten. This finding illustrates the fact that a sound understanding of particular content does not necessarily link with knowledge of effective ways to teach that content.

To identify those teachers who would most benefit from professional development support, the distribution of teacher scores was divided into four quadrants. Figure 3 shows these divisions.

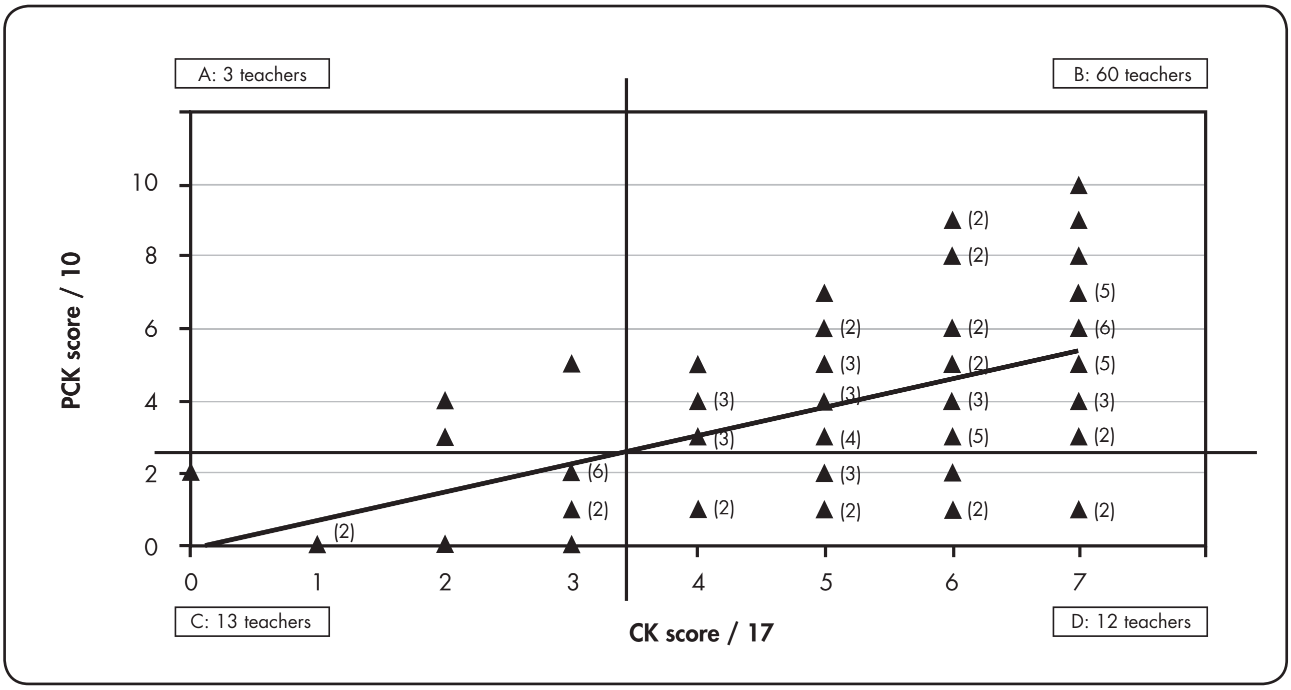

Figure 3. Quadrants reflecting teachers' CK and PCK scores

The quadrant lines were placed to identify two groups of teachers, with one-third of teachers in the lower group. Quadrants A, C, and D combine to make up group one, those teachers with the lowest scores in either or both of the CK and PCK components of the assessment. This group comprises 28 teachers and represents 32% of the sample. The 60 teachers in quadrant B make up group two, those with at least moderate scores in both components of the assessment.

The quadrant divisions represent teachers with varying levels of CK and PCK. Because teacher responses to the assessment scenarios varied on this basis, it is useful to investigate the types of responses provided by teachers in each of the quadrants. Figure 4 shows one of the assessment scenarios, and figures 5–7 illustrate and describe typical teacher responses from each of the quadrants.

Following a similar pattern to all seven scenarios in the assessment, the first question in the pizza scenario asks teachers to provide the correct answer to the problem. Teacher responses to this question provide a measure of their CK. The second question probes the teachers' PCK by asking what action they would take. The assessment instructions encourage teachers to be as specific as possible in their responses, and it is suggested they may like to mention what they would say to the student, how they would use any materials, and any further examples they would draw on.

Cameron is working on the following problem:

There is $\frac{3}{4}$ of a pizza left after the party. $\frac{1}{3}$ of the leftovers are given to Sarah to take home. How much pizza does Sarah take home?

You hear Cameron say "One-third of three-quarters; that's the same as one-third times three-quarters ..."

1. What is the answer to the problem?
2. What action, if any, do you take?

Figure 4. The pizza question

Figure 5 shows the response from a year 9 teacher with both high CK and high PCK (quadrant B) to this question. This year 9 teacher's response is conceptually sound because it encourages the student to reason about the relative size of the pieces, using simple part–whole relationships to solve the problem. The teacher steers the student away from an application of rules and towards a real understanding of the size of the pieces involved.

A diagram should be drawn showing $\frac{3}{4}$ of the pizza. $\frac{1}{3}$ of the $\frac{3}{4}$ will be $\frac{1}{4}$.

Of means times is not understanding the deeper meaning of fractions.

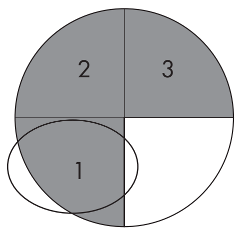

Figure 5. The pizza question, quadrant B teacher response (high CK, high PCK)

In contrast to this were responses from teachers with high CK and low PCK (quadrant D). These teachers could answer the content question, correctly identifying that Sarah took one-quarter of a pizza home, but were unable to direct students towards a meaningful solution. Figure 6 illustrates a response from a year 4–6 teacher with high CK and low PCK. The response indicates that the teacher believes no deeper understanding is required, and it would be sufficient in this situation to ensure that the student can successfully apply the method for multiplication of fractions. Clearly, this approach does not develop the student's conceptual understanding.

No action needed, as long as he knows how to "do" $\frac{1}{3}$ x $\frac{3}{4}$.

Figure 6. The pizza question, quadrant D teacher response (high CK, low PCK)

The teachers in quadrant C demonstrated both low CK and low PCK. Three of these teachers' responses to the pizza question are provided in Figure 7. None of these teachers could provide an accurate answer for the amount of pizza taken home. The descriptions of the actions they would take to help Cameron illustrate the range of difficulties they had with the question. The first teacher, who teaches in a year 7 classroom, converts the fractions to equivalent fractions with common denominators but cannot go further with the problem. The second teacher, from a year 8 class, shows some familiarity with the terminology of the word problem, and the third teacher, who works with students in year 3, inappropriately attempts to apply the "invert and multiply" rule. Despite showing some understanding, these teachers did not know enough to complete the question.

1. Back to common denominators.
   $\frac{4}{12}$ of $\frac{9}{12}$

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2. Cameron must realise that "of means times" can only be applied when we know how many pieces the pizza has been cut into.

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3. Use his statement to work out his answer.
   $\frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$ Equivalent fractions $\frac{2}{3}$

Figure 7. The pizza question, quadrant C, three teachers' responses (low CK, low PCK)

The final three teachers were in quadrant A, with low CK scores but reasonable PCK scores. The responses of these teachers were highly fragmented, with some questions having both CK and PCK components answered correctly and other questions either not attempted or incorrectly answered. These three teachers have been combined with the teachers in quadrants C and D to make up group one, the lowest scoring teachers.

Demographic information from the teachers in the two groups was compared. Figure 8 shows the percentage of teachers at each year level in each group. In general, teachers in group one tend to teach at younger year levels, with 50% of teachers in group one teaching students in year 3 or younger, compared with 21% of teachers in group two. Eighteen percent of the teachers in group two teach in year 9 classrooms, while none of the teachers in group one teach at this level.

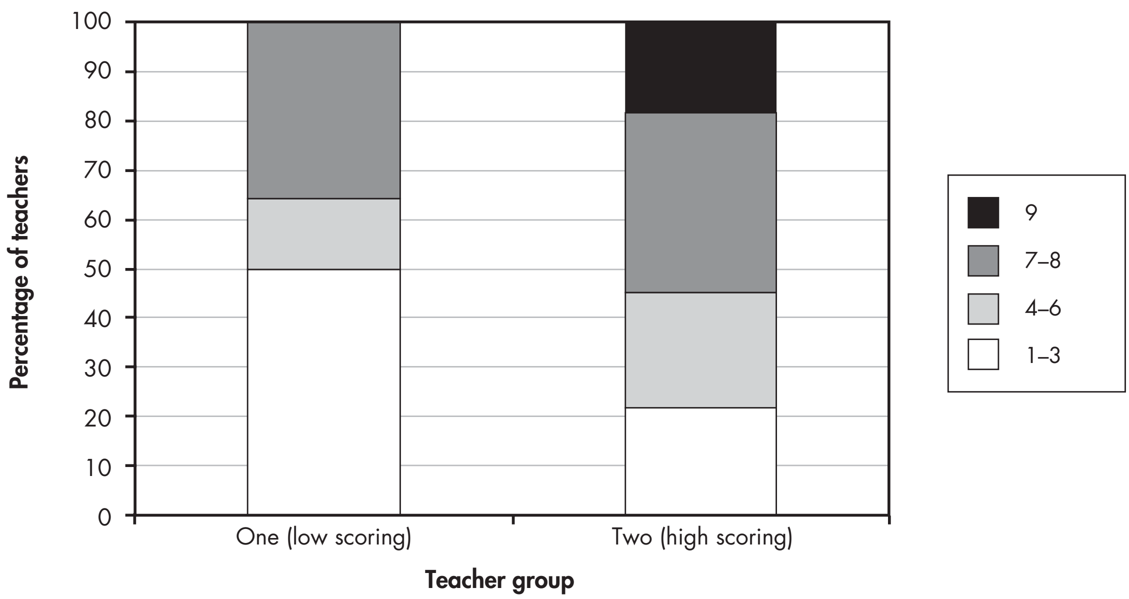

Figure 8. Year levels taught, teacher groups one and two

Figure 9 shows the number of years of teaching experience for the group one and group two teachers. The teachers in group one tend to be less experienced. Just over 60% of group one teachers have fewer than six years' experience in the classroom, compared with just over 40% of group two teachers.

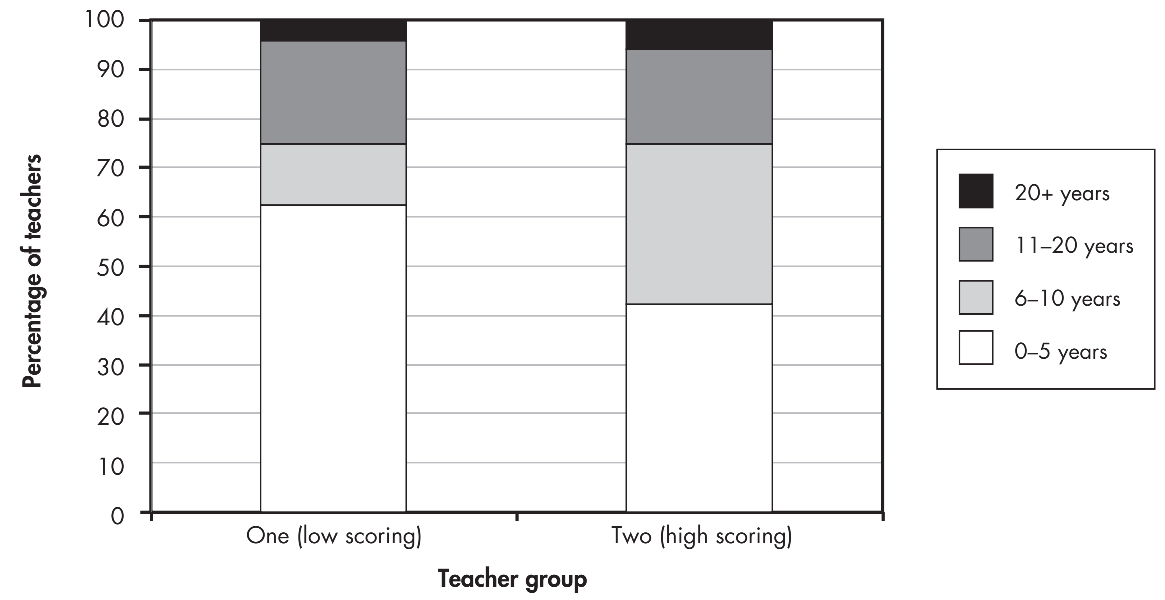

Figure 9. Years' teaching experience, teacher groups one and two

Figure 10 shows the number of stages gained by participating students for the group one and group two teachers. Gains are displayed from each starting stage because student gains across the stages are not linear and therefore mean gain does not provide a valid measure of progress for students at different stages (Irwin, 2003; Irwin & Niederer, 2002; Ward & Thomas, 2002). A generated GloSS score was used to measure gain, as described in the analysis section of this paper. Note that the numbers in brackets give the numbers of students at each starting stage for each teacher group.

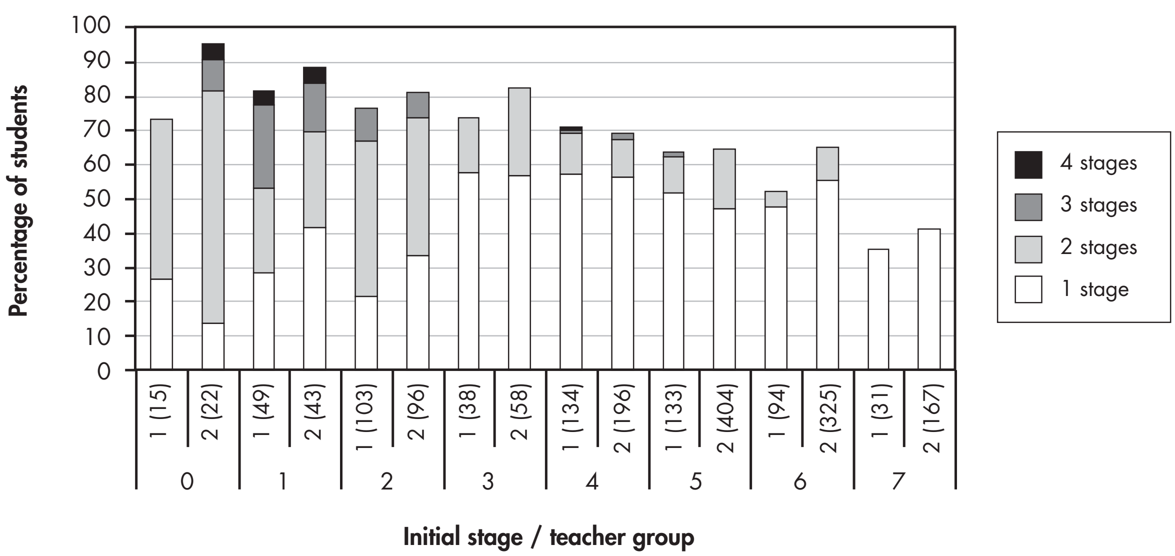

Figure 10. GloSS gains from starting stage, teacher groups one and two

Significant differences in student achievement ($p < 0.01$) between the two teacher groups were found for students starting at stages 0 and 6. Seventy-four percent of the students who started the year at stage 0 with a group one teacher gained at least one stage over the course of the year, with 47% gaining two stages. In contrast, 96% of the students that started at stage 0 with a group two teacher gained at least one stage, with 82% making gains of two or more stages.

Fifty-two percent of the students that were initially rated at stage 6 with a group one teacher made gains during the year, whereas 65% of students starting at this stage with a group two teacher made progress. Of the students initially rated at stage 6, 4% of those with a group one teacher gained two stages, compared with 10% of those with a group two teacher.

The lack of a linear scale for student progress across the Number Framework (Ward & Thomas, 2002) results in difficulties comparing student performance across teachers. A reliable measure of mean student gain would enable those teachers whose students are making the least progress to be identified as a cohort, and this would make it possible to identify patterns of CK and PCK scores among this group.

Concluding Comment

This study highlights several issues of significance within the wider context of the NDP. The relationship between teacher CK and PCK scores found here is significant. It cannot be assumed that teachers who understand the content of the mathematics and statistics learning area of the new curriculum also know effective methods to teach this content. This finding points to teachers' differing needs and highlights the importance of targeted professional development for teachers.

The links between teacher CK or PCK and student achievement are inconclusive at this stage. Further and more detailed analysis is continuing.

It is widely recognised that multiplicative and proportional reasoning are challenging topics to teach and learn (Lamon, 2007). In the context of the NDP, it is well established that both students and teachers have difficulties in this area (Ward & Thomas, 2007; Young-Loveridge, 2006, 2007; Young-Loveridge et al., 2007). This more recent study provides some evidence of the impact of teacher knowledge on student achievement. In addition, it has been suggested that proportional reasoning is the area of primary school mathematics that is "the most essential to success in higher mathematics and science" (Lamon, 2007, p. 629) and that it is also an area of mathematics that is valuable for an adult in everyday life. Without proportional reasoning, it is difficult to compare prices, understand a mortgage, or calculate loan repayments. This is one of the most challenging and important areas of the mathematics and statistics learning area, so it is clear that emphasis needs to remain on improving the teaching and learning of fractions, proportions, and ratios in the NDP.

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