Mathematics in the New Zealand Curriculum
Second Tier

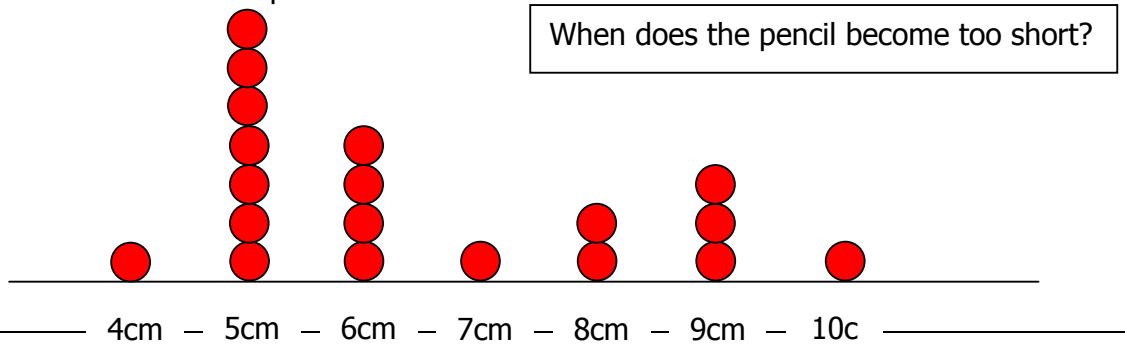
Strand: Statistical Thinking Thread: Statistical Investigations Level: Two

Achievement Objectives: Conduct investigations using the statistical enquiry cycle by:
- Posing and answering questions;
- Gathering, sorting and displaying category and whole number data;
- Communicating findings based on the data.

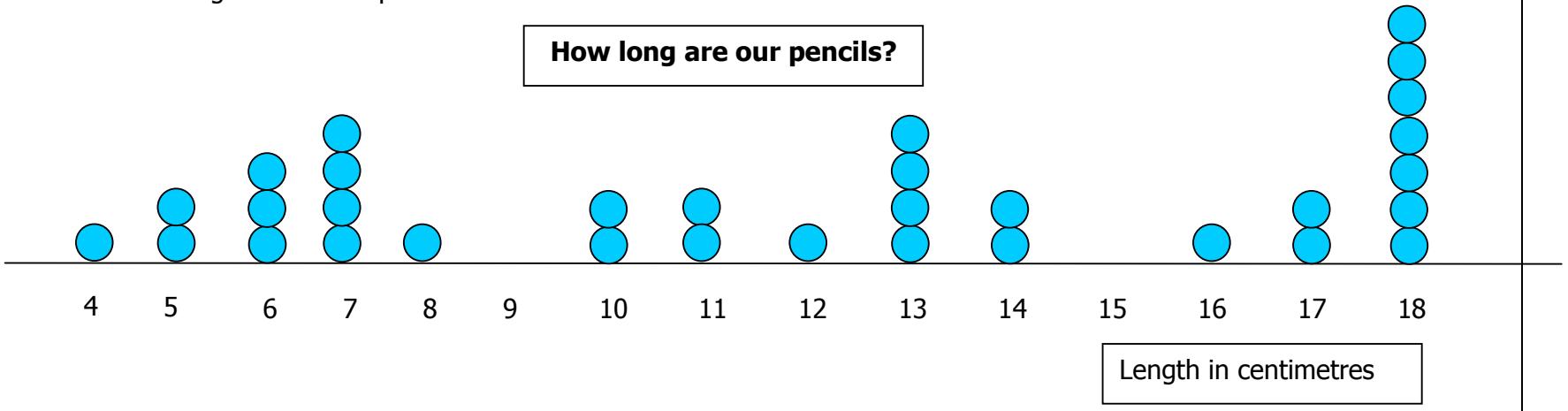
Exemplars of student performance:

Exemplar One: Painful Pencils

By mid-year a teacher is concerned about the length of the pencils his/her students are working with.
He/She asks, "How many times can you sharpen the pencils in this class before they will be too short to use?"
In groups the students decide what information they will need to answer the question, including:
How much pencil is used between each sharpening?
How long is a pencil that is no use for writing?
How long are the pencils in our class?
They gather data about how much pencil is lost between sharpenings (sharp pencil to sharp pencil), and find that it is about 3 millimetres. They also test out each person in the class with pencils of various lengths to find the length at which a pencil becomes too short. Each student places a sticky dot plot above the matching number of centimeters to form a dot plot.



Students observe that most people believe that a pencil that is either five or six centimetres is too short.
Following a discussion about sensible rounding, all the pencils in the class are measured to the nearest centimetre. The lengths are recorded using another dot plot:



Students note that 18 centimetres is the most common length and conclude that this is the length of a new pencil. They also note that 6 pencils are in the "too short" range of six centimetres or less.
With support from the teacher they find that the difference between 18 centimetres and 6 centimetres is 12 centimetres and this is 120 millimetres. Using repeated addition with place value materials they establish that a pencil can last about 40 sharpenings (10 sharpenings is 30 mm, 20 is 60 mm, etc.)
The students ask "I wonder" questions to investigate like:
"I wonder if room 5 takes better care of their pencils than us?"
"I wonder if the machine sharpener takes more or less off a pencil than a little hand-held sharpener?"
"I wonder if a pencil lasts longer than a pen?"

This exemplifies Level Two because the students are gathering and displaying discrete numeric data to answer their questions.

Exemplar Two: Animal Antics

A class is planning a trip to the zoo. They receive information about the animals at the zoo such as, the name of the animal, the location of their cages, feeding times, type of food, the continent they come from, their natural lifespan, and how many animals are in each enclosure.

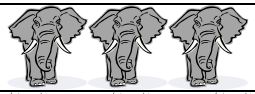
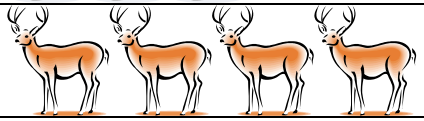
Students pose questions that might be answered from this data, such as:
How often are different animals fed?
What kinds of animal are fed … meat? …plants? …both meat and plants?
What animals are kept in each enclosure? Why?
What continents do the monkeys, cats, birds, and insects come from?
Which kind of animal lives the longest?

Some students investigate the question about what animals eat. They gather data on each animal during feeding time in a table like this:

Animal name	**Eating plants, meat, or both**
Zebra	Plant
Tiger	Meat
Parrot	Plant
Snake	Meat
etc.	etc.

The students organise their data in a two-way table using pictures from the zoo catalogue. Each picture shows a specific animal they observe:

Animal	Plants	Both Plants and Meat	Meat
Tiger			
Parrot			

Zebra			
Pig			
Snake			
Elephant			
Deer			

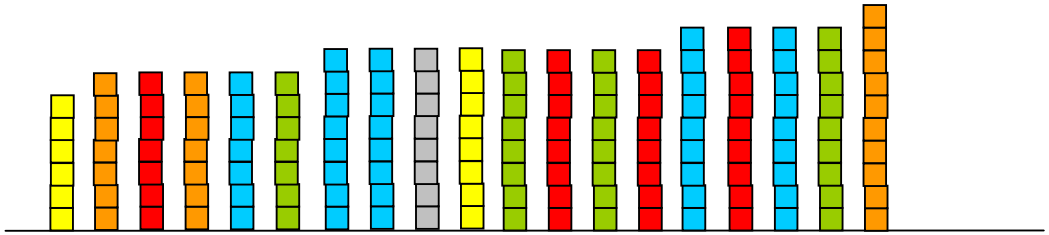
From the table the students make statements like:
Seventeen animals ate plants and only five animals ate meat. That is, there were 12 more plant eaters than meat eaters. Only the three pigs ate both meat and plants.

The teacher extends the students' connection to the context of animals with questions like:
Is it true that there are more plant eaters than meat eaters in the whole zoo?
What is the same about… meat eating animals?...plant eating animals?
(Carnivores have sharp, piercing teeth, and forward looking eyes. Herbivores have pulling and grinding teeth and sideways facing eyes)

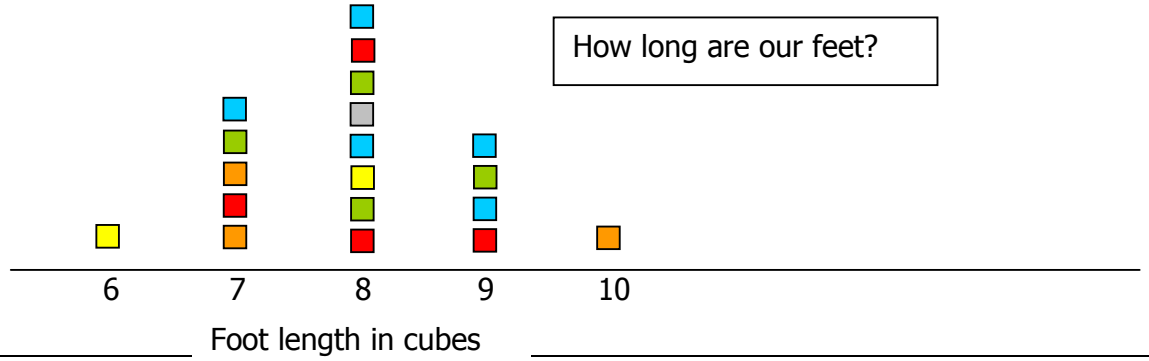
This exemplifies Level Two because the students are posing questions and answering these questions by collecting category data, i.e. Animals are categorized as meat eaters, plant eaters, or both. Students are using tables to organise and display their data.

Exemplar Three: If the Shoe Fits

Motivated by a fairy story of Cinderella, students investigate the question, "When Cinderella was seven or eight how much smaller was her foot than other people her age?" They discuss shoe sizes and decide that foot length is the best way to measure size. Using connecting unifix cubes they create a stack that is closest to the same length as their foot. The students place their stacks along a line in order of length.



The students decide that a small foot for a seven or eight year old is six cubes long. A long foot is four cubes longer at ten cubes long. They then generate further questions like:
What is the most common foot length in our class?
How long would the feet of nine and ten year olds be? (leads to comparing different ages)
How long is a very short or very long seven or eight year olds foot? (leads to comparing to more samples from the same age)
By grouping the stacks and standing them up the students create a dot plot like this:



The students make statements like, "Most feet are 7, 8, and 9 cubes long" and "The middle foot length is 8 cubes."
They pose further "I wonder" questions such as:
"I wonder if girls' feet are smaller than boys' feet?"
"I wonder what the most popular show size is for each age group?" Shoe shops would need to know this.

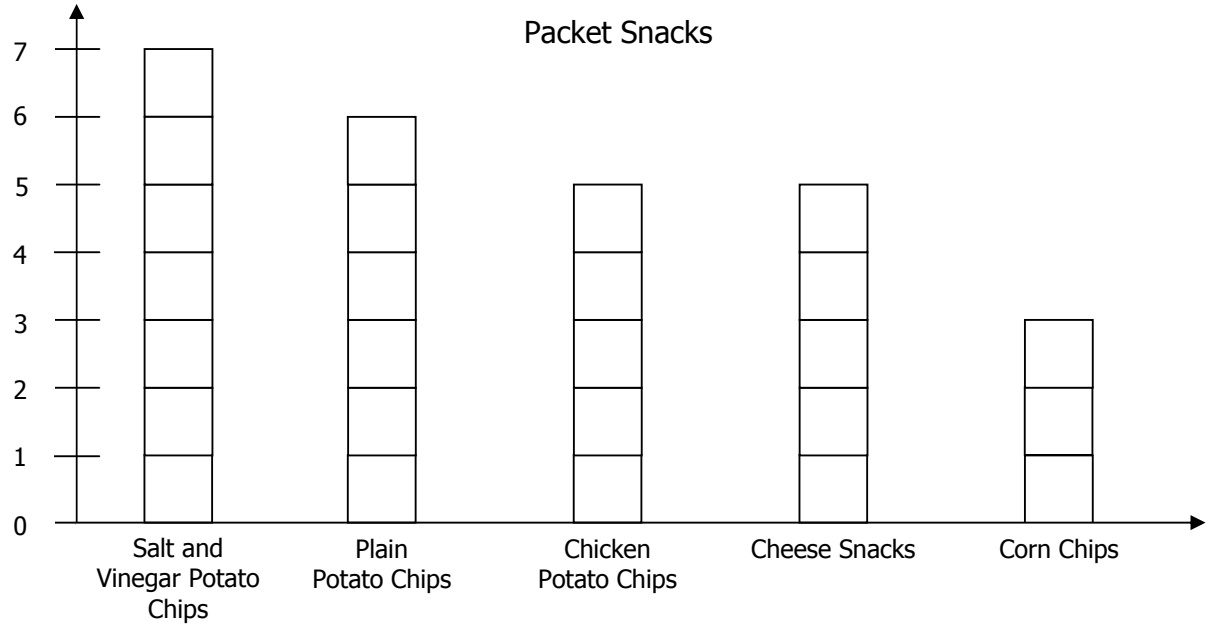
This exemplifies Level Two because the students are answering a question by gathering and displaying discrete numeric data, i.e. Foot lengths in whole numbers of cubes. They are analyzing the data by displaying it using appropriate graphs, e.g. Individual Bar Graph, Dot Plot.

Exemplar Four: Party Packs

A class is planning a class Christmas party. Each student in the class receives three *post-its*™. On two of the *post-its*™ they draw and write two different types of food they would like and on the other post-it they record their preferred drink. Students process the responses collectively by defining categories (e.g. Potato chips) and sub-categories (e.g. Flavour). The teacher allocates categories to different groups for decisions about what to supply for the party. The allocation should include reference to how many individual responses are in each category. For example:

Type of food:	Number of responses:	Group Leader:	Your group chooses:
Packet Snacks	26	Michael	9 favourites
Hot foods	15	Tabitha	5 favourites
Sweet foods	19	Jazz	6 favourites
Drink	30	Lisa	5 favourite bottles

The students sort the *post-its*™ for their category then display them in a way that supports their decision about what to buy. For example, the packet snack group decides that they should buy two packets of each flavour of potato chips, salt and vinegar, plain, chicken, cheese, and one packet of corn chips. They support this decision with reference to their graph (below) saying that the three potato chip flavours have about the same number of responses, while corn chips are a lot less popular.



This exemplifies Level Two because the students are defining and redefining categories to answer their questions, i.e. Packet snacks are separated into potato chips and corn chips, then into flavours. The students also use a formal data display to answer their questions, i.e. Bar graph.

Important teaching ideas (working at):

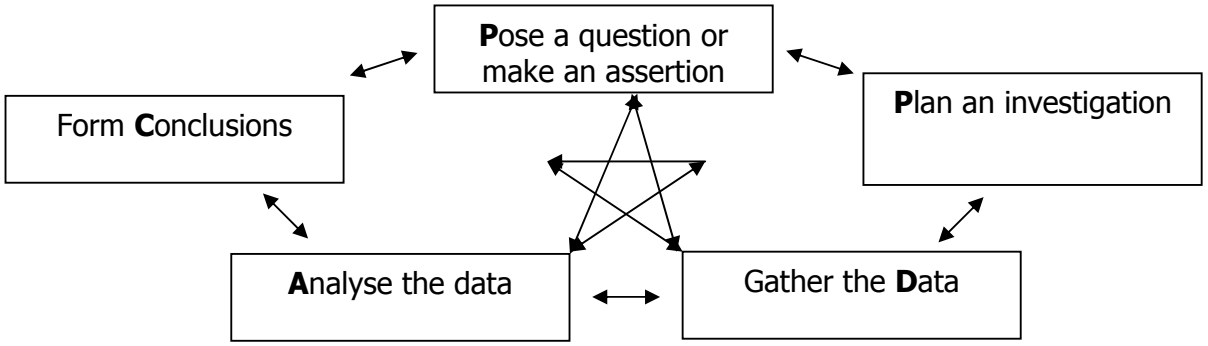
- Investigations at Level Two begin with posing questions that are answered by gathering **category** or **discrete** numeric data.

Category data is organised in distinct groups by classification. For example, pets might be organised into categories such as cats, dogs, birds, etc. Of most interest with category data is frequency, that is the number of occurrences in each category, e.g. 12 cats, 8 dogs, etc.

Discrete numeric data comes as numbers that can only take up particular values. For example, the number of people living in a household can only be a whole number, like 6 people. Measurement at levels 2 and 3 are mostly restricted to rounded whole numbers, e.g. A student is 132cm tall. Data in which the measurements can take up any value within a range, e.g. 15.67 seconds, is continuous rather than discrete, and is dealt with at later levels.

The focus at level two is analysing univariate (one variable) data rather than looking at relationships between variables, e.g. How tall are we? (one variable: height), as opposed to, Do tall people have big feet? (two variables: height and foot length).

- Students at Level Two will need teacher support in applying the Statistical Enquiry Cycle (see below). It is important that students have opportunities to carry out their own investigations to reflect on the processes they used as well as being involved in teacher-lead models of data collection.



- Posing questions and making assertions is the essence of Statistical Investigation. These questions can arise from topical issues, like road safety, literature or film that students are reading/viewing, or other contexts that are engaging, like toys, pets, games, or food. Assertions are statements based on observation validated by collecting, displaying and analysing data, e.g. Dogs are heavier than cats.

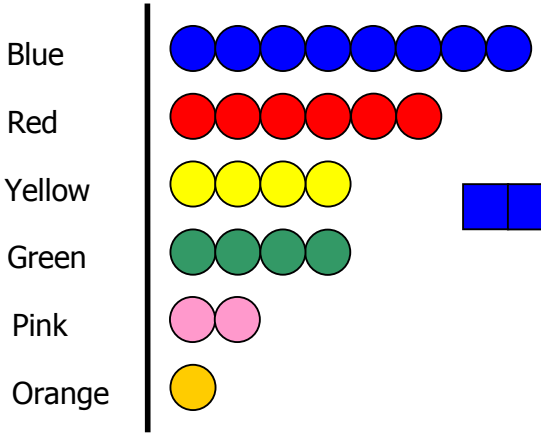
Questions and assertions can lead to gathering data. It can also be that looking at data that is already gathered can provoke investigations (see censusatschool.org.nz).

- Planning involves thinking about how to gather the data. It includes issues such as:
  - Who or what to ask or find out about (the sample)?
  - How many who's or what's to find out about (sample size)?
  - What to find out to answer the question or assertion (measures)?
  - How to collect the data (data collection process)?

It is tempting at the planning stage to anticipate the responses that will be received, e.g. The common pets will be ctas, dogs, birds, and fish, and pre-organise the responses into these categories. While this approach can make the data collection process easy it often masks unexpected responses in the data and deprives students of the opportunity to classify the data into categories.

- Gathering data involves carrying out the plan for data collection and ensuring that data is safely stored. Students need to have systems of data collection modeled, such as using tallying to ensure enough people have been asked or checking off class lists to ensure every student has been asked.
- Analysing the data involves a process of sorting and resorting the data. In this phase of the cycle data displays are used to detect patterns and relationships, not just to communicate the patterns and relationships at the end of an investigation. Many simple mini-tools are freely available on the internet that allow students to import and display data quickly (see for example: http://nlvm.usu.edu/en/nav/category_g_1_t_5.html and http://peabody.vanderbilt.edu/depts/tandl/mted/Minitools/Minitools.html). The exercise of drawing their own graphs is helpful in alerting the students to issues of scale and the conventions of creating titles, axes, and keys. However, use of computer technology plays a critical role in statistics education allowing students to view and analyse multiple displays of the same data rapidly.

At level two students need to be familiar with using and constructing the following data displays:
Category Data Displays



Pictograph

Strip Graph

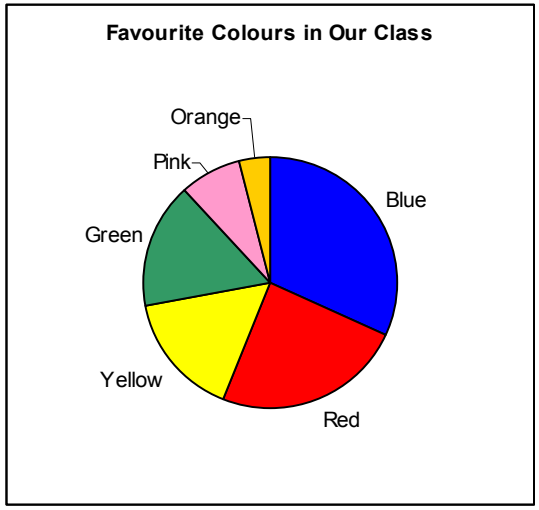
Colour	Number of People
Blue	𝍸 III
Red	𝍸 I
Yellow	IIII
Green	IIII
Pink	II
Orange	I

Tally Chart…

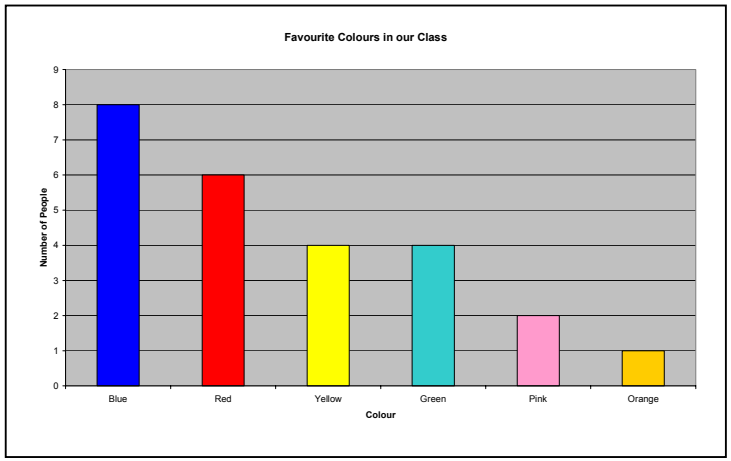
leading to…

Colour	Number of People
Blue	8
Red	6
Yellow	4
Green	4
Pink	2
Orange	1

Frequency Table



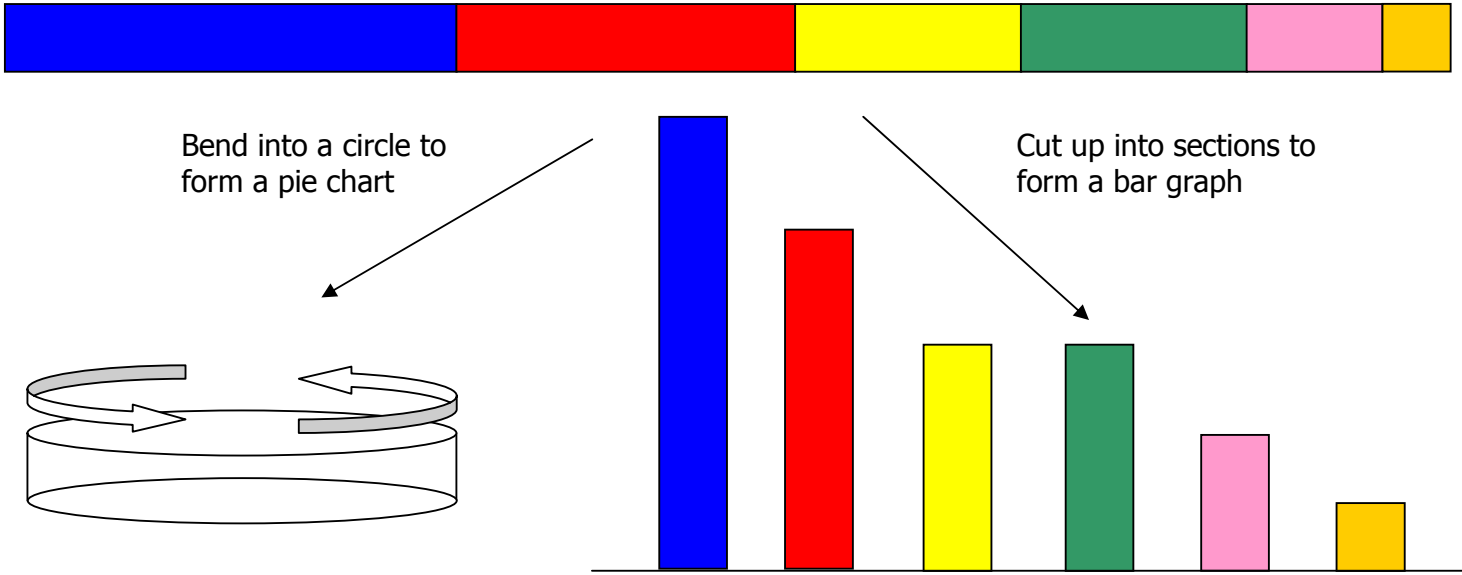
Pie Chart



Bar Graph

Note that in some of these graphs the individual items of data are visible while in others, like the bar graphs and pie charts the items are grouped. This represents a significant challenge for students so the emphasis should be on using the individual data items to build up displays before they are grouped. For example, each student might colour in square of card their favourite colours and these paper squares grouped to form the bars.

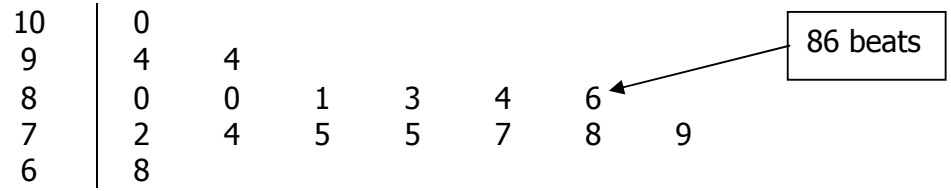
It is also important that students experience how one type of category data graph can be transformed into another.



Student should be encouraged to think critically about why a particular display is used. Pie charts and strip graphs are good for showing how parts in the dataset relate to the whole while pictographs and bar graphs show differences and similarities in frequency.

Discrete Numeric Data Displays

Dot Plot



Heartbeats in one minute for our class

Stem and leaf graph

Both of these graphs allow display of discrete numeric data as it is collected and give a nice picture of how the data is distributed (clustered and spread).

- In the conclusion part of the Statistical enquiry Cycle the focus is on students making statements about the data. These statements need to connect with the original question or assertion and be validated with reference to features of the data displays. With category data, differences in frequency need to be noticed, e.g. There are two more people who like blue than red. With numeric data, main clusters and outlier need to be noticed, e.g. Most students have four to six people living at home but one person had ten people in their household.

Useful resources

Figure It Out, Level 2-3 Statistics, Pages 1-16

nzmaths.co.nz units:

http://www.nzmaths.co.nz/statistics/Investigations/Party.aspx,
http://www.nzmaths.co.nz/statistics/Investigations/MyFavourite.aspx,
http://www.nzmaths.co.nz/statistics/Investigations/planninginvestigationL2.aspx,
http://www.nzmaths.co.nz/statistics/Investigations/datasquares.aspx,

Website links:

http://illuminations.nctm.org/WebResourceList.aspx?Ref=2&Std=4&Grd=0