Mathematics in the New Zealand Curriculum
Second Tier

Strand: Geometry and Measurement Thread: Measurement Level: Two

Achievement Objectives:

- Create and use appropriate units and devices to measure length, area, volume and capacity, weight (mass), turn (angle), temperature, and time.
- Partition and/or combine measures and communicate them, using number and units.

Important Teaching Ideas

Developing as a Measurer

The key idea of measurement at **Level One** is **comparison** within a chosen measurement attribute such as length, weight etc. Comparing, and therefore ordering, is the first step in a person becoming a mature measurer. Hence the language of measurement at this stage is 'longer', 'shorter', 'smaller', 'bigger', 'heavier', 'lighter', 'hotter', 'colder', etc.

Perception of the attributes of measurement (length, weight, time etc.) is fundamental and in fact precedes, or is developed in conjunction with, the comparative concepts. For example the **concept** of distance must be developed before any attempt to measure distance is made. Appropriate use of language such as 'shorter' and 'longer', 'further and 'closer' will develop and strengthen the concept of distance. The definitions of the measurement attributes of length, area, volume, capacity, weight, angle, temperature and time are given in the glossary. The metric system (SI) is also well covered in the glossary.

An important distinction is between **'direct comparison'** and **'indirect comparison'**. As an example, we could compare the lengths of two objects by placing them next to each other and thus observing which is the longer. That would be an example of direct comparison. Alternatively, we could take a piece of string the length of one object and place it against the other object and so compare their lengths that way. That would be an example of indirect comparison. It is important that children experience both forms of comparison at Level One. Indirect comparison is an important step towards the use of non-standard and standard measurement units and the use of measurement tools like rulers and jugs.

Students at level one should generalise the principles of counting to measurement. Counting describes the total number of objects in a set so measuring describes the total number of measures in a space. For example, a student measuring the width of a room using footsteps needs to understand that the count, 1,2,3,.., is describing the number of steps to that point not naming each step.

The key idea of measurement at **Level Two** is the use **of units** and 'devices' to measure length, area, volume etc. These units may be standard or non-standard. A non-standard unit is a unit of measurement that has been chosen by a group of people because it is convenient to use. Non-standard usually means that the unit is not universally accepted. Historically non-standard units have usually been decreed by or taken from body parts of an important person like a tohunga. A standard unit is one within the SI (metric) system, e.g. litre, metre, that are universally accepted if not always used, e.g. USA still uses pounds and miles.

So for length, examples of non-standard measures would be paces, hand spans or pencil lengths; for area, exercise books or sheets of newspaper. Non-standard units should be selected from the student's environment and experiences. The use of non-standard units and whole numbers of standard units introduces the student to the concept of measurement units without requiring the rigour of understanding the metric system, particularly the multiplication and division required, e.g. 1000 metres is 1 kilometre. Further it gives the student the opportunity to develop skills of estimation which are required for operating effectively in the adult world.

In measuring the length of, for example, a desk, students can choose an appropriate unit and see how many of those units can be fitted into the length of the desk. Length is a continuous measure but in order to measure the length of a desk we need to partition the length into a sequence of the chosen units, counting how many of those smaller subdivisions will fit into the length of the desk. We may need to partition the unit itself and consider halves and quarters to finish the task. What we call the length of the desk is the combination of those units and part units. Measurement is a critical context for the connection of whole (units) to parts (fractions). Once again, it is the completing of such tasks and the appropriate discussion that is associated that will lead the students to such understandings.

Students at **Level Two** should be able to apply the addition and subtraction, simple multiplication and division understandings they have from number to measurement problems involving whole numbers of units, e.g. 6 metres. Most children will have encountered some basic metric units, such as metres, kilometres, litres and kilograms, without realising how they relate to one another. For example, they should be able to recognise that a 20 centimetre length could be cut into two ten centimetre lengths, or a nine centimetre and an eleven centimetre length, etc. They should realise that if two litres of water weighs two kilograms then ten litres should weigh ten kilograms.

At **Level Two** students should be asked to create their own measurement instruments. For example, they might be given a strip of paper and cubes to measure the length of many object thus promoting the creation of a ruler. They may be asked to develop a time measurement device having been shown examples of sandtimers, candle and water clocks, or asked to find the weight of other children given a see-saw.

Effective use of non-standard units will lead students naturally to an understanding of the need for standard units. This can be seen through the effective selection of activities that expose the two main reasons for having standard units. Firstly, there is the difficulty that arises through the use of non-standard units such as hand spans or pencils. That is, that there can be many different hand span or pencil lengths and so the measure of the length of an object will vary according to whose hand span or pencil is being used. Such an understanding can be developed through the use of comparison activities. For example, having the students measure the length of something using their pencils and then comparing results and discussing the reason for the variation in results.

Secondly, if we wish to communicate the result of a measurement to a person in another classroom, city, or country we will all need to be using exactly the same units. This understanding can be assisted by using 'fixed non-standard units' such as the length of a piece of A4 paper as opposed to 'variable non-standard units' such as pencil lengths. Through appropriate examples students will see the need for a single standard system such as the metric system.

Level Three measurement sees a strong focus on **standard units** and specifically the **metric system (SI)**. Appropriate measurement experiences at **Level Two** will have prepared students for the necessity of having standard units so that people all over the world can communicate measurement values and understandings to each other.

Students should be immersed in measurement experiences that are rich in investigation and the use of scales and instruments. As well as gaining a good feel for the size of the metric units, students need to know the names of the units, understand the prefixes and know the symbols for each unit. Teaching students to use scaled instruments such as rulers and protractors effectively, and to read graduated containers is important at this level.

Note that some of the standard units in use today are not actually SI units but can be used in conjunction with SI units. For example, the second is the standard unit of time but the minute and the hour are not actually part of the metric system. However as a society we accept kilometres per hour as a standard unit.

Level Three also sees the start of the use of relationships between length and area, or length and volume, for figures such as rectangles and cuboids. Cuboids are portions of space bounded by rectangles. For example, nearly all packaging boxes are cuboids. So students find equations for the area of a rectangle in terms of the lengths of its sides in whole numbers of units, and the volume of a cuboid in terms of the lengths of its edges in whole numbers of cubes.

Level Four sees a consolidation of understanding and use of the metric system and the further use of scales and measuring devices. Students need to be able use their multiplicative thinking and emerging understanding of decimals to convert between metric units such as grams and kilograms, millimetres and centimetres, most of which they should have met at Level Three. They reinforce their use of formulas to find the areas of rectangles and the volumes of cuboids and extend that to figures such as parallelograms and triangles.

At **Level Five** students should have achieved a level of maturity as measurers that enables them, when given a practical measurement problem, to develop and use a method of solution and discuss the degree of accuracy of the result. Students should apply their understanding of decimals to converting between measures of the same attribute, e.g. 1.276t = 1276kg (t means tonne, 1000kg).

They should be capable of finding ways of measuring the perimeters (circumferences) and areas of circles and using their measurement knowledge to determine the areas of figures that are a composition of known shapes.

In doing so students should connect their understanding of classes of geometric shapes to measurement. For example, a cylinder can be seen as an example of a prism, a solid with constant cross section. The volume of all prisms is found by multiplying the area of its cross section by its height hence the formula $v = \pi r^2 h$.

Teaching Measurement at Level Two:

Providing a rich learning experience for measurement at Level Two requires involving students in many measuring activities that progress their understanding from the use of non-standard measures to simple standard (metric) units (a diversity of metric units is required at **Level Three)**. In **Level Two** they will develop the understanding that measurement requires the selection of a unit which has the same attribute as the object being measured. For example, a piece of string, which has the measurement attribute of length, could be used to measure length but it would be of no use in measuring area or capacity. The following characteristics of units need to be developed:

1. A unit is a part to the attribute being measured, e.g. length is measured with a part of length
2. Units are identical, e.g. All handspans are the same length
3. Units tile, i.e. fit together with no gaps or overlaps (this is also true of units of weight, and time in a figurative sense)
4. Units can be partitioned into parts of units, e.g. halves and quarters of cups are allowed
5. Measures can be joined and separated, e.g. 6 handspans and 8 handspans combine to make a length of 14 handspans.

Area provides an excellent context for Level Two students to enhance their understanding of units of measure. The square is chosen as the measure of area because the units tessellate with not gaps or overlaps. Note that all of the square units are the same size. This means that the measure is consistent. Fundamental to this is that the squares are arranged in rows and columns. The array structure is not easily grasped by students and connects strongly to their potential understanding of multiplication and division. A unit is always a part of the attribute being measured. This can cause confusion. Many students believe that “curvy shapes” should be filled with “curvy units” such as counters. This idea is correct but it will lead to inconsistent measurements since the gaps will vary as the space is filled. Use of the unit square as the unit of measure for area should be developed by comparing the number of different shaped units that can be fitted in a given area, and realising that the gaps and overlaps give different results.

Level Two is an opportunity for students to make or select from their own environment, objects that have the required measurement attribute and that are a suitable size. By a suitable size we mean that they are small enough that measurements can be reasonably accurately made within whole number values (and possibly simple fractions) and large enough that the number of units required for measuring is not impractically large. Through experience and practice students will develop an ability to select appropriate units and to say why they are appropriate.

Students at Level Two also need experience with creating their own measurement devices before being exposed to more structured instruments. Creating their own tools to measure length, volume, weight, and time can greatly enhance their understanding. Investigating how instruments such as thermometers and scales work helps students to recognise that measurement is a technological creation to make life easier. Fundamental to all devices is some scale that is created by additively joining units, e.g. the marks on a ruler are at the endpoints of the units. Scales always have an arbitrary zero, i.e. a place to start counting units from.

Exemplars of Student Performance:

Exemplar One: Students measure the capacity of a variety of rectangular boxes (cuboids) using non-standard units:

Students are given a variety (in shape and size) of rectangular boxes and appropriately sized rectangular blocks such as cubes. Initially they estimate the number of cubes that will fit into each box and record their estimations. They then fit the blocks into the boxes. They compare the actual capacity of the boxes with their estimations. They verbalise their findings with statements such as: "This box holds 30 blocks." Students at Level Two should use their additive knowledge and strategies to find the total number of cubes, e.g. Make a tower of cubes and iterate (repeat) it to find the volume. "My tower is five cubes high so the total number of cubes is 5 + 5 = 10, 3 lots of ten is 30.

Exemplar Two: Students measure the area of a region using a variety of non-standard units:

Students discuss how they could find out which of three regions drawn on the concrete with chalk is biggest. Students discuss the kind of unit that could be used to measure the areas of the regions. This discussion gives the teacher the opportunity to bring out the fact that the unit of measure must have the same attribute as the article being measured. Students then look for something that could be used as a unit of measure. They return to the discussion with their unit and say why it is an appropriate unit of measurement to measure the areas of the drawn regions. The students then measure the regions with their chosen unit of measurement and report to the discussion group the number of times they needed to place the unit to cover the region. This activity not only helps to develop ideas of area and its measurement, but also the need for standard units.
Note: A possible follow-up for this activity would be to use different sizes of paper, given that an A3 sheet is twice the area of an A4 sheet and an A5 sheet is half the area of an A4 sheet, students could measure with A3 sheets and halves and quarters.

Exemplar Three: Students measure the weight of a variety of objects using non-standard units:

Students are given a selection of objects to weigh and a pan balance to weigh them on. They are given a unit of weight such as large steel bolts, stones of the same size, small bottles of water or fishing sinkers. They will need several of the units so that they can weigh a variety of objects. They feel the weight of the units and estimate the weight of the objects in terms of the chosen units. They use the balance to find how many units each object weighs.
Students could write the weight on each object its weight (in units) and weigh heavier objects by using a combination of previously weighed objects and units. This activity would emphasize the combining of weights to give other weights.

Exemplar Four: Students learn to estimate area:

Students are given a collection of boxes. They are also given newspaper, scissors and tape. In groups they discuss the amount of paper that will be needed to wrap each box. They cut out the required amount of paper and compare with other groups paper. The objective is to fully cover each box with the minimum amount of paper. The group that has the smallest amount of paper attempts to wrap the box. Students will need to overlap the pieces of paper, and possibly cut and shift them, to decide which piece is smallest.

Exemplar Five: Students compare and order boxes by capacity.
Students are given five or six small boxes of fairly similar size and asked to put them in the order of smallest to biggest according to how much they think they will hold. The students are then given blocks of an appropriate size (such as cubes) and pack each box with the blocks in the most efficient way possible. They use additive strategies to count the blocks as they remove them. From this they discuss which box has the greatest capacity and compare their findings with their initial estimations. The students are operating at Level Two because they are combining units to find a measure of the capacity of each box. This activity could then be extended by having the students choose a different unit of measurement to find the capacity of each box. This will help develop the important understanding that more smaller units and less larger units fit into the same space.

Exemplar Six: Students measure the length of a table using pencils.
Students are given a collection of pencils of varying lengths. In groups they lay the pencils along the chosen table and count how many pencil lengths long the long side of the table is. It is quite likely that different groups will get different answers depending on the pencils that they choose. That should generate a discussion as to why they did not all obtain the same answer. Such a discussion prepares students for the standard units of Level Three. Students then use the information from pencils of known size to estimate how many of a different pencil will measure the length of the table.

Exemplar Seven: Students explore length and turn by pacing outdoors.
Students are given an outdoors trail with measurements in paces. Before starting the activity, the teacher needs to mark out a 30 metre length along a line on a surface such as a tennis court. Students walk along the line counting their paces. Most students at age 7 and 8 years will take about 60 paces to cover the 30 metres.
The teacher gives the students instructions for a planned trail. Instructions would include statements such as:
Start at the red cone. Face the school office. Walk 20 metres. Do a quarter turn to the left. Walk 40 metres. Find the next clue.
Terms could include: quarter turn, half turn, three quarter turn, left, right. Distances should be given in numbers of metres easily related to thirty, e.g. 15, 10, 60. This will help to develop students concept of rate, e.g. 60 steps is 30 metres so 20 steps is 10 metres.

Exemplar Eight: Students explore time by making a sand timer and comparing it with minutes.
Students are given cardboard and pegs or clips and given the task of making a sand-timer. They could be shown a manufactured sand-timer so that they can see how it works. They also discuss the minute as a unit of time. An analogue clock with a large second hand needs to be visible to the students. The students also have a container of sand and something such as a tray or box for the sand to pour into. Students are given the task of making a two-minute timer or whatever the teacher considers appropriate for the students. They experiment with the shape of the timer and adjust it until it is pouring close to the required time interval. This could be made into a competition to see who can make a timer that is closest to the set time.

Useful resources

Figure It Out:
Measurement Level 2-3: Length (pp. 1,3, 24), Area (pp.2,4), Volume (pp.6,8-9,24), Mass (pp.7), Time (pp.18-23)

Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 3-15.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/measurement/Length/gingerbread.aspx
http://www.nzmaths.co.nz/measurement/Length/MakingBenchmarks.aspx
http://www.nzmaths.co.nz/measurement/Length/PiratePlays.aspx
http://www.nzmaths.co.nz/measurement/Length/MathTrail.aspx
http://www.nzmaths.co.nz/measurement/Length/AllAboutMe.aspx
http://www.nzmaths.co.nz/measurement/Length/paperplanesl2.aspx
http://www.nzmaths.co.nz/measurement/Length/sacavengerhunt.aspx
http://www.nzmaths.co.nz/measurement/Area/OutliningArea.aspx
http://www.nzmaths.co.nz/measurement/Volume/howmuchcereal.aspx
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http://www.nzmaths.co.nz/measurement/Mass/weighingstations.aspx
http://www.nzmaths.co.nz/measurement/Mass/GreatGrams.aspx
http://www.nzmaths.co.nz/measurement/Mass/benchmarks.aspx
http://www.nzmaths.co.nz/Geometry/Shape/Clockwise.aspx
http://www.nzmaths.co.nz/measurement/Time/JustAMinute.aspx
http://www.nzmaths.co.nz/measurement/Time/HowLongDoesItTake.aspx
http://www.nzmaths.co.nz/measurement/Time/calibratingclocks.aspx

Digital Learning Objects (These are accessed through the Ministry of Education Digi-Store and are the result of a collaborative project run by The Learning Federation, Australia)
http://www.tki.org.nz/r/digistore/protected/objects/?id=3548&vers=1.0

Other Website links:
http://nlvm.usu.edu/en/nav/category_g_1_t_2.html