Mathematics in the New Zealand Curriculum
Second Tier

Strand: Geometry and Measurement Thread: Measurement Level: One

Achievement Objectives:

- Order and compare objects or events by length, area, volume and capacity, weight (mass), turn (angle), temperature, and time by direct comparison and/or counting whole numbers of units.

Important Teaching Ideas

Developing as a Measurer

The key idea of measurement at **Level One** is **comparison** within a chosen measurement attribute such as length, weight etc. Comparing, and therefore ordering, is the first step in a person becoming a mature measurer. Hence the language of measurement at this stage is 'longer', 'shorter', 'smaller', 'bigger', 'heavier', 'lighter', 'hotter', 'colder', etc.

Perception of the attributes of measurement (length, weight, time etc.) is fundamental and in fact precedes, or is developed in conjunction with, the comparative concepts. For example the **concept** of distance must be developed before any attempt to measure distance is made. Appropriate use of language such as 'shorter' and 'longer', 'further and 'closer' will develop and strengthen the concept of distance. The definitions of the measurement attributes of length, area, volume, capacity, weight, angle, temperature and time are given in the glossary. The metric system (SI) is also well covered in the glossary.

An important distinction is between **'direct comparison'** and **'indirect comparison'**. As an example, we could compare the lengths of two objects by placing them next to each other and thus observing which is the longer. That would be an example of direct comparison. Alternatively, we could take a piece of string the length of one object and place it against the other object and so compare their lengths that way. That would be an example of indirect comparison. It is important that children experience both forms of comparison at Level One. Indirect comparison is an important step towards the use of non-standard and standard measurement units and the use of measurement tools like rulers and jugs.

Students at level one should generalise the principles of counting to measurement. Counting describes the total number of objects in a set so measuring describes the total number of measures in a space. For example, a student measuring the width of a room using footsteps needs to understand that the count, 1,2,3,.., is describing the number of steps to that point not naming each step.

The key idea of measurement at **Level Two** is the use **of units** and 'devices' to measure length, area, volume etc. These units may be standard or non-standard. A non-standard unit is a unit of measurement that has been chosen by a group of people because it is convenient to use. Non-standard usually means that the unit is not universally accepted. Historically non-standard units have usually been decreed by or taken from body parts of an important person like a tohunga. A standard unit is one within the SI (metric) system, e.g. litre, metre, that are universally accepted if not always used, e.g. USA still uses pounds and miles.

So for length, examples of non-standard measures would be paces, hand spans or pencil lengths; for area, exercise books or sheets of newspaper. Non-standard units should be selected from the student's environment and experiences. The use of non-standard units and whole numbers of standard units introduces the student to the concept of measurement units without requiring the rigour of understanding the metric system, particularly the multiplication and division required, e.g. 1000 metres is 1 kilometre. Further it gives the student the opportunity to develop skills of estimation which are required for operating effectively in the adult world.

In measuring the length of, for example, a desk, students can choose an appropriate unit and see how many of those units can be fitted into the length of the desk. Length is a continuous measure but in order to measure the length of a desk we need to partition the length into a sequence of the chosen units, counting how many of those smaller subdivisions will fit into the length of the desk. We may need to partition the unit itself and consider halves and quarters to finish the task. What we call the length of the desk is the combination of those units and part units. Measurement is a critical context for the connection of whole (units) to parts (fractions). Once again, it is the completing of such tasks and the appropriate discussion that is associated that will lead the students to such understandings.

Students at **Level Two** should be able to apply the addition and subtraction, simple multiplication and division understandings they have from number to measurement problems involving whole numbers of units, e.g. 6 metres. Most children will have encountered some basic metric units, such as metres, kilometres, litres and kilograms, without realising how they relate to one another. For example, they should be able to recognise that a 20 centimetre length could be cut into two ten centimetre lengths, or a nine centimetre and an eleven centimetre length, etc. They should realise that if two litres of water weighs two kilograms then ten litres should weigh ten kilograms.

At **Level Two** students should be asked to create their own measurement instruments. For example, they might be given a strip of paper and cubes to measure the length of many object thus promoting the creation of a ruler. They may be asked to develop a time measurement device having been shown examples of sandtimers, candle and water clocks, or asked to find the weight of other children given a see-saw.

Effective use of non-standard units will lead students naturally to an understanding of the need for standard units. This can be seen through the effective selection of activities that expose the two main reasons for having standard units. Firstly, there is the difficulty that arises through the use of non-standard units such as hand spans or pencils. That is, that there can be many different hand span or pencil lengths and so the measure of the length of an object will vary according to whose hand span or pencil is being used. Such an understanding can be developed through the use of comparison activities. For example, having the students measure the length of something using their pencils and then comparing results and discussing the reason for the variation in results.

Secondly, if we wish to communicate the result of a measurement to a person in another classroom, city, or country we will all need to be using exactly the same units. This understanding can be assisted by using 'fixed non-standard units' such as the length of a piece of A4 paper as opposed to 'variable non-standard units' such as pencil lengths. Through appropriate examples students will see the need for a single standard system such as the metric system.

Level Three measurement sees a strong focus on **standard units** and specifically the **metric system (SI)**. Appropriate measurement experiences at **Level Two** will have prepared students for the necessity of having standard units so that people all over the world can communicate measurement values and understandings to each other.

Students should be immersed in measurement experiences that are rich in investigation and the use of scales and instruments. As well as gaining a good feel for the size of the metric units, students need to know the names of the units, understand the prefixes and know the symbols for each unit. Teaching students to use scaled instruments such as rulers and protractors effectively, and to read graduated containers is important at this level.

Note that some of the standard units in use today are not actually SI units but can be used in conjunction with SI units. For example, the second is the standard unit of time but the minute and the hour are not actually part of the metric system. However as a society we accept kilometres per hour as a standard unit.

Level Three also sees the start of the use of relationships between length and area, or length and volume, for figures such as rectangles and cuboids. Cuboids are portions of space bounded by rectangles. For example, nearly all packaging boxes are cuboids. So students find equations for the area of a rectangle in terms of the lengths of its sides in whole numbers of units, and the volume of a cuboid in terms of the lengths of its edges in whole numbers of cubes.

Level Four sees a consolidation of understanding and use of the metric system and the further use of scales and measuring devices. Students need to be able use their multiplicative thinking and emerging understanding of decimals to convert between metric units such as grams and kilograms, millimetres and centimetres, most of which they should have met at Level Three. They reinforce their use of formulas to find the areas of rectangles and the volumes of cuboids and extend that to figures such as parallelograms and triangles.

At **Level Five** students should have achieved a level of maturity as measurers that enables them, when given a practical measurement problem, to develop and use a method of solution and discuss the degree of accuracy of the result. Students should apply their understanding of decimals to converting between measures of the same attribute, e.g. 1.276t = 1276kg (t means tonne, 1000kg).

They should be capable of finding ways of measuring the perimeters (circumferences) and areas of circles and using their measurement knowledge to determine the areas of figures that are a composition of known shapes.

In doing so students should connect their understanding of classes of geometric shapes to measurement. For example, a cylinder can be seen as an example of a prism, a solid with constant cross section. The volume of all prisms is found by multiplying the area of its cross section by its height hence the formula $v = \pi r^2 h$.

Teaching Measurement at Level One:

Providing a rich learning experience for measurement at Level One involves students engaging in a variety of comparison activities, such as determining which object is **longer**, or the longest (or shortest). These activities should range across the eight measurement attributes of the curriculum (see below). This can be done by direct observation or indirect comparison. Students could throw a ball and compare the distance that they throw it with either another person's throw or a previous throw of their own. Measurements could be set out on the ground with students able to count the number of steps that their throw passed. Decisions as to who threw the longer (or longest) could be made by observation or by number comparison.

Students could place ice-cubes in different places and see which ice-cubes take the greatest **time** to melt. The time taken could be measured to the nearest five minutes using an analogue clock.

Area is about how much 'stuff' is required to cover a given "flat" region. Students could look at a variety of rectangular shapes and estimate pair-wise which has the greater area and will therefore require the greater amount of covering material. They would then check their estimations by covering the regions using newspaper or some other appropriate material. The copies could be cut up and overlaid to determine which rectangle is largest. Student could estimate and then check to see how many exercise books it would take to cover a desk.

Recognising the cardinality (How many?) of numbers of **non-standard units** such as pencils, shoes, foot lengths etc is largely the focus for students at Level One. Objects, such as rods, blocks etc., should be used as measurement units with the aim of developing key ideas about how units work:

1. Units must be a part of the attribute being measured, e.g. lengths are measured with a part of length
2. Units must be the same size
3. Units must fit together with no gaps or overlaps

The development of the concept of **weight** can be aided through activities such as comparing the weights of containers holding different masses of articles. Students could make their own judgements and confirm these through the use of a pan balance or see-saw.

Volume and **capacity** (interior volume) can be explored by using cardboard boxes of varying sizes and students can estimate the relative capacity of the boxes and confirm their estimations by using sand, blocks, or small boxes to fill the boxes. The capacity of plastic containers could be checked using water.

Angle (turn) concepts could be developed by following trails outside the classroom. The concept of turning to the left or to the right by, initially, a quarter-turn, and later a half-turn can be deployed in following a trail with clues to find along the way and a destination to arrive at.

Temperature concepts can be established by daily consideration of how the students feel. "Is it a hot day today?" "Is it hotter than yesterday?" "Was it colder last night?" etc. Cups of water with different temperatures (max. 40◦C) could be ordered from hottest to coldest using the "finger dip" test.

Exemplars of Level One Student Performance:

Exemplar One: Students compare and order their heights by length.

Each student is given a piece of material whose length is the same as their height. The material could be strips of ribbon, cardboard, string or whatever is suitable and available. The students compare their height with the height of another student by comparing the length of their strips. Appropriate instructions would be: “Find someone who is shorter than you”. “Find someone who is the same height as you.” “Find someone who is taller than you.” These could be confirmed by direct comparison.

An extension of the concept of indirect measurement is the involvement of the transitivity relation, namely that if the length of A is greater than B and the length of B is greater than C then the length of A is greater than C. Hence if Mary’s strip is longer than Paul’s and Paul’s strip is longer than Jane’s then we know that Mary’s strip is longer than Jane’s even if we have not directly compared Mary’s and Jane’s strips. Discussion and activity involving this concept is important as it is an essential part of maturing as a measurer. After making a judgement based on the transitivity relation students should confirm their judgement by direct comparison.

Note: Variations on this are:

(i) To use an object that each student can bring to class such as a drink bottle and make the comparisons with those.

(ii) Instead of having the students have a piece of material they can lie on a large sheet of paper and have a cut-out of their silhouette. They can then compare the cut-outs. As an extra, if two cut-outs are made they can be stapled or taped together after the measurement lesson and stuffed with paper to form a person.

Exemplar Two: Students compare and order shapes by area.

(a) Students directly compare the areas of items of the same shape but different sizes. These could be made out of cardboard, or a variety of materials. Circles and squares would be good shapes to start with as they are easily recognised. Students could sequence a set of circles (or squares) labelled by colours (or some other attribute that does not have an established sequence) and discuss the comparisons within the sequence.

“Which is bigger, Red or Green?” “Which took the most cardboard to make?”

“I’ve got another circle and it is bigger than Green. Do you think it is bigger than Blue? Why.”

“Do you think it is bigger than Orange?” It might be, but look, it’s smaller than Orange! Where will we put it in our order? That’s right. It is in between Blue and Orange.”

(b) Now indirectly compare **different** shapes made of cardboard by cutting out paper that fits exactly over one shape and cutting it up to see how it fits another shape. Rectangles of different shapes (that is, their sides are in different ratios) would be best for that at this level. Again the rectangles could be sequenced and similar questions could be asked.

Exemplar Three: Students compare and order containers by capacity.

Students are supplied with a variety of containers. Initially they handle the containers and say which they think will hold the most, the least etc. “Do you think this one will hold more than this one?” How could we find out which one holds the most?” Led by the teacher the children will be guided towards the idea of filling one container with suitable material (sand, water, etc.) and then pouring it into the container with which the first is being compared. With repetition of this method the students will be able to sequence the containers by capacity.

Exemplar Four: Students develop their understanding of area by comparing areas.

Students are told that it is gift wrapping time. They are given a sequenced variety of about five boxes to wrap, adhesive tape, and wrapping paper that will just cover the box for which is intended. They discuss which piece of paper should cover which box. Because the boxes are not simply enlargements of the one shape, this will require some thinking out. When they are sure that they have matched the paper with the correct box, they attempt to wrap the box with the paper.

Exemplar Five: Students compare and order containers by weight.

Students are given a variety of sealed containers. The containers are of different weights. (Variously shaped drink bottles containing different amounts of water could be used.) The students feel the containers and order them from lightest to heaviest by feel. They then check their ordering by using a pan balance.
They discuss the ordering using appropriate language such as “This one is heavier than this one”; “This one is the heaviest”; “This one is the lightest”.

Exemplar Six: Students compare time by doing tasks.

Students are put into groups and each group is given a task to do in the classroom. They discuss which task they think will take the longest, the shortest, in between etc. They are then sent off to do the tasks as quickly as possible and return when completed. They compare the time taken with the original estimation. The teacher could time the tasks (to the nearest minute) or by counting and share the time taken with the students.

Exemplar Seven: Students compare temperatures and record daily temperatures.

Students are given a collection of containers having water of varying temperatures. Obviously the temperatures would have to be carefully tested before the children touch the containers (max. 50◦C). They discuss the temperatures and order them according to feel. The teacher then measures the temperature of each container using a thermometer and records the result.
The class records the morning, midday, and afternoon temperatures for the next month. Students could be asked to think about how a thermometer works. They may work out that increased temperature expands the alcohol causing it to fill up the tube.

Exemplar Eight: Students explore angle by examining a variety of turning objects.

Students are exposed to a variety of turning things such as taps, on/off switches, thermostatic control gauges etc. Some of these could be in existence in the classroom while others could be collected for the experience. They explore the instruments and discuss their findings in terms of the amount of turn required or possible for each object. Language would include statements such as "This one turns right around a whole turn"; "This one only turns a half turn."; "This one has four places it stops." – (an ideal opportunity to discuss quarter turns and half turns.)

Students could be given a set of instructions involving distances and turns that map out a letter of the alphabet, e.g. Walk 10 steps, turn left, walk three steps, half turn, walk six steps (capital "T").

Useful resources

Numeracy Project Book 9: Teaching Number through Measurement, Geometry, Algebra, and Statistics, pages 3-15.
nzmaths.co.nz units (This website is sponsored by the Ministry of Education)
http://www.nzmaths.co.nz/measurement/Area/wormsandmore.aspx
http://www.nzmaths.co.nz/measurement/Area/GreatCoverUp.aspx
http://www.nzmaths.co.nz/measurement/Area/PrintsAndOutlines.aspx
http://www.nzmaths.co.nz/measurement/Volume/Spoonfuls.aspx
http://www.nzmaths.co.nz/measurement/Volume/Spoonfuls.aspx
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http://www.nzmaths.co.nz/measurement/Volume/TrickyBags.aspx
http://www.nzmaths.co.nz/measurement/Mass/Seesaws.aspx
http://www.nzmaths.co.nz/measurement/Mass/MeasuringBeads.aspx
http://www.nzmaths.co.nz/Geometry/Shape/Turns.aspx
http://www.nzmaths.co.nz/Geometry/Shape/Clockwise.aspx
http://www.nzmaths.co.nz/measurement/Time/PassingTime.aspx
http://www.nzmaths.co.nz/measurement/Temperature/I'mFreezing.aspx

Other Website links:
http://illuminations.nctm.org/Activities.aspx?grade=all
http://nlvm.usu.edu/en/nav/category_g_1_t_4.html