An Evaluation of the Numeracy Project for Years 7–9 2003

Final Report

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Acknowledgments

I am grateful to several people for their contributions to this report. Foremost among these are the facilitators, teachers, and children who participated in this different approach to learning mathematics and who took part in the annual evaluations of the Numeracy Project in the three years that are the subject of this report. Several people also agreed to participate in additional assessments to evaluate the broader effects of the Numeracy Project.

The evaluation of the case study of year 9 students was undertaken by Angelika Anderson, Research Associate. Considerable assistance was given by Jan Wallace, the facilitator concerned, by the mathematics staff of the college where this case study took place, and by the teacher of the class studied.

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Executive Summary

1. The final numeracy stages reached by students evaluated in 2003 are very similar to those of 2001 and 2002 for most topics, when compared by deciles, but superior in multiplicative thinking to the 2002 cohort.

1a. Recommendation: Continue the emphasis on multiplicative thinking for years 7 through 9.

1b. Recommendation: Continue to emphasise improvement in the performance of students from low decile schools.

2. There were few significant differences between full primary and intermediate schools or between schools in the first and second year of the project, when matched by decile. The only significant differences between schools in the first and second year of the project favoured the decile 3 school in the first year of the project. This is likely to relate to other factors within the schools' mathematics programme.

2a. Recommendation: Facilitators need to build on and encourage those other factors that appear to be helping schools to be more successful. These include administrative support and the importance placed on mathematics within the school.

3. A sample of year 7 students did significantly better than the norm sample on asTTle tests of number knowledge and number operations but not on patterns in number.

3a. Recommendation: The Ministry of Education should continue to monitor the effects of the Numeracy Project against other measures of mathematical competence. After the booklet on algebraic thinking has been widely used, patterns in number should be reassessed.

4. Year 8 students in the Numeracy Project did significantly better than a comparison cohort on a test of numerical generalisation in which they used numbers as quasi-variables; this should lead to improved performance in secondary school algebra. This is similar to results found for the 2002 sample.

4a. Recommendation: The Ministry of Education should monitor the effect of this benefit of the Numeracy Project on performance in algebra in secondary schools.

5. The project for year 9 students focused on classes of students with low attainment. The project was successful for these students who most need assistance. Gain was shown by 50%–60% of these students on all the major scales that are appropriate for their age group. This compares favourably with results for all students in other years. A case study of one class showed a high

level of student engagement. The teaching style furthered by the Numeracy Project successfully transferred to other areas of the mathematics curriculum.

5a. Recommendation: The project is valuable for low attaining year 9 students and should be continued.

5b. Recommendation: The model of transferring the teaching style to other areas of the mathematics curriculum fostered by the facilitator of the school that was studied in depth should be emulated in primary schools and in other secondary schools.

6. The understanding of decimals, percentages and fractions was not assessed fully for most students, because some items were only on Form C of the assessment. Where decimals were assessed, all items could be answered using rules, without the necessary understanding. Therefore, it is not possible to know how much students in this age range know about decimals nor how much progress students are making.

6a. Recommendation: All year 7–9 students should be assessed fully on their understanding of fractions and decimals. In the short term the easiest way to do this is to use the 2003 Form C of the assessment for all students in this age range, so teachers know the extent of students' understanding**.**

6b. Recommendation: In the future, an assessment of decimals needs to be separated from place value for the earlier stages, and taught with fractions and percentages.

6c. Recommendation: The project would benefit from an integration of the knowledge scales for fractions and decimals with the strategies for addition, multiplication, and proportional reasoning.

1. Introduction

This is the third year in which the Numeracy Project has been offered to students between years 7 and 10 and independently evaluated (see Irwin and Niederer; 2002; Irwin, 2003). These reports should be read in conjunction with those by Thomas and colleagues (2001, 2002, 2003) and Higgins (2001, 2002, 2003) who have evaluated the project for years 1–3 and years 4–6. In this three-year period the project has moved from being exploratory to being established, as an excellent way in which teachers can understand the needs of their students and can help them move to more advanced stages in mathematical thinking. In that period facilitators have become highly professional in their role of helping teachers and teachers have become more adept in their role of helping students learn.

While earlier reports have covered all students in facilitated schools from years 7–10, the cohort for years 9 and 10 was different in 2003 from that of earlier years. Only low achieving students from years 9 and 10 were involved in 2003. Almost no final returns were received for year 10 students. Therefore this report reviews the success of year 9 students separately from year 7 and 8 students, and does not report on the two year 10 students for whom final results were received by 18 December 2003.

The project was initially developed for younger students, and the assessments as well as some of the supporting teaching documents are more appropriate for younger students. Teachers of students in years 7–9 have adapted materials, with considerable help from their facilitators. However, the assessment forms have not been adapted, and the majority of these older students have not been assessed on areas that are important in their education. The most crucial of these is understanding of decimals.

Only 81% of schools had sent in their final results by the end of the school year, on 18 December 2003. Others could not be included in this evaluation. This is of some concern for evaluators of the project. The end of a year is a busy time for schools and not one in which teachers are strongly interested in redoing a formative assessment.

The Numeracy Project has been outlined in previous reports. Although the project has different names for different age ranges, it is the same continuous project. The Early Numeracy Project (ENP) is for years 1–3, the Advanced Numeracy Project (ANP) is for years 4–6, the Intermediate Numeracy Project (INP) is for years 7 and 8 although years 7 and 8 in full primary schools often consider that they are doing ANP, and the Secondary Numeracy Project (SNP) is the same project in years 9 and 10.

The following summary is repeated from these reports. Note that I have chosen to use titles that describe the attributes of each stage on the scales, rather than the numerals or the additive strategy stage, as used by many who discuss development of younger students on the project. Analyses have shown that few children of this age range are on the same stage in different topics. As problems become more difficult a higher proportion of students revert to lower strategy levels which they are sure of. Similarly, there is a difference in importance of topics for these older students. For example, initially, stage 6 was called “advanced additive part-whole strategies / early multiplicative part-whole strategies”. I use the phrase “advanced additive part-whole” for addition but because of the importance of multiplicative thinking in this age range, for strategies for multiplicative problems, I use the phrase “early multiplicative part-whole” to emphasise that its focus is multiplication.

Assessment forms B and C are presented in Appendix A.

Scales and stages

Scales

The **strategy scales**, dealing with computation, were:

Strategies for addition and subtraction. These operations are called “additive strategies” in the figures in this report, as well as in general writing about this field. In assessing this scale, students were given addition and subtraction problems to do. The teacher noted whether the student: counted all objects to obtain an answer; counted on or counted back from one of the numbers; had a “part-whole” strategy in which they broke up one of the numbers being added or subtracted parts to make the problem easier; or had a range of such part-whole strategies.

Strategies for multiplication and division. Together, these are referred to as “multiplicative strategies”. In assessing this scale, the teacher noted whether the student completed a problem that could have been solved using multiplication by using a counting strategy or by repeated addition (for example, if the student knows that 13 x 7 can be solved by multiplying 10 x 7 and then adding 7 three more times: “77, 84, 91”); derived the answers to unknown multiplication questions from known facts in addition and multiplication (for example, 32 x 7 is the same as 30 x 7 plus 2 x 7); or used a range of part-whole strategies.

Strategies for solving ratio and proportional problems. This scale took students into fractional, ratio, and proportional problems. In this report, it is referred to as “proportional strategies”. These problems required similar skills to those needed to solve multiplicative problems, but at the upper stages they also required at least two multiplicative processes, such as the division and multiplication required to find three quarters of 24. At the lower stages, the student is asked to find a fraction of a whole number, like one quarter of 24. At the more advanced stages, students are required to find the relationship between two numbers and then apply this relationship to a third number (for example, if 16 bags of apples weigh 10 kg, what would be the weight of 24 bags of apples?). This scale appeared on assessment Form B for Stages 1–6 and on Form C for Stages 2–8.

The **knowledge scales** were:

Whole number identification. In assessing this scale, students were asked to identify printed numerals. The numbers ranged from two-digit to six-digit figures. This scale was only on Form A.

Forward number word sequence. This scale is often referred to as "FNWS". In assessing this scale, students were asked to name the number directly following a written numeral. This appeared on Forms B and C.

Backward number word sequence. This scale is often referred to as "BNWS". In assessing this scale, students were asked to name the number directly before a written numeral. This appeared on Forms B and C.

Knowledge of fractions. In assessing this scale, students were required to match fractions to samples from a pie diagram, to read unit fractions less than one (1/2, 1/4, 1/3), to indicate the meaning of a fraction greater than one, and to order fractions with different numerators and denominators. This scale appeared on Form B for Stages 2–6 and on Form C for Stages 2–8.

Knowledge of decimals and percentages. Assessing this scale required students to read, order, and round decimals and translate between decimals and percentages. In 2001, some of these skills were included in the scale for knowledge of fractions. It was separated from fraction knowledge in 2002. It appeared only on Form C, Stages 4–8.

Knowledge of grouping and place-value. In early stages of this scale, students were required to tell how many dots were in groups of five and 10. In Stages 4–6, they were required to name the number of 10s in numbers between two and five digits long and to give the number of 100s in numbers from six to seven digits long. At Stages 7 and 8, students were required to name the tenths and hundredths in numbers that included both whole numbers and decimal fractions. Form A covered Stages 0–4, Form B covered Stages 0–6, and Form C covered Stages 4–8.

Stages

Stages are defined in relation to strategy scales. When used in the lower year levels, they are used in relation to addition stages.

Stage 0. Pre-counting. Students at this level cannot count a small group of objects.

Stage 1. Count from one on materials. Students at this stage can count and form a set of up to 10 objects by counting each one. They cannot solve simple adding problems by joining these sets.

Stage 2. Adding by counting from one with materials. These students can add four counters and two counters by counting all of them.

Stage 3. Counting from one by imagining the objects to be counted. These students use counting but do not need to see objects in order to add.

Stage 4. Advanced counting. Students at this stage solve addition problems by counting on. For example, for 8 + 3 they say "8, 9, 10, 11" to get the answer 11.

Stage 5. Early additive part-whole thinking. At this stage, students recognise that addition problems can be solved efficiently by breaking up numbers into their component parts. For example, students who do not automatically know that 8 and 5 is 13 can see that 5 can be broken up into 2 and 3, and that since 8 + 2 = 10, 3 more make 13.

Stage 6. Advanced additive/early multiplicative thinking. Students at this stage use a variety of ways to break up numbers for doing addition problems and may do multiplication problems by using these part-whole addition strategies. For example, they may mentally work out that 63 – 29 can be worked out by thinking that 63 – 30 = 33, and adding one would be 34.

Stage 7. Advanced multiplicative/early proportional thinking. At this stage, students can use their understanding of multiplication to break up numbers. For example, they may realise that 50 x 124 is the same as 100 x 62, so the answer will be 6200.

Stage 8. Advanced proportional thinking. Students at this stage can use a range of multiplication and division strategies to solve proportion problems. This includes finding a percentage of a whole number. Students who can do this might find 15% of 240 by first finding 10% (24) and then adding half of this (12). When these two percentages are added together, they would get 36 as 15% of 244.

Overview of this report

The sections in this report relate to specific research questions.

Sections 2 and 3 report on the final stages of the year 7 and 8 students who participated in the project in 2003 and in previous years. Because initial assessments were seen as relatively unreliable, as teachers were learning the Number Framework at the start of the year, only final assessments were used.

Section 4 reports on the relative performance of students in intermediate and full primary schools, and on year 8 intermediate school students in the first and second year of the project.

Section 5 reports on a comparison of achievement of year 7 students in the Numeracy Project on an asTTle test with the group used to provide norms for asTTle in a period that preceded the Numeracy Project.

Section 6 reports on a comparison of the ability of students in, and not in, the Numeracy Project to use principles that can simplify mental operations. This is similar to an assessment used in 2002. It demonstrates the superior ability of students in the Numeracy Project to use numbers as quasi-variables in a manner similar to algebraic thinking.

Section 7 reports on the progress made by the low attaining classes in year 9. It includes a case study of one teacher who, with the aid of her facilitator, used the teaching principles of the project for all areas of the mathematics curriculum.

Section 8 focuses on fractions and decimals, and the need for teachers and students in these years to focus on these topics.

2. Results of final assessments for 2004 for year 7 and 8 students

In 2003 the final assessment results were analysed for years 7, 8, and 9. While initial results were entered for 62 year 10 students, only two students had results entered for a final assessment. These results have not been analysed.

Results analysed for year 7 through 9 were those that had been entered in the national database by 18 December 2003. At that time results had been entered for 12 203 year 7 and 8 students and 762 year 9 students. A further 2957 students had not had their final results entered. This amounted to 19% of those for whom initial results were entered.

In 2003 the Numeracy Project for year 9 was focused on the students in most need of catching up in their understanding of numeracy. This was a different population from previous years, and different from the population who took part in the Numeracy Project in years 7 and 8 in 2003. The manner in which the project was used for year 9 students also differed from previous years. Year 9 results and a case study of one example of this specialised project and its results are presented in Section 7 of this report.

Data for years 7 and 8 are presented below. The percentage of students at each stage is given in Appendix B. A comparison of students from full primary schools and intermediate schools within this sample is given in Section 3 of the report.

Characteristics of the year 7 and 8 sample

There were 6390 year 7 students and 5823 year 8 students. The decile rank and ethnic distribution of the 2003 sample is given in Table 2.1 and Table 2.2. In both year groups 49% of the students were female and 51% were male. Throughout this report deciles are grouped, with deciles 1–3 described as low, 4–7 described as medium, and 8–10 described as high.

Table 2.1. Percentage of students from schools of different decile ranking in years 7 and 8

	Year 7	Year 8
Low decile	50%	48%
Medium decile	40%	42%
High decile	10%	11%

Table 2.2. Percentage of students of ethnic group in years 7 and 8

	Year 7	Year 8
European	50%	50%
Māori	30%	29%
Pasifika	12%	13%
Asian	4%	4%
Other	4%	3%

The percentage of year 7 and 8 students in each ethnic category was virtually the same.

Performance on key subtests of the Numeracy Assessment

For students in years 7 and 8, key indicators of numeracy were seen as additive strategies, multiplicative strategies, proportional strategies, knowledge of fractions and knowledge of decimals. Results given are for the second assessment in each case.

The percentage of students who were not given the second assessment is presented first, Table 2.3.

Table 2.3. Percentage of students, for whom other results were returned, who were not given the second subtest in the fields listed

	Additive strategies	Multiplicative strategies	Proportional strategies	Knowledge of fractions	Knowledge of decimals
Year 7	3%	5%	6%	7%	62%
Year 8	4%	5%	5%	7%	52%

In the additive strategies, the students not assessed were largely those who were scored as competent on the initial assessment. Therefore the percentages in Table 2.4 could underestimate the percentage of students at the top stages. For multiplicative strategies, students who were not scored on this scale had a variety of initial scores, and it is hard to predict the reason for not giving the final assessment. For proportional reasoning strategies, nearly half of the students (43%) who were not given the second assessment had not been given the initial assessment in this field, but the others had a variety of initial scores. Similarly, nearly half (43%) of the students who were not given the second fraction assessment were not given the initial assessment but the rest had a variety of initial scores. The large percentage of students not given the knowledge of decimals test is related to the fact that this scale is not on Form B of the assessment, which was given to most students. The percentages given below are of students for whom results were returned. For understanding of decimals and percentages this is a minority of the students in the numeracy project.

Additive strategies: Most students in years 7 and 8 were using part-whole strategies for addition. See Table 2.4.

Table 2.4. Percentage of students in years 7 and 8 using different strategies for addition

Strategies for Solving Additive Problems	Year 7	Year 8
Counting strategies, including counting on	18%	11%
Early additive part-whole	44%	38%
Advanced additive part-whole	39%	51%

From these results it can be seen that 82% of year 7 students and 89% of year 8 students have at least one part-whole skill that they can use in doing addition. This proportion might have been higher if all students had been given the second assessment. This indicates that there should not be a concern about addition strategies for most students in this age range. There should be some concern for the 11% of year 8 students who still use a counting strategy for adding.

Multiplicative strategies: Multiplicative strategy stages are presented in Table 2.5.

Table 2.5. Percentage of students in years 7 and 8 using different strategies for multiplication

Strategies for Solving Multiplicative Problems	Year 7	Year 8
Counting strategies, including counting on	17%	10%
Early additive part-whole	24%	20%
Early multiplicative part-whole	39%	37%
Advanced multiplicative part-whole	20%	32%

Of these students, 59% of year 7 and 69% of year 8 are using multiplicative mental strategies. The need to increase the percentage was emphasised in the report on the 2002 cohort (Irwin, 2003). Results presented in Section 3 of this report, Figures 3.4, 3.5, 3.6, and 3.11 indicate that the proportion has increased for year 7 students in all decile ranges, and for year 8 for middle decile students.

Proportional strategies: Strategies for proportional reasoning are presented in Table 2.6. Note that only students who were given Form C of the assessment had the chance to demonstrate understanding of the top two levels, where proportional reasoning is required.

Table 2.6. Percentage of students in years 7 and 8 using different strategies for proportional reasoning

Strategies	Year 7	Year 8
Counting strategies, including counting on	25%	18%
Early additive part-whole	28%	24%
Early multiplicative part-whole	28%	29%
Early proportional part-whole	16%	20%
Advanced proportional part-whole	4%	9%

These results show that 20% of the year 7 students and 29% of the year 8 students were judged to use mental proportional reasoning, as shown in the bottom two categories. However, only students given form C of the assessment were able to demonstrate achievement at this stage, so we don't know how accurate this assessment is. It is important that intermediate aged students be asked about knowledge at these top stages, so that we can know about the extent of the need for improvement here.

The scales for fractions and decimals are the two areas designated as Knowledge Scales that are particularly relevant for this age range. They are more complex than other knowledge areas. They are also fields which many adults fail to understand fully. The knowledge underlying fractions and decimals, beyond identification, requires the application of multiplicative thinking. Calling these knowledge assessments, in the same category as knowing forward and backward number word sequences, underrates difficulty in understanding fractions and decimals. It can be argued that an understanding of fractions is covered in strategies for proportional reasoning. Understanding of decimals, as opposed to surface knowledge of the numerals, is not well covered in the Numeracy Project in my view.

Knowledge of fractions: Table 2.7 shows the stages of knowledge of fractions displayed by these students.

Table 2.7. Knowledge of fractions demonstrated by students in years 7 and 8 on the second assessment

Knowledge Stage for Fractions	Year 7	Year 8
Cannot order unit fractions (Stages 2 through 4)	27%	19%
Orders unit fractions	37%	32%
Coordinates numerator and denominator	21%	22%
Recognises equivalent fractions	10%	14%
Orders fractions with unlike numerators and denominators	6%	13%

It would be reasonable to assume that only the three last categories reflect understanding of fractions, as ordering of unit fractions can be done by a rule in the absence of understanding. Using this criterion, 37% of the year 7 students assessed and 49% of the year 8 students assessed were judged to have an understanding of fractions.

Knowledge of decimals: The assessment of decimals and percentages as presented in the Numeracy Assessment was given to less than half of the students on the numeracy project. This assessment can largely be achieved by applying rules without underlying understanding. Most of the items relate to surface knowledge of the numerals rather than what these numerals represent. It is not nearly as comprehensive a test of understanding as is the Chelsea Diagnostic test (Hart, Brown, Kerslake, Kuchemann, and Ruddock, 1985). However, it is a starting point for assessing what students understand. Table 2.8 shows the stage that the students assessed were seen to be at by the end of 2003.

Table 2.8. Knowledge of decimals and percentages demonstrated by students in years 7 and 8 on the second assessment

Knowledge Stage for Decimals	Year 7	Year 8
Cannot identify decimals	12%	7%
Identifies decimals	39%	34%
Orders decimals	25%	21%
Rounds decimals	15%	19%
Converts decimals to percentages	10%	19%

Only the more competent students were likely to have been given Form C and, therefore, asked about decimals. If the more competent students in the numeracy project perform at these relatively low levels, the project is not helping students to understand decimals.

Understanding of fractions and decimals is essential for most year 7 and 8 students. Many adults lack these understandings which are unlikely to be taught after year 8. These topics are covered separately in Section 8 of this report.

Conclusions

There are five scales of the numeracy project that are especially important for year 7 and 8 students. Of these, two scales (proportional strategies and fractions) do not include the top stages in Form B. Decimals were not covered at all on Form B in 2003. This means that there is inadequate information on how well the Numeracy Project is meeting the needs of these year 7 and 8 students.

Results available do indicate that most of these students are performing well in additive strategies, although the fact that 11% of year 8 students are still using counting on to add is of concern.

Performance on multiplicative strategies was better in 2003 than in 2002, especially for year 7 students. However, 31% of year 8 students are still not using multiplicative strategies by the end of their primary school years. These students cannot use their multiplication tables flexibly to solve multiplication problems.

Proportional reasoning strategies and fractions are being attended to but the assessment forms treat them as separate topics, although the teaching suggestion booklet includes teaching fractions at an advanced proportional stage. Students would benefit from more work on integrating these topics.

The decimal knowledge of all year 7 and 8 students needs to be assessed. This assessment should be done in a manner that assesses understanding rather than rules that can be memorised. Until a different assessment can be written for this, I would recommend using the assessment in Form C for all year 7 and 8 students.

3. Comparison of final assessment results in 2003 with those of 2002 and 2001

The Numeracy Project has been offered to years 7, 8 and 9 for three years now. As doubt has been cast on the accuracy of initial assessments, given while the teachers were learning to understand the framework, it was decided to compare only the final results to see what change in outcomes there might have been over this three-year period.

Table 3.1 gives the number and characteristics of students involved in the Numeracy Project for years 7 and 8 differed in 2001, 2002, and 2003. In 2001 only three deciles were represented in the schools that participated: deciles 3, 4 and 10. In 2002 and 2003 all 10 deciles were represented but in 2003 there was a markedly higher representation of low decile schools and a corresponding lower representation of high decile schools. Only 15 schools were represented in both the 2002 and 2003 data, which is surprising given that the project is a two-year one. The number of schools, students and decile representation is outlined in Table 3.1. Low deciles are deciles 1, 2, and 3. Middle deciles are decile 4, 5, 6, and 7. High deciles are 8, 9, and 10.

Table 3.1. Number and decile range of year 7 and 8 students assessed in the Numeracy Project in 2001, 2002 and 2003

	Number of schools	Number of students	Low decile	Middle decile	High decile
2001	6	1871	37%	33%	30%
			decile 3 only	decile 4 only	decile 10 only
2002	186	11842	33%	47%	20%
2003	196	12213	49%	41%	10%

The following figures compare the assessment of addition and multiplication strategies, given at the end of the school year, given by decile groups. Differences in achievement for different decile groups are fairly consistent. These are the main two scales for which the items were similar over the three years.

Year 7 students for additive and multiplicative strategies

The fact that very few schools appear in more than one year's data means that most of the students were participating in the Numeracy Project for the first year. Thus the following figures compare similar students, although the percentage from different deciles is different in 2001 from later years. It is not possible to tell how many of the students in 2002 and 2003 may have had the project before reaching year 7.

The main finding of this comparison of final results for 2001, 2002, and 2003 was the similarity of the percentage ending the year at each stage. A main difference was in the percentage reaching the top stages from schools of different deciles.

Strategies used for addition

The percentage of students ending each year at different stages is given in Figures 3.1 to 3.3.

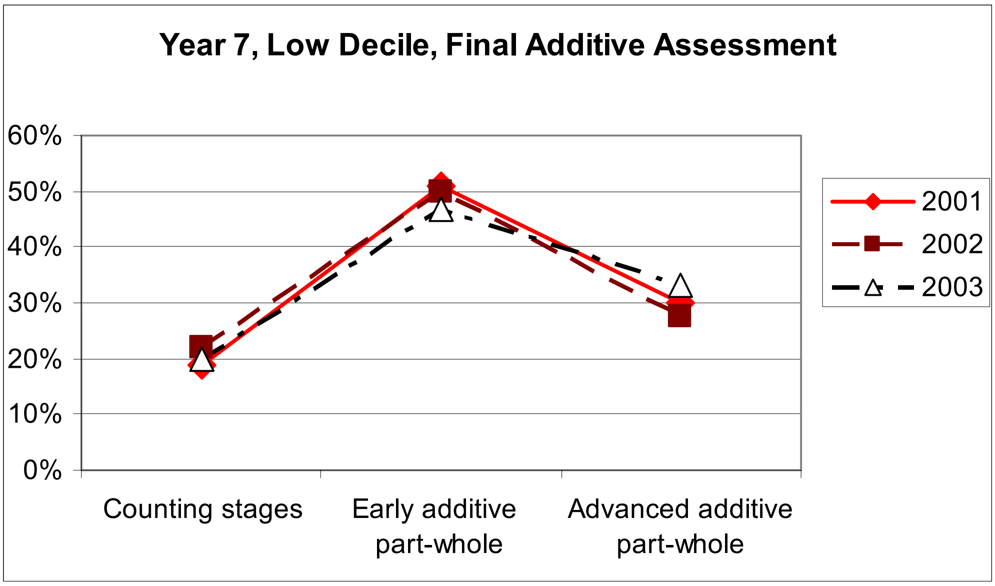

Figure 3.1. Percentage of year 7 students from low decile using each strategy for addition at the end of 2001, 2002 and 2003

This figure shows that a very similar percentage of low decile students from each year finished the year using the named strategies. About half of the students from low decile schools ended each year being able to use at least one part-whole strategy for mental addition. About 20% of each year group continued to use some type of counting strategy for adding. Percentages for all of these figures are given in Appendix B.

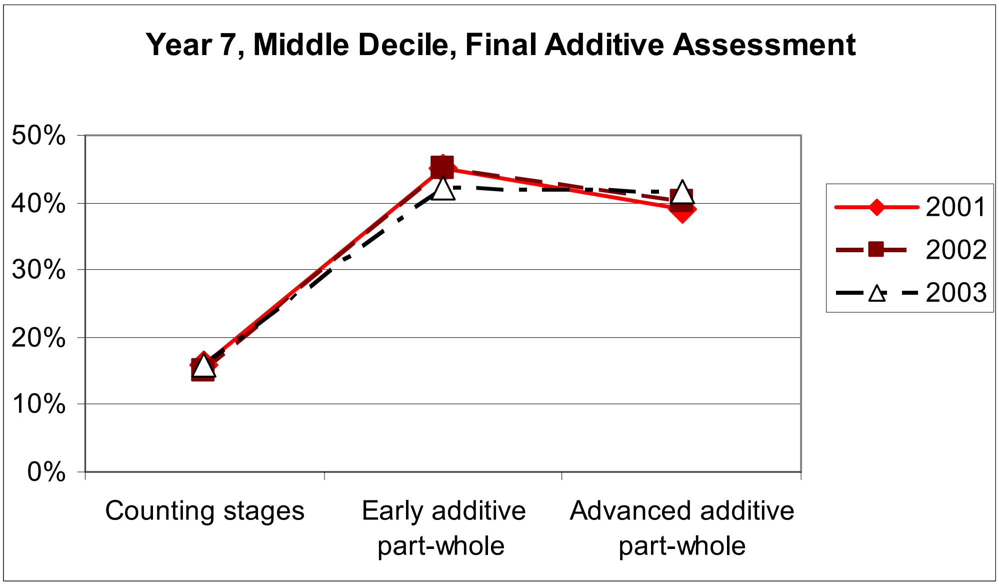

Figure 3.2. Percentage of year 7 students from middle decile schools using each strategy for addition at the end of 2001, 2002, and 2003

As for the low decile schools, these figures show that the percentage finishing each year using each strategy are very similar. The difference from the low decile schools is that a lower percentage continued to use counting strategies (15-16 % versus 19-22%), and the majority ended the year being able to use a variety of additive part-whole strategies.

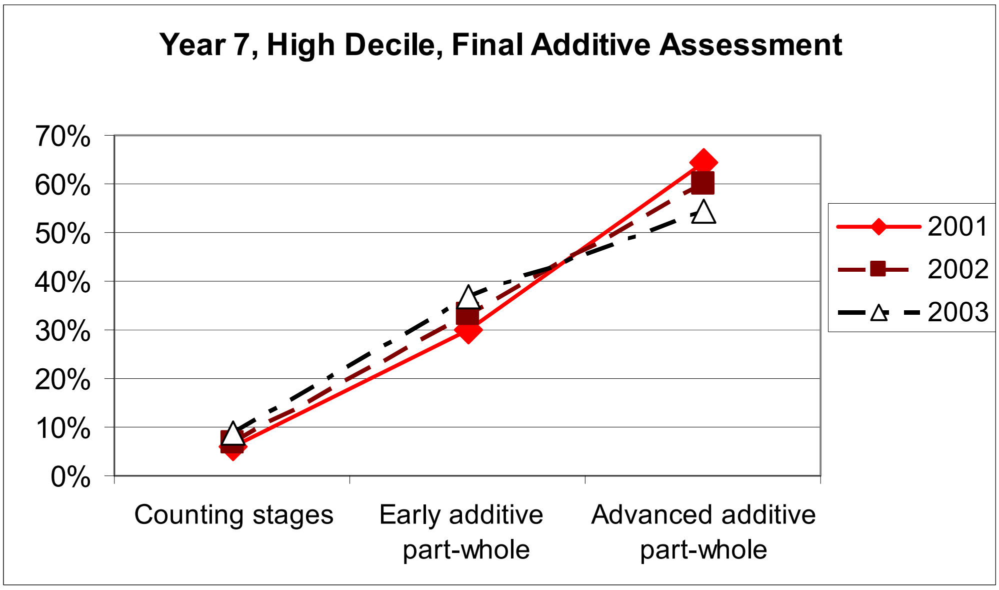

Figure 3.3. Percentage of year 7 students from high decile schools using each strategy for addition at the end of 2001, 2002 and 2003

The students from high decile schools differed from those from low and middle decile schools in the even lower percentage that continue to use counting for addition (6-9%), and the high percentage that use advanced additive part-whole strategies. Thus in each year the proportion of students using a variety of part-whole strategies for addition is higher for upper decile groups.

Strategies used for multiplication

The end-of-year evaluations for multiplicative strategies were less similar over the years than were the results for additive strategies. These are presented in Figures 3.4 through 3.6.

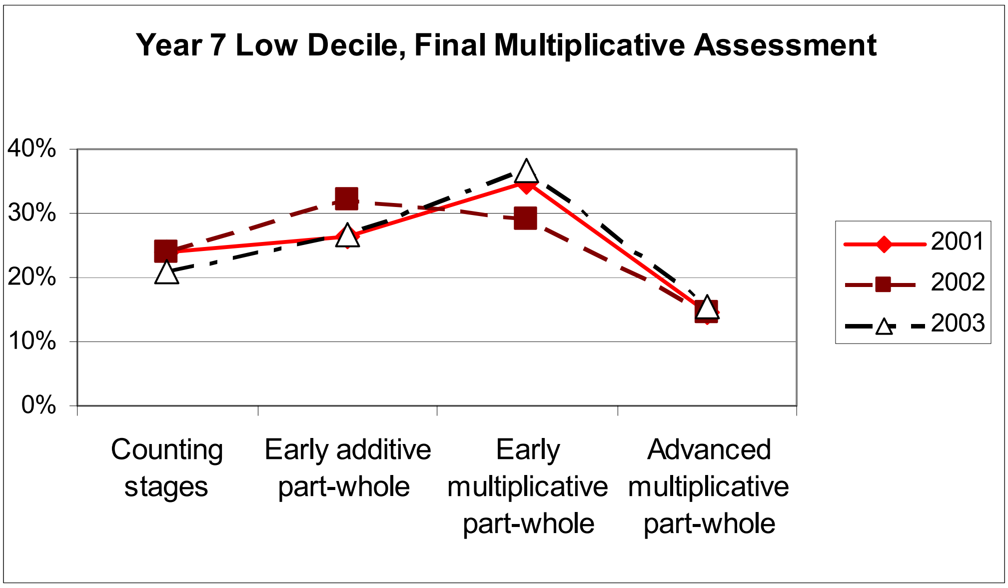

Figure 3.4. Percentage of year 7 students from low decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

Figure 3.4 shows that the percentage of students using each of these strategies for multiplication differed over the three years although the proportions for years 2001 and 2003 were similar. In saying this, note that in 2001 all of these students were from decile 3 schools, while in 2003 they covered all three lower deciles. The final results for 2002, in comparison to other years, show that a lower percentage of students in this decile range came to use multiplicative part-whole strategies, and a higher percentage continued to use additive strategies for multiplication problems. The increase in the proportion of low decile students using multiplicative strategies in 2003 in comparison to 2002 is pleasing. The fact that 21-24% in these three years used various types of counting strategies for multiplication problems is of concern.

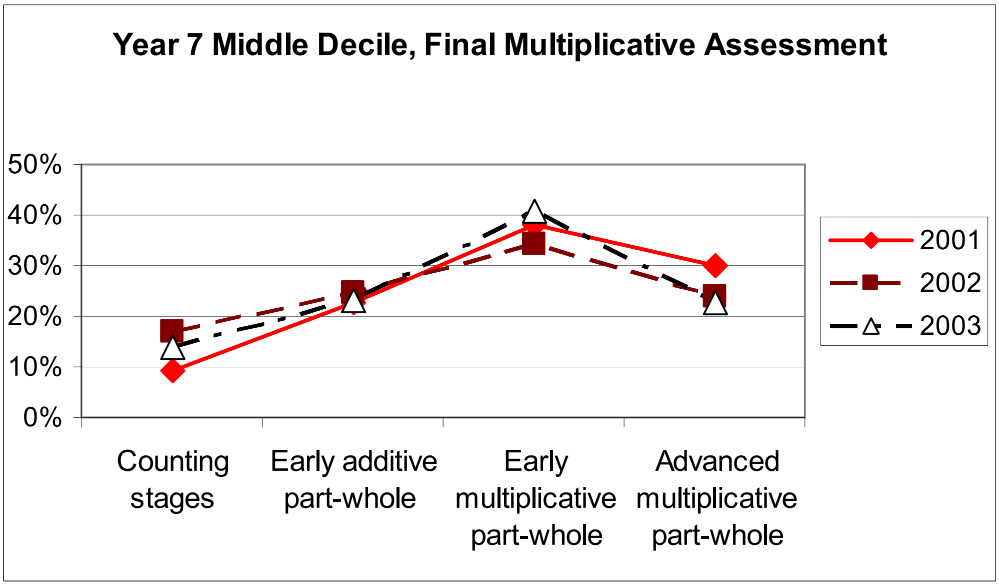

Figure 3.5. Percentage of year 7 students from middle decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

Fewer students from middle decile schools (7-17%) used counting skills for multiplying at the end of these three years. A higher proportion of these students than of the students from low decile schools were judged to be using the advanced multiplicative part-whole strategies.

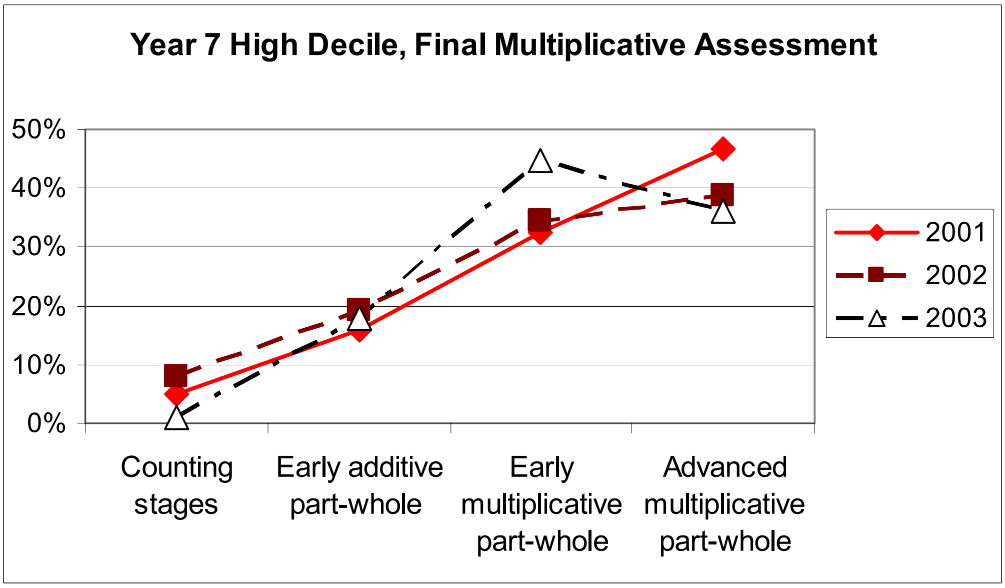

Figure 3.6. Percentage of year 7 students from high decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

This figure for high decile students shows a steady increase in the percentage of students at more advanced multiplicative stages. In noting the high proportion of

2001 students who were judged to be using advanced multiplicative strategies, it should be noted that these students were from one decile 10 school. The larger percentage of 2003 students using at least one multiplicative part-whole strategy, combined with the very low percentage of the 2003 students using a counting strategy for multiplying, is commendable.

Year 8 students for additive and multiplicative strategies

Results for the final assessment of year 8 students for 2001, 2002, and 2003 also showed more similarities than differences. Figures 3.7 through 3.12 show comparisons of additive and multiplicative stages reached in these years by students from low, middle, and high decile schools.

Strategies for additive problems

A higher percentage of year 8 students than year 7 students from every decile bracket reached the part-whole stages for addition.

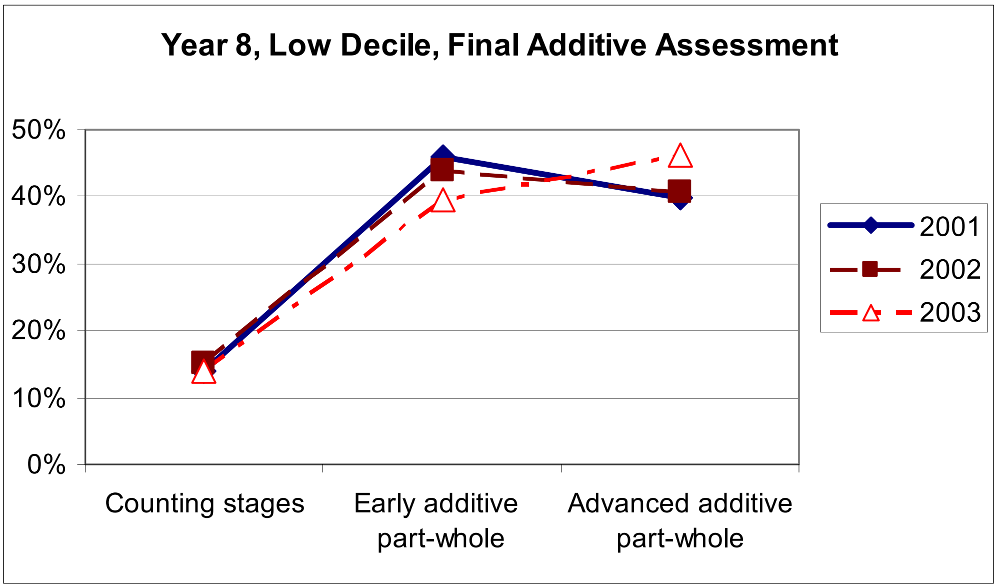

Figure 3.7. Percentage of year 8 students from low decile schools using each strategy for addition at the end of 2001, 2002, and 2003

In 2003, the proportion of year 8 low decile students who came to use a variety of part-whole strategies exceeded the proportions that ended the year using one part-whole strategy for adding. In 2001 and 2002, a higher proportion of students had finished at the early part-whole stage, using only one part-whole stage for additive problems. For these three years, all of the averages for year 8 students from low decile schools showed more students finishing at the advanced additive part-whole stage.

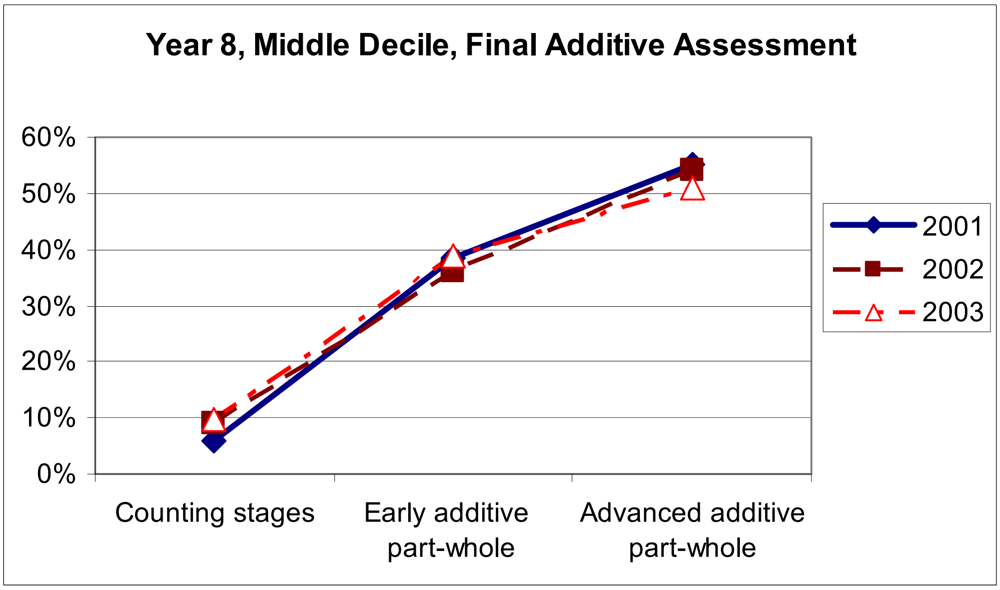

Figure 3.8. Percentage of year 8 students from middle decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

By year 8 the majority of students from middle decile schools were judged to be at the advanced part-whole stage in adding, in that they were using a variety of part-whole strategies.

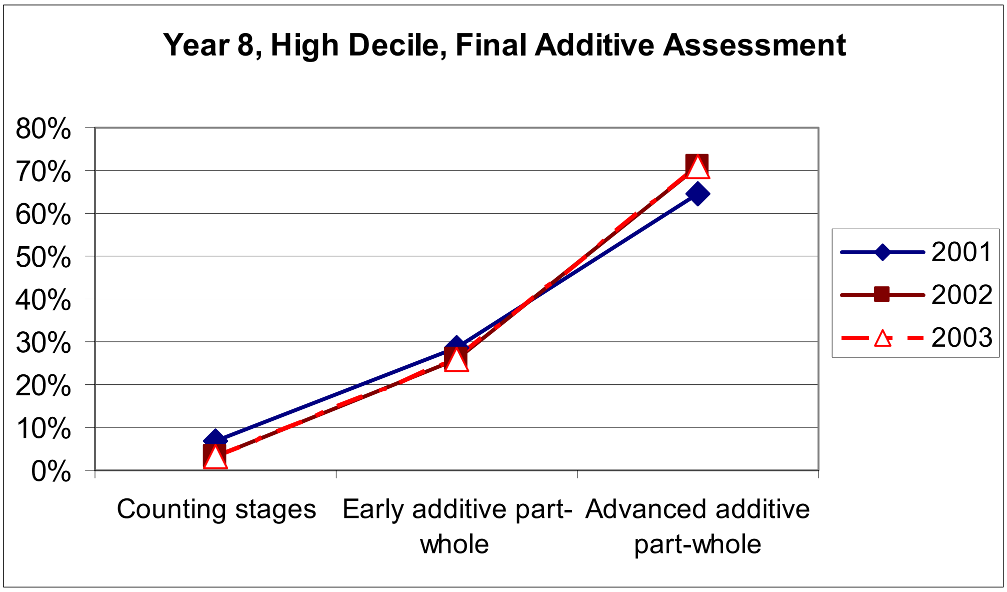

Figure 3.9. Percentage of year 8 students from high decile schools using each strategy for addition at the end of 2001, 2002, and 2003

This figure is similar in slope to that of high decile year 7 students. In 2002 and 2003, 70% of students in the Numeracy Project were judged to be working at the advanced part-whole additive stage, demonstrating a variety of ways in which they could manipulate numbers in order to add them mentally. The percentages for years

2002 and 2003, when rounded, were identical and therefore appear on the figure as a single line on this figure.

Strategies for multiplicative problems

The percentage of year 8 students using multiplicative strategies for multiplication problems was higher for all deciles than for year 7 students. The majority of students in all groups could use either early or advanced multiplicative thinking. See Appendix B.

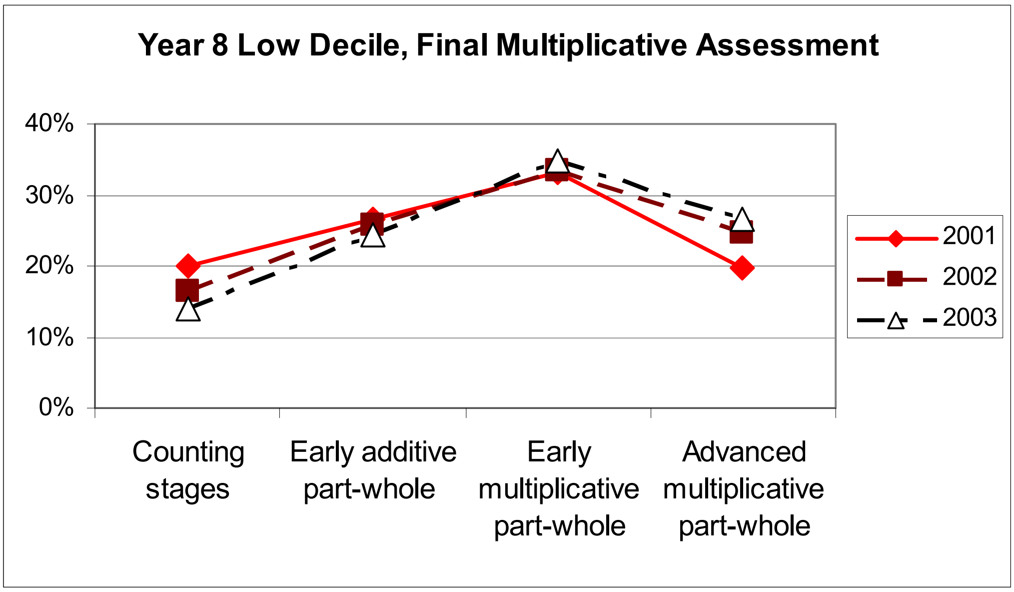

Figure 3.10. Percentage of year 8 students from low decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

Figure 3.10 shows that a higher percentage of students from low decile schools could use advanced multiplicative thinking with each successive year of the project.

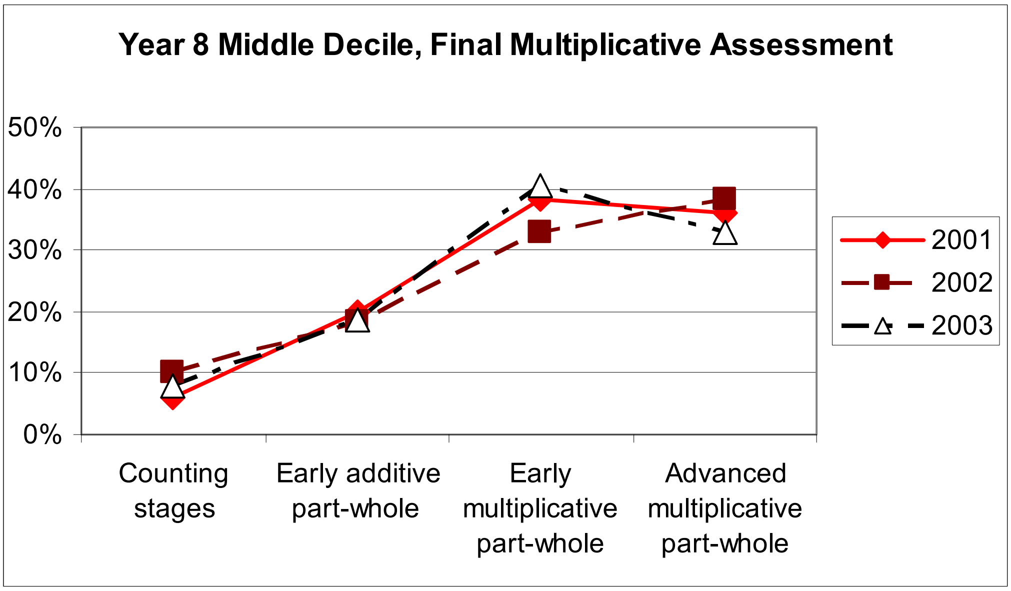

Figure 3.11. Percentage of year 8 students from middle decile schools using each strategy for multiplication at the end of 2001, 2002 and 2003

Figure 3.11 shows that while the percentage of students using early multiplicative thinking in 2002 was lower than that for 2001; that percentage increased in 2003. Fewer students reached the advanced multiplicative stage in 2003.

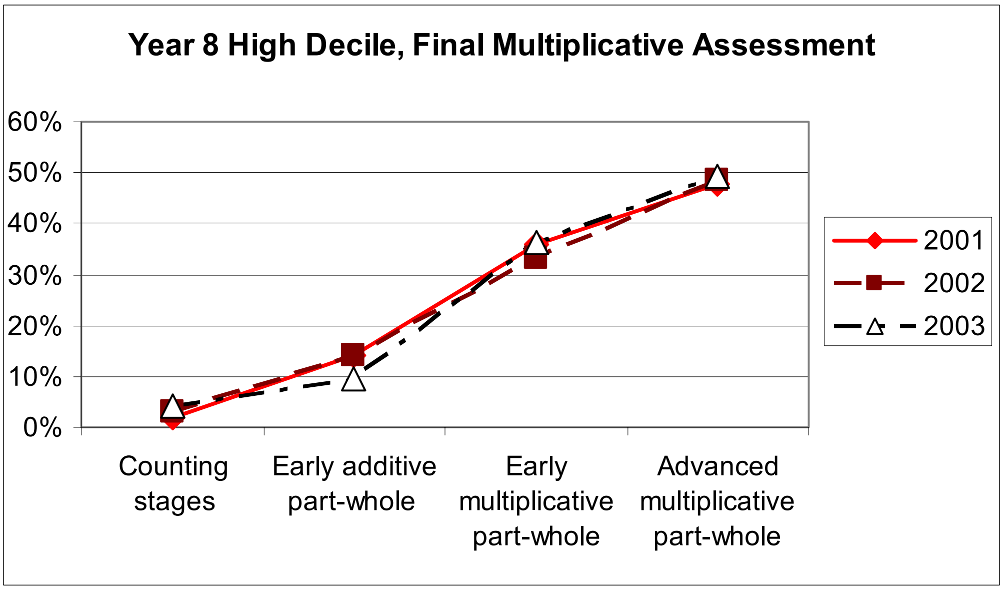

Figure 3.12. Percentage of year 8 students from low decile schools using each strategy for multiplication at the end of 2001, 2002, and 2003

The percentage of students from high decile schools was very similar in each of these three years. In these schools 48% or 49% of students were judged to be at the advanced multiplicative stage.

Other scales

Scales for proportional reasoning, fractions, and decimals are also important in the years covered by this analysis. However, the different manner in which data were collected across these three years makes comparisons of little value. The type of items in these scales changed between 2001 and 2002. For all three of these scales, students had to be assessed on Form C of the assessment to be credited with the higher stages.

Conclusions

These data show that students have been judged to finish the year at similar stages in addition and multiplication in 2001, 2002, and 2003, when compared with students from a similar decile band. Where differences do occur, they show a higher percentage of year 7 students in low and middle decile schools becoming early multiplicative thinkers in 2003 and more high decile year 7 students becoming advanced multiplicative thinkers, than in 2002. Similarly, the percentage of year 8 students from middle decile schools being judged to be multiplicative thinkers increased.

These data show that the Numeracy Project has been effective in enabling year 7 and 8 students to use part-whole additive thinking, and from 61% (low decile) to 85% of students to become multiplicative thinkers.

Differences in attainment between students from schools in different decile bands persist.

4. Comparison of students in full primary schools or intermediate schools, and in the first or second year of the Numeracy Project

Full primary versus intermediate schools

A question has been raised about the comparative effectiveness of the Numeracy Project in full primary schools versus its use in intermediate schools. I believe that this concern is related to how familiar teachers in the two types of institutions are with the teaching style that emphasises students working in groups, or assuring that the needs of individual students are met. Intermediate schools are usually much larger than full primary schools and this may lead to administrative requirements that could make the Numeracy Project more difficult to implement.

Students attending these types of schools differ in their demographic characteristics. Table 4.1 and 4.2 show the percentage of students from different deciles in these schools and the ethnicity of students in different schools. Deciles have been grouped as low (deciles 1–3), medium (deciles 4–7), and high (deciles 8–10).

For the purpose of this analysis, composite schools (year 1–13) and contributing schools that had year 7 and 8 students were grouped with full primary schools, as they were thought to be similar to primary schools in character. Year 7–13 schools were grouped with intermediate schools, as it was believed that their nature was more like intermediate schools than like primary schools. The number of students from composite, contributing and year 7-13 schools was small. Given this breakdown, there were 4862 students labelled as being in full primary schools and 7346 students labelled as being in intermediate schools. See Appendix C for a table of numbers of students in each category.

Table 4.1. Percentage of students from full primary schools and intermediate schools in different decile bands

	Full Primary Schools			Intermediate Schools		
	Year 7	Year 8	Total	Year 7	Year 8	Total
Low decile	50%	47%	49%	50%	52%	51%
Medium decile	30%	31%	31%	47%	44%	45%
High decile	19%	21%	20%	4%	4%	4%

About half of the students in both types of schools came from low decile schools (deciles 1–3). This reflects the focus of the Numeracy Project in its attempt to raise the achievement of these students. However, the percentage of middle decile students was larger in intermediate schools (45% versus 31%) and the percentage of students from high decile schools was higher in full primary schools (20% versus 4%).

The ethnicity of students differed in the two types of schools, again largely as a function of their location. Table 4.2 gives these data. About half of the students in both types of schools were of European descent. There was a higher proportion of Māori students in full primary schools than in intermediate schools and a higher proportion of Pasifika students in intermediate schools.

Table 4.2. Distribution of ethnicity in year 7 and 8 classes in full primary schools and intermediate schools

	Full Primary		Intermediate Schools	
	Year 7	Year 8	Year 7	Year 8
European	54%	52%	47%	50%
Māori	34%	34%	28%	26%
Pasifika	7%	8%	15%	17%
Asian	3%	2%	5%	4%
Other	3%	3%	4%	4%

To investigate the question of comparative success of the project in these two types of schools, the second addition and second multiplication assessments were compared. One school of 7 students is left out of this analysis as its type could not be determined at the time of analysis. All students for whom this second assessment was not given were eliminated. This may have had the effect of lowering the mean scores, as students who achieved a high score initially were usually those who were not reassessed. For assessment of addition strategies, most of these non-tested students came from one low decile intermediate school.

An analysis of variance showed that there were no significant differences between scores (shown in Appendix C) at these two types of schools on addition. For year 7, the analysis yields $F(1, 373) = 1.1177$, $p = 0.29$ and for year 8, $F(1,766) = 0.9522$, $p = 0.33$. However, there was a significant difference by decile groups. For Year 7 this was $F(2,372) = 50.67$, $p < 0.001$, and for year 8 $F(2, 763) = 107.93$, $p < 0.001$. There was also a significant interaction between type of school and decile for both age groups: for year 7 this was $F(2,372) = 4.3237$, $p = 0.0139$ and for year 8 $F(2,764) = 6.9501$, $p = 0.001$. The large difference in degrees of freedom between the year 7 and year 8 cohorts was the result of the need to use the Brown-Forsythe formula because of unequal variance between the groups.

Additive stages

Figures 4.1 and 4.2 present the interaction of school type and decile.

Note that the scores on the y axis of these figures is not the stage number. They are the number assigned to the stage in the computer program that presented the data to me for analysis, and differ for different scales.

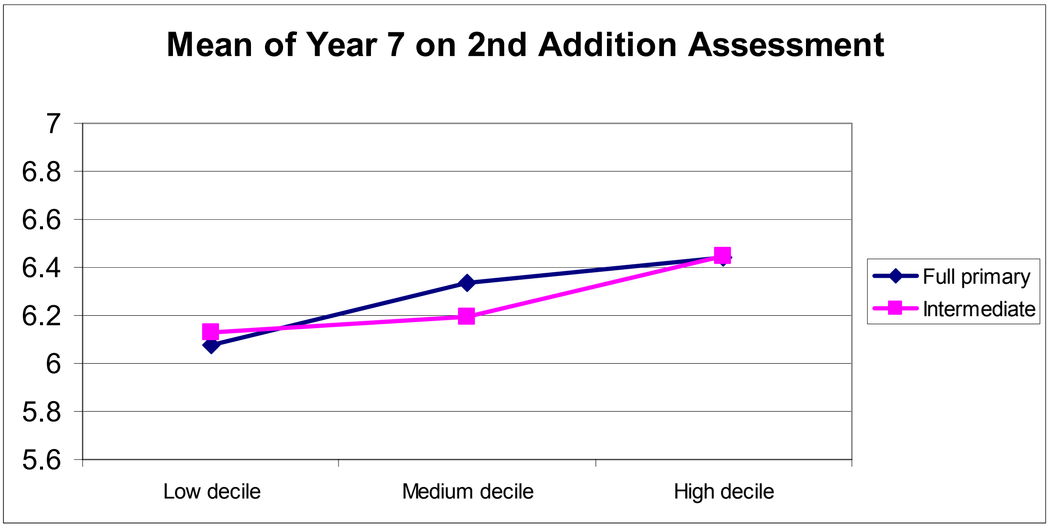

Figure 4.1. Means for year 7 students attending full primary or intermediate schools. On this scale 6 indicates the early additive part-whole stage and 7 indicates the advanced additive part-whole stage

It can be seen that students in full primary schools of medium decile did better than those in intermediate schools, but the difference is not large enough to make the overall result significant. There is little difference for other deciles.

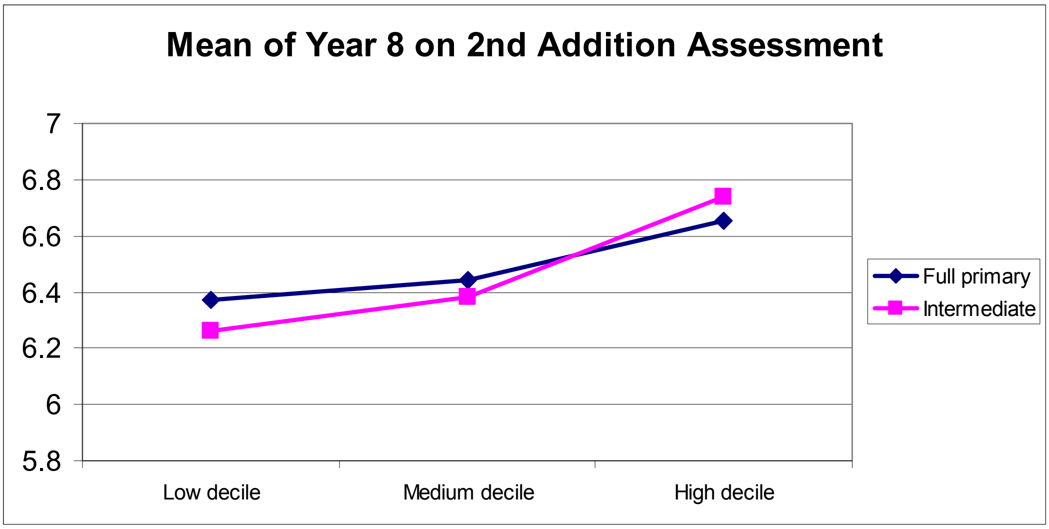

Figure 4.2. Mean of year 8 students from full primary and intermediate schools. On this scale, 6 indicates the early additive part-whole stage and 7 indicates the advanced additive part-whole stage

For year 8, the students in high decile intermediate schools performed better than year 8 students in full primary schools, but in low and medium decile schools those in full primaries performed better than those in intermediates.

Multiplicative stages

There was no significant difference on the second assessment of multiplicative strategy for year 7 students (1,305) = 1.6206, $p = 0.204$) but there was a significant difference favouring full primary schools for year 8 students. ($F(1,388) = 9.9909$, $p = 0.002$). There was a significant effect for decile as there was for the additive tests: year 7 ($F(2,304) = 50.3511$, $p < 0.001$), year 8 ($F(2,387) = 79.4980$, $p < 0.001$). There was not a significant interaction between school and decile for either year 7 or year 8 students.

Figures 4.3 and 4.4 show the means by decile for each group on multiplicative strategies

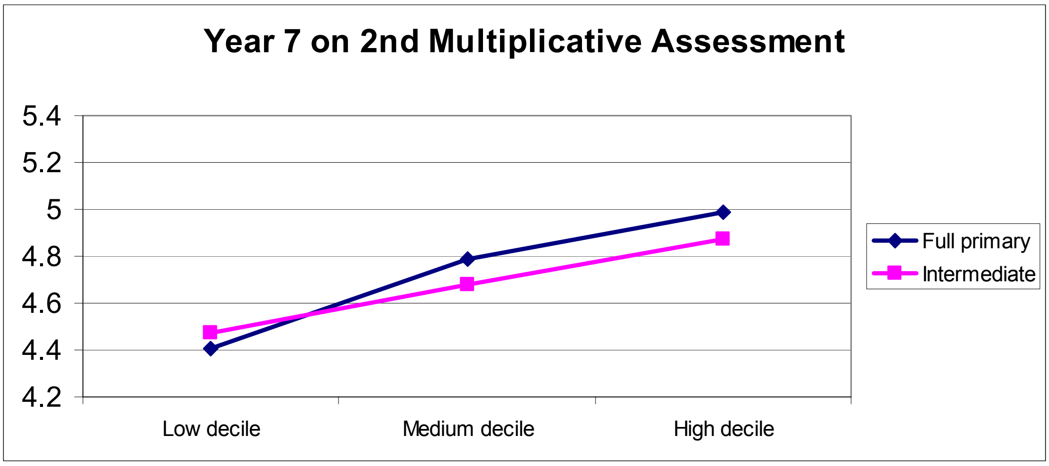

Figure 4.3. Mean scores of year 7 students in full primary and intermediate schools by decile. A score of 4 on the y axis indicates early additive part-whole strategies, and a score of 5 indicates early multiplicative part-whole strategies

This figure shows that students from low decile intermediate schools did slightly better, on average, than did students from low decile full primary schools. As low decile students made up about 50% of each group, this difference is worth noting. However, students from middle and high decile full primary schools performed better than students from these deciles in intermediate schools.

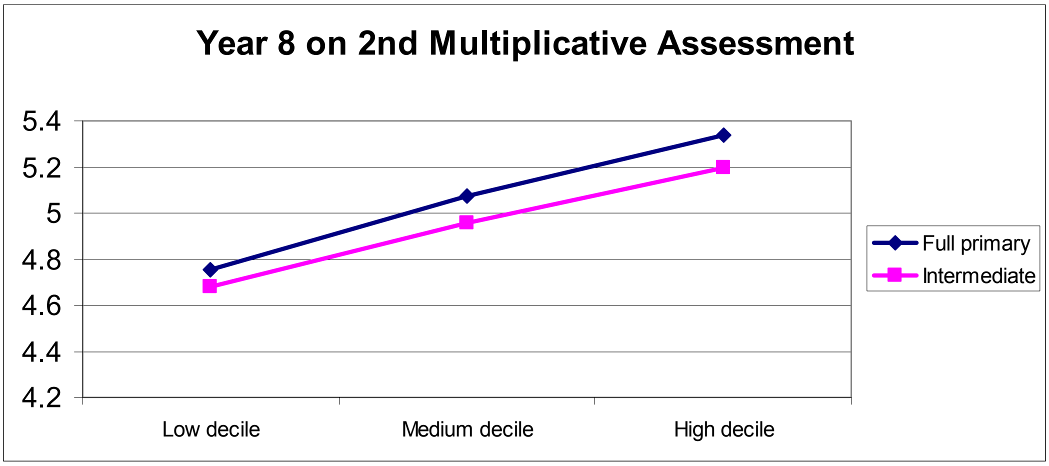

Figure 4.4. Mean scores for year 8 students on the second multiplicative assessment. A score of 4 on the y axis indicates early additive part-whole strategies, and a score of 5 indicates early multiplicative part-whole strategies

Year 8 students from all deciles of full primary schools scored more highly on this assessment than did students from intermediate schools.

Year 8 students in intermediate schools that were in the first or second year of the project

One hope of the project is that it will have a cumulative effect on students' level of strategies. If that were true, year 8 students in the second year of the project would perform at a higher level than year 8 students in the first year of the project, other factors being relatively equal. It is apparent from the figures throughout this report that decile rank is a strong determiner of level of performance on the second assessments. Therefore, year 8 students whose schools were in the second year in the project were compared with students from schools in the same deciles. This could be done for deciles 2, 3, and 5. Although there were schools in the project for the second year in deciles 4 and 8, there were no intermediate schools in their first year of the project from these deciles.

The second assessments were compared for additive and multiplicative strategies for decile 2, 3, and 5 schools. The only significant differences for schools in the first or second year of the project were for the decile 3 schools, where the school in the first year of the project did significantly better than the school in the second year of the project on both scales (decile 3 schools: additive p=0.004; multiplicative p=0.030).

Additive strategies

Figures 4.5, 4.6, and 4.7 show the percentage of students in schools in the first or second year of the project at each stage for additive strategies.

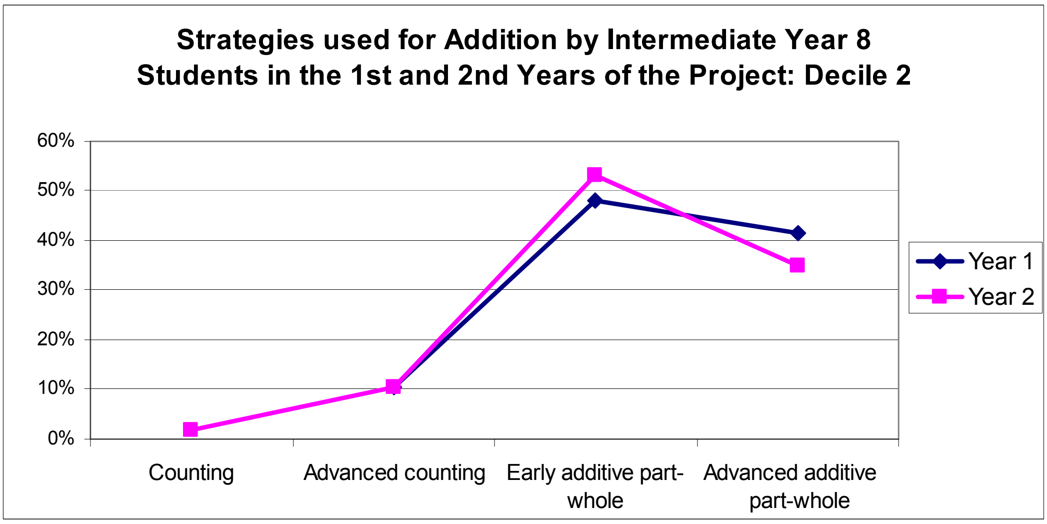

Figure 4.5. Percentage of year 8 students from decile 2 schools in the first and second years of the project for additive strategies. This figure represents the students from one school in the project in the first year and one in the project for the second year

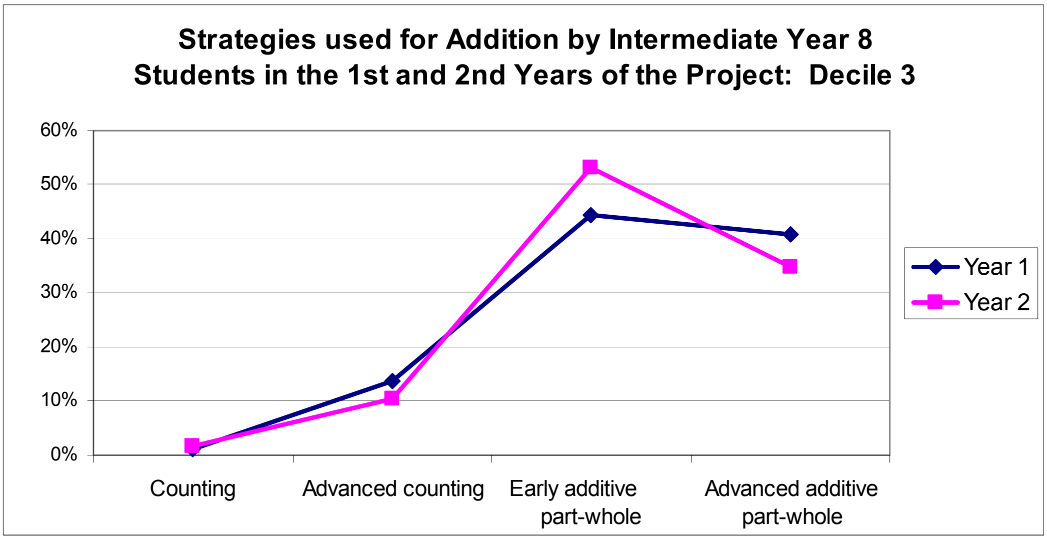

Figure 4.6. Percentage of year 8 students from decile 3 schools in the first and second years of the project for additive strategies. This figure represents the students from one school in the project in the first year and one in the project for the second year

The decile 3 school in the first year of the project did significantly better on the second assessment of addition than did the comparable school in the second year of the project.

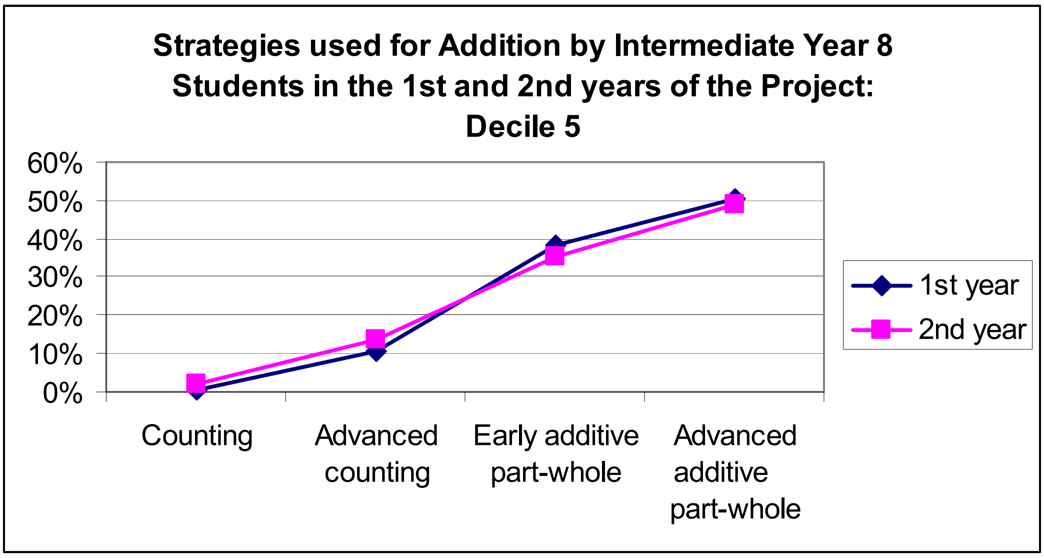

Figure 4.7. Percentage of year 8 students from decile 5 schools in the first and second years of the project for additive strategies. This figure represents the students from four schools in the project in the first year and two schools in the project for the second year

The percentage of students at different stages in the decile 5 schools in the first and second year of the project is virtually identical.

Multiplicative strategies

The pattern of stages for multiplicative strategies in the three comparisons is shown in Figures 4.8, 4.9, and 4.10.

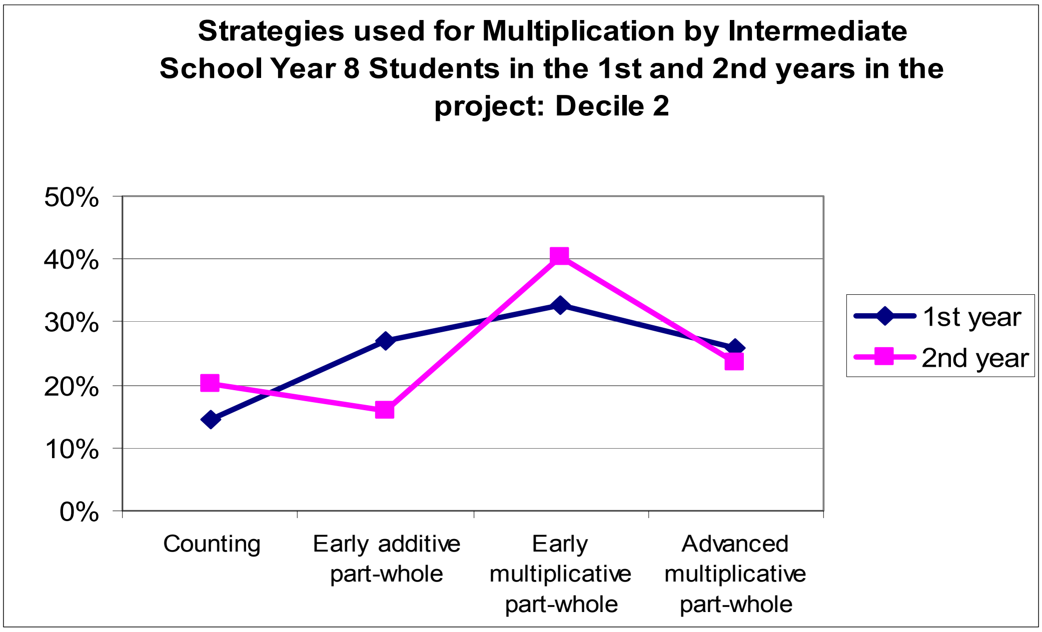

Figure 4.8. Percentage of year 8 students from decile 2 schools in the first and second years of the project at different levels for multiplicative strategies. This figure represents the students from one school in the project in the first year and one school in the project for the second year

The school in the project for the second year had a higher percentage of students using early multiplicative strategies, but the difference was not significant.

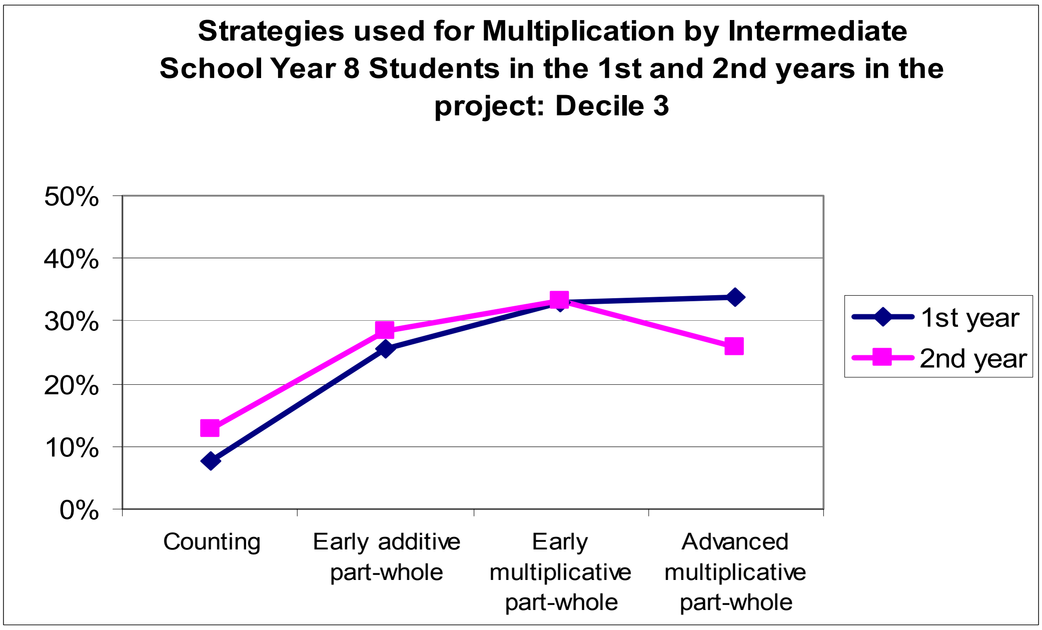

Figure 4.9. Percentage of year 8 students from decile 3 schools in the first and second years of the project at different levels for multiplicative strategies. This figure represents the students from one school in the project in the first year and one school in the project for the second year

The school in the first year of the project had more students at the advanced multiplicative stage. This school was significantly better than the one it was compared with.

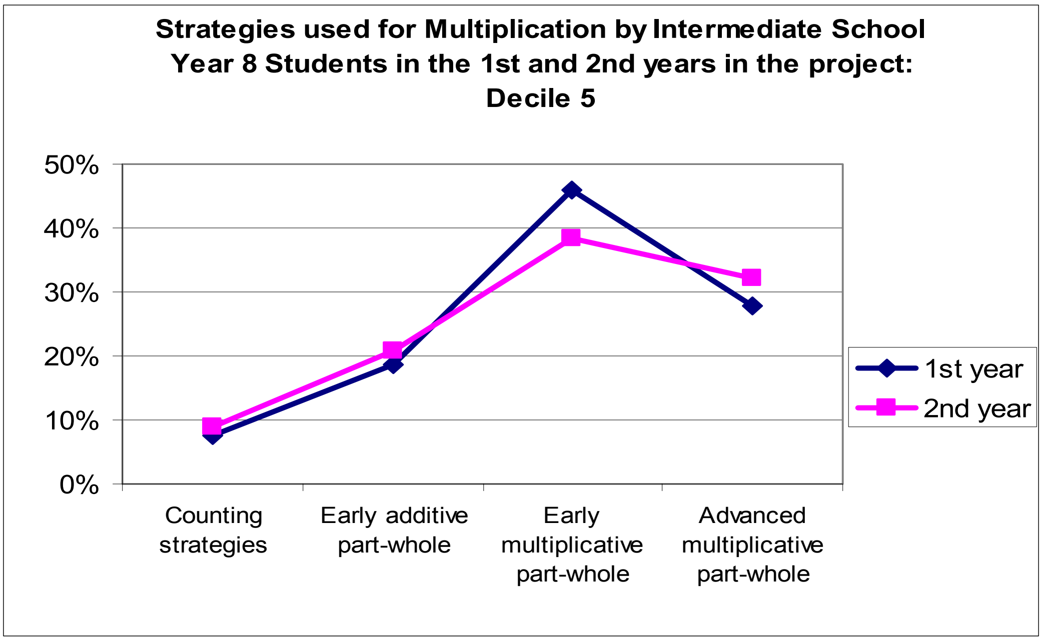

Figure 4.10. Percentage of year 8 students from decile 5 schools in the first and second years of the project for additive strategies. This figure represents the students from four schools in the project in the first year and two schools in the project for the second year

The four schools in the first year of the project had a higher percentage of students using early multiplicative strategies, and the schools in the project for the second year had a slightly higher percentage of students using advanced multiplicative strategies, but the difference was not significant.

Conclusions

Year 8 students in full primary schools did significantly better in multiplicative strategies than did comparable students in intermediate schools. There were no significant differences between types of school for year 7 students in additive or multiplicative strategies, or for year 8 students in additive strategies.

When year 8 students in schools that were in the project for the first or second year were compared with schools from the same decile on additive and multiplicative strategies, the only significant differences favoured the decile 3 school in the first year of the project. This was particularly true for additive strategies. When the facilitator for this school was asked about factors that could have led to the success of these students, she reported that the deputy principal took a special interest in mathematics. Mathematics was taught in every class between 9 and 9:45 a.m., and there were no interruptions during this period. This draws attention to factors within the school that aid the progress of a numeracy project.

5. Comparison of year 7 students in the Numeracy Project with norms for the asTTle mathematics test

This comparison covers a sample of 408 year 7 students from nine schools that administered an identical asTTle test and returned results, although not all students appear in all analyses.

AsTTle is a New Zealand designed test of students' skills in several mathematical domains. It is based on curriculum levels as outlined in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992). Teachers can select the curriculum level and the mathematical topics that interest them. It is particularly useful as a diagnostic test for teachers wishing to see where each student's strengths and weaknesses lie. It is delivered on a CD-ROM that selects items in relation to preferences that a teacher specifies. The software will then print out for the teachers a report for each child, for a class, or for several classes that took the same test. The report for a class or group of classes is represented in the form of a "console" that presents information on how the group did in comparison with "Schools like Mine". This includes pointers on a dial or other indicators for performance on the mathematical areas assessed, on scales of depth of thinking, on overall mathematical competence, and on attitude, all in relation to a comparable group. Comparable groups are described by location (e.g., South of Taupo) by school size, type of location (e.g., small rural), by decile group (low, medium, high) and by their ethnic makeup. This last category is described only with terms like "high minority". Minority refers to ethnicities other than New Zealand European. Tests of curriculum levels 2 through 4 were standardised before the onset of the Numeracy Project.

The students are compared with the appropriate norms for asTTle on three subtests. These norms come from a period before students had participated in the Numeracy Project (November 2001 to June 2002), so the norms act as a comparison group. Because norms for subtests on this test are available for 16 different clusters of New Zealand schools, determined by area, decile, size, and proportion of minority students, it is possible to compare each school that returned their results with year 7 students in similar schools in asTTle's "Schools like Mine" report.

Initially, 44 schools in the Numeracy Project, constituting a stratified sample, were approached to participate in this comparison. Following a second approach, 26 schools agreed to administer the asTTle test sent to them and to return results. An identical asTTle test was sent to each school both electronically and by post. However, for a series of reasons for which schools apologised, only nine schools eventually returned results. These reasons related to general school issues, not to difficulties with downloading or scoring the asTTle test.

For this assessment I selected a test for the schools that covered the subtests of Number Knowledge, Number Operations, and Patterns in Number. It covered curriculum levels 2, 3, and 4 with 17 of 32 items at level 3. Half of the items were designated as surface items and half deep items, a distinction based on the SOLO taxonomy. See Appendix D for the test used and the number of items in each category.

Sample assessed

The schools in the sample varied in size from having 2 to 242 year 7 students. The schools and their comparison groups for asTTle clusters are given in Table 5.1.

Table 5.1. Schools returning results of an AsTTle test made up of Number Operations, Numerical Reasoning and Patterns in Number

School	Number of year 7 students	Decile group	Ethnicity	Area and size of school
1	16	Medium –high	High majority	Country, South of Taupo
2	2	Low	High minority	Auckland city, small
3	242	Low	High minority	Auckland city, large
4	7	Medium	Majority	South Island country
5	10	Medium–high	High majority	South of Taupo, country
6	35	Medium–high	High majority	South of Taupo, country
7	45	Low	High minority	Auckland city
8	9	Middle	2/3 majority	North of Taupo, smaller country school
9	38	High	Majority	South Island country

Results

The relationship of each student to the means of the matching asTTle cluster could be taken from the pictorial "console" for that school. It was possible to compare the three sub tests with appropriate means, although not for the whole test or for the deep and surface items.

When compared with the means for each relevant subgroup using "Schools like Mine", all schools except school 7 scored above the mean on the full mathematics scale. All schools except school 5 scored above the mean for deep thinking, and all schools except for schools 5, 7, and 8 scored above the mean for surface thinking. Because of the way asTTle statistics are presented it is not possible to measure the significance of these differences.

After all students who had not answered all items were eliminated, there were results for subtest scores for 360 students. A comparison of the difference between the mean for each student and the mean for their sector on each subscale indicated that students in the Numeracy Project were significantly above the norm for Number Knowledge ($p<.001$) and Number Operations ($p<.001$). There was not a significant difference from the mean for Patterns in Number.

The following figures show the relationship of students to the means for year 7 in their cluster. Figure 5.1 relates to the Number Knowledge scale and Figure 5.2 relates to the Number Operations scale. The standardised score for the mean of each subtest is 500 for the entire sample, differing for different year levels and different clusters, with a standard deviation of 100. For example, a difference of +100 indicates a student who is one standard deviation above the mean.

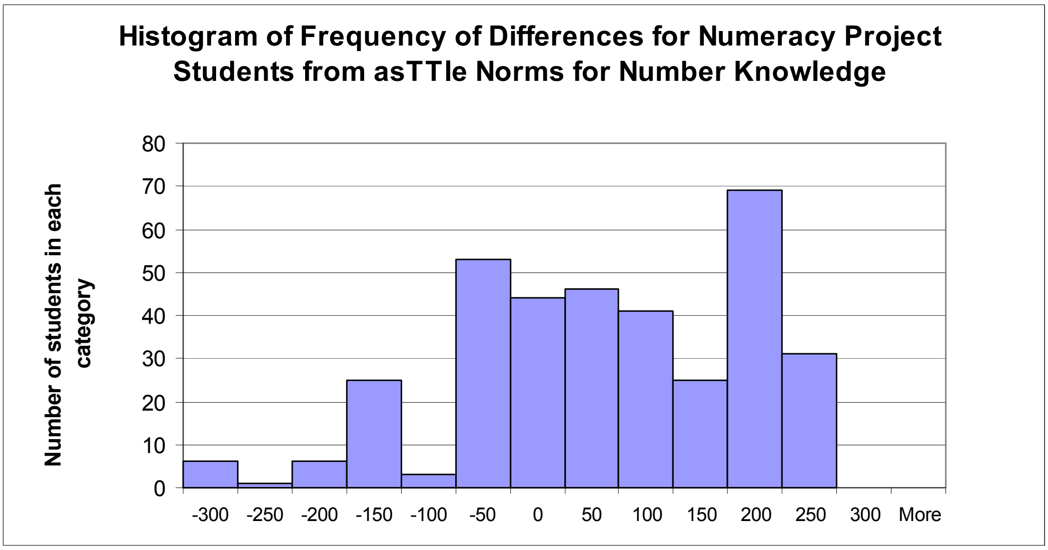

Figure 5.1. Distribution of the differences between the asTTle mean for a cluster and year 7 students' scores for Number Knowledge

In this figure, the bar designating the numerical difference between a student's score and the mean is indicated by the lower limit. Hence the bar labelled "-300" includes all students whose scores were between -300 and -249.9999 below the mean. This figure shows the majority of students to be above the mean of the norm.

Figure 5.2 shows the comparison of means for the Numeracy Project students with the mean of the normative group on Number Operations.

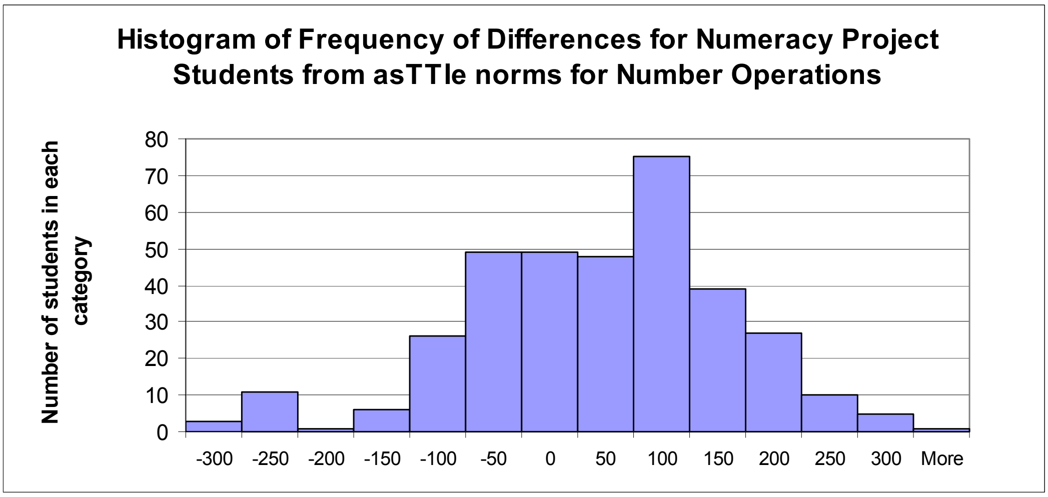

Figure 5.2. Distribution of the differences between the asTTle mean for a cluster and year 7 students' scores for Number Operations

As with Figure 5.1 the distribution of the means of individual students' scores is well above that of the norm group.

Attitude

An analysis of attitude for a subgroup of 273 of these students, from whom results were initially available in Excel format, showed attitudes to be well above the mean for the entire norm group.

The Pearson correlation coefficient between attitude and total mathematics achievement (not assessed above) was 0.20, which, although significantly different from zero at the 0.01 level, represents a very weak positive relation. Similarly, the correlation between each of the three subtests and attitude was small: the correlation of attitude with Number Knowledge was 0.11 (not significant), with Number Operations it was 0.22, and with Patterns in Number it was 0.20.

Conclusions

As the items in the asTTle test were designed to measure achievement on levels in the 1992 Mathematics Curriculum (Ministry of Education, 1992) they do not specifically measure strategies or knowledge emphasised in the Numeracy Project, as do the final numeracy assessments given in schools. This AsTTle test therefore provided a good test of whether or not these students could be said to have improved in numerical skills and knowledge generally.

Although this study involved a smaller sample of students than initially desired, the nature of the norms for asTTle tests allow meaningful conclusions to be drawn from this sample. These year 7 students in the Numeracy Project did significantly better, on average, than a sample of students who had not had the Numeracy Project on Number Knowledge and Number Operations, but not on Patterns in Number. The results suggest a better general understanding of mathematics than that held by the group on which the asTTle test was normed, although the range of difference is large as shown in Figures 5.1 and 5.2.

The reason for lack of difference between the norm and results for these students on the Patterns in Number subtest is unclear. It may be that this type of item has not been emphasised in the Numeracy Project, or actually de-emphasised, as teachers became used to the new focus of the Numeracy Project. It would be interesting to see if this lack of significant difference persisted after teachers made more use of the booklet on "Teaching Number Sense and Algebraic Thinking" (http://www.nzmaths.co.nz/Numeracy/2004numPDFs/Book%208%20Number%20Sense.pdf).

The very positive attitude expressed by these students, and its low correlation to success, carries two messages. Positive attitude does encourage students to participate in mathematics classes. It may relate to their teachers' enthusiasm about a new project, their own enthusiasm at having more interesting tasks to do in mathematics, and/or to their own success. The teaching policy of the Numeracy Project requires that each student work at an appropriate level, with just enough challenge to move to the next level. It does not present students with tasks at which they frequently fail. Therefore they may have a more positive attitude than would be justified by results on this test that goes across three curriculum levels and includes items that most students could not complete. This has been the case in New Zealand in previous studies, as in the results of the TIMSS (Beaton et al., 1996).

While we value positive attitude, we must continue to ensure that our students are challenged.

6. Assessment of generalisation of strategies for year 8 students

The year 8 students in six intermediate schools took a test prepared by Murray Britt that assessed their ability to understand a method of manipulating the numbers in an addition, subtraction, multiplication, or division problem that would make the problem easier to do. Two examples showing the manipulations were given at the top of the page in the following format:

Jason uses a simple method to work out problems like 47 + 25 and 67 + 19 in his head.

Problem	Jason's calculation
47 + 25	50 + 22 = 72
67 + 19	66 + 20 = 86

This was followed by three problems that the student was to do "using Jason's method". The first two of these in each section dealt with whole numbers and the third dealt with numbers that included a decimal fraction. The test is in Appendix E.

Each page modelled a different strategy that was appropriate for addition, subtraction, multiplication, or division. Students were marked by whether or not they used the method modelled in the example at the top of each page. The model problems included the use of compensation and distribution although these were not stated. Minor calculation errors were disregarded, but if a student got an answer using any other method, such as the vertical algorithm, they were not credited. In this analysis we were looking for an understanding of the appropriate strategies for operating with numbers in a manner that is expected to lead to a greater understanding of algebraic manipulation (see Irwin and Britt, under review).

Schools included in the assessment

Results from a similar analysis in 2002 (using a somewhat different test) demonstrated that students in schools using the Numeracy Project outperformed students from similar schools that were not in the project. Similarly, upper decile schools performed better than middle decile schools.

In 2003 we assessed three schools that were in the first year of involvement in the Numeracy Project and three similar schools that were not in the project. Schools were from deciles 1, 3, and 5. The two schools in decile 5 were in fact different sections of a large intermediate school that was broken up administratively and had half of the school involved and half not involved with the project. In this report they are referred to as separate schools. All students took the test in one class period. Table 6.1 shows the number of students in each school. As these were all intermediate schools they are referred to as being in, or not in, the Intermediate Numeracy Project (INP).

Table 6.1. Number of students in each school who took the test of generalisation

	Schools in INP	Schools not in INP	Totals
Decile 1	167	383	550
Decile 3	225	254	479
Decile 5	159	168	327
Total	551	805	1356

Results by decile and status in the project

Schools within the Intermediate Numeracy Project were significantly more successful on this test of generalisation than were similar schools that were not in the project (F (1,976) = 19.00, $p < 0.001$). There was also a significant effect of decile (F (1,975) =63.11, $p < 0.001$), but there was not a significant interaction between these two factors (F (1,976) = 0.65, $p = 0.52$). That can be interpreted as indicating that each school in each decile group benefited from involvement in the project, but schools from different decile groups did not differ significantly in the amount that they benefited. (The above results are based on the Brown-Forsythe analysis, rather than the standard analysis of variance, because the variances of the groups were significantly different.) The mean scores and standard deviations for each school were as follows:

Table 6.2. Mean number correct and standard deviation for each school

	Schools in INP		Schools not in INP	
	Mean	Std. Dev.	Mean	Std. Dev.
Decile 1	3.62	4.26	2.73	3.41
Decile 3	6.72	4.95	5.18	4.83
Decile 5	6.96	4.66	6.01	4.56

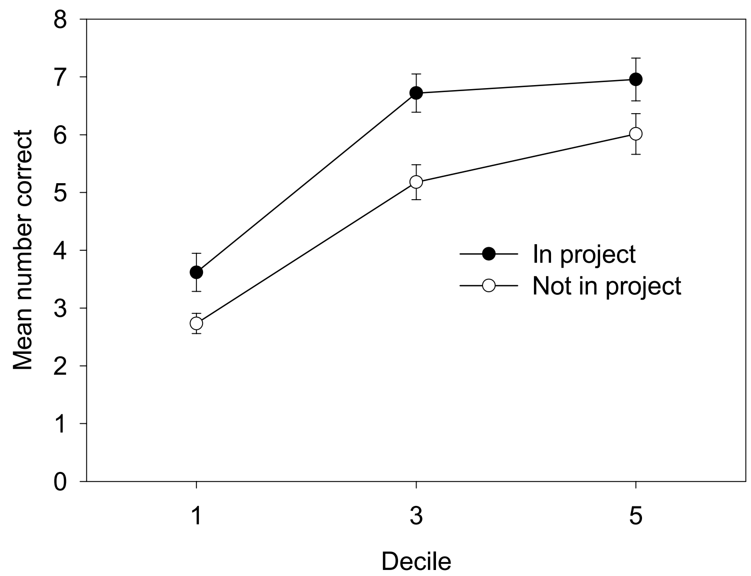

Figure 6.1. Mean scores and standard error bands of schools that took the test of generalisation of strategies

Figure 6.1 shows the mean scores for the schools. Note that a maximum score was 15, but the average for all groups was well below this. Some students in five of the six schools reached this maximum score, but in the schools with lower mean scores there were many students with a total score of 0, because they did not imitate the strategy modelled at the top of the page. In some cases this was because they had recently been introduced to more basic strategies, such as the use of a number line.

Item analysis

The percentage of students receiving credit for each item showed that a higher percentage of students in the project than those not in the project was successful on every item. Figures 6.2, 6.3, and 6.4 show the percentage passing for each item.

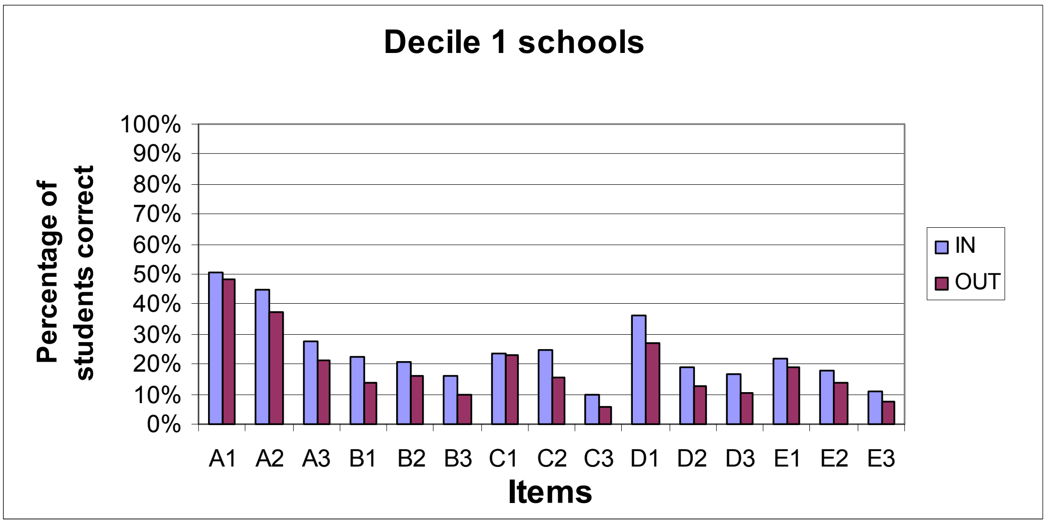

Figure 6.2. Percentage of students in decile 1 schools passing each item

Although the decile 1 schools had the lowest percentage of students answering each question correctly, those in project schools outperformed those from a parallel decile 1 school that was not in the project.

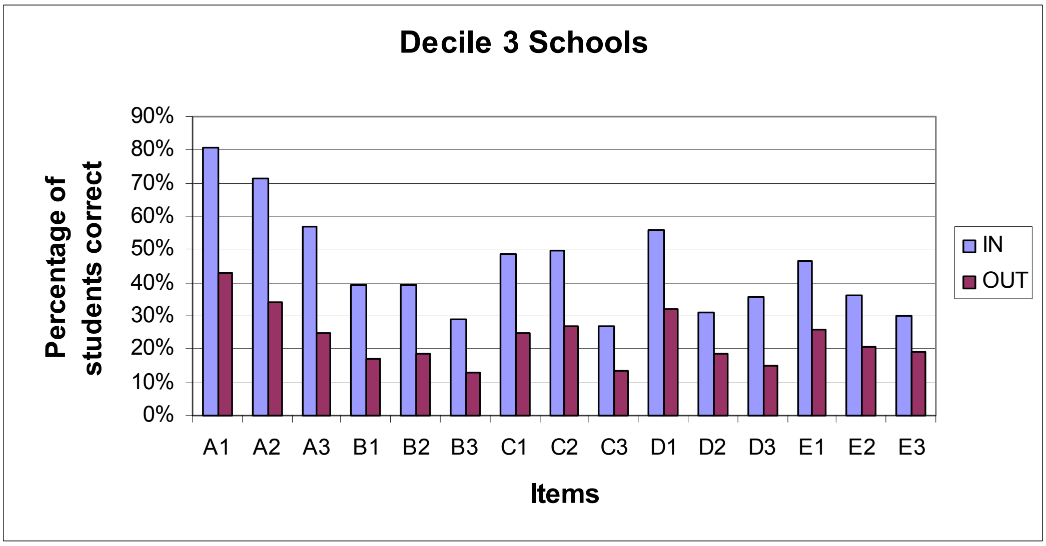

Figure 6.3. Percentage of students in decile 3 schools passing each item

Students in the decile 3 school that was in the project showed markedly better performance than did the school of a similar decile that was not in the project.

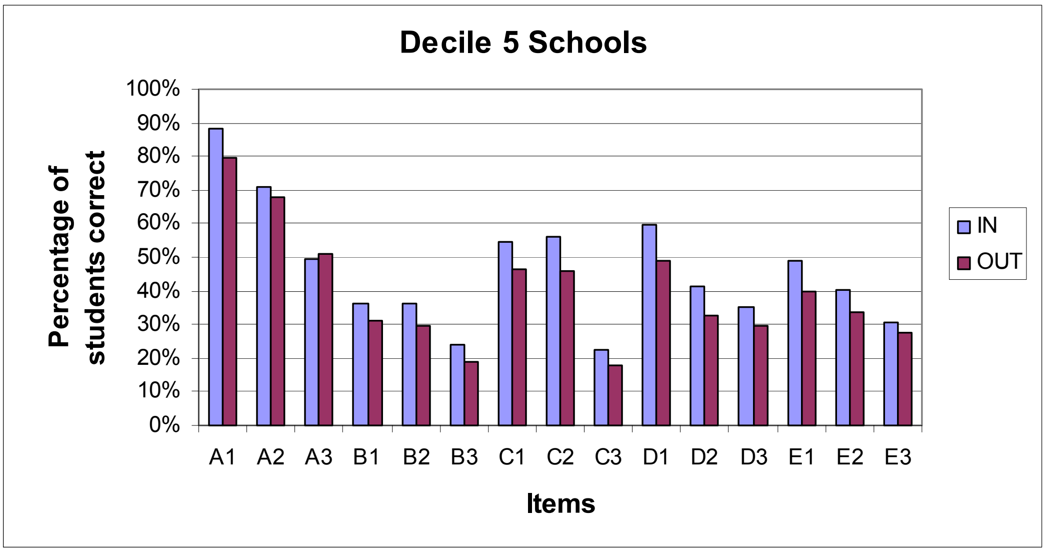

Figure 6.4 Percentage of students in decile 5 schools passing each item

Figure 6.4 shows that students from decile 5 schools, both in and not in the project, showed a good understanding of the strategies used in these questions. However, those in the project did better at recognising and incorporating the strategies into their problem solutions than did the students not in the project.

Relationship between the use of a strategy in problems using whole numbers and its use in problems with decimals

As the Numeracy Project is now presented, decimals are only assessed in Book C, the booklet used for the most successful students. In 2003, 62% of year 7 and 52% of year 8 students were not assessed on this topic. In the booklets that provide a framework for teaching, decimals are linked with fractions and percentages in a separate booklet from strategies for addition or multiplication of whole numbers. They are presented as an outgrowth of fractions, a very reasonable approach. However, there could be benefits to linking them to the strategies used for calculating with whole numbers. In this evaluation, Murray Britt and I were interested in seeing whether or not year 8 students could use a strategy that they used successfully with whole numbers with numbers that included decimal fractions. This requires students to understand that decimals are divisions of whole numbers and an extension of the whole-number system.

Of the various items that assessed whether or not the students understood decimals well enough to generalise them to the whole-number system, one example demonstrates both the power of this generalisation and the misconceptions that can lie in the way of a true understanding of decimals. This item was in Section D, requiring multiplicative compensation.

The example was 48 x 5 can be done as 24 x 10

The decimal item was 48 x 0.5

Students who understood that 0.5 was a half, and two of them equalled 1.0, did this item well. However, a number of students changed the problem to 24 x 0.10 or 24 x 10. The first error treats the decimal point as "a decorative dot" (Swan, 1983) where numbers on different sides of the decimal point are treated as unrelated. The other error disregards the decimal point. Both are common errors that show that students do not understand decimals.

As the students were in year 8, they were in the last year of school in which they were likely to receive any instruction in decimals. It is known that many adults have a weak understanding of this area. An understanding of decimals that allows simple transformations like the one above, in which 48 x 0.5 can be transformed into 24 times 1, can make decimals much easier to deal with.

For every subtest, the first two items involved transformations of whole numbers, and the third item involved the same transformation with decimals.

Table 6.3 shows the percentage of students at each school who correctly manipulated at least one of the whole number problems and of those, the percent that correctly manipulated the decimal items. See Appendix F for the numbers of students in each category.

Table 6.3. Percentage of students from each school who were accurate on at least one whole number item in each section and were correct on the decimal item

		In INP		Not in INP	
School	Test section	% correct on at least one whole number problem	% correct on decimal problem	% correct on at least one whole number problem	% correct on decimal problem
Decile 1	A	55%	28%	53%	21%
Decile 3	A	85%	53%	69%	38%
Decile 5	A	91%	50%	82%	51%
Decile 1	B	24%	16%	21%	10%
Decile 3	B	43%	29%	33%	20%
Decile 5	B	42%	24%	36%	19%
Decile 1	C	29%	17%	26%	6%
Decile 3	C	56%	27%	49%	20%
Decile 5	C	60%	23%	53%	18%
Decile 1	D	37%	16%	28%	10%
Decile 3	D	58%	36%	49%	23%
Decile 5	D	52%	35%	52%	20%
Decile 1	E	23%	11%	20%	7%
Decile 3	E	52%	30%	43%	29%
Decile 5	E	52%	31%	45%	27%

The percentage of students who could generalise a principle from an example of problems involving only whole numbers to problems that included decimal fractions was greater for all INP groups on almost all items.

When the number of students who were correct on the decimal item was taken as a percentage of the same students who were correct on at least one whole number item, the results, in Table 6.4, show that a mean of 55% of the students in the Numeracy Project could apply the principle to problems including both whole numbers and decimals, while 48% of the students not in the Numeracy Project could do this. Comparison with Table 6.3, above, shows the extent to which putting all the decile groups together disguises the difference between low and medium decile schools.

Table 6.4. Percentage of students in and not in the Numeracy Project who were successful in applying a principle to at least one whole number problem, the problem with decimals and those who were successful on both one whole number and the decimal problems

Sections	Percent successful on at least one whole number item		Percent successful on the decimal item		Percent of those who were successful on a whole number item who also used this strategy for decimals	
	INP	Not in INP	INP	Not in INP	INP	Not in INP
A	78%	64%	44%	33%	57%	51%
B	37%	28%	24%	15%	64%	54%
C	49%	40%	18%	13%	36%	32%
D	55%	39%	30%	18%	54%	47%
E	43%	32%	24%	18%	57%	57%
Mean	78%	64%	28%	19%	55%	48%

Section A was the one that most students were most successful on, while Sections D and E, using multiples for addition and division were next easiest. Students could succeed on at least one item in these sections by doubling, or halving and doubling. This may have accounted for the success rate shown in these data.

Thus, a higher percentage of students in the INP were successful in the use of a strategy in at least one item, and a higher percentage of INP students were successful in using this strategy with an item involving decimals. Although the percentage for both the INP students and the non-INP students who were successful was smaller than desirable, the percentage of students from both groups who understood the strategy for whole numbers who could transfer that understanding to decimals is worth noting. See Figure 6.5.

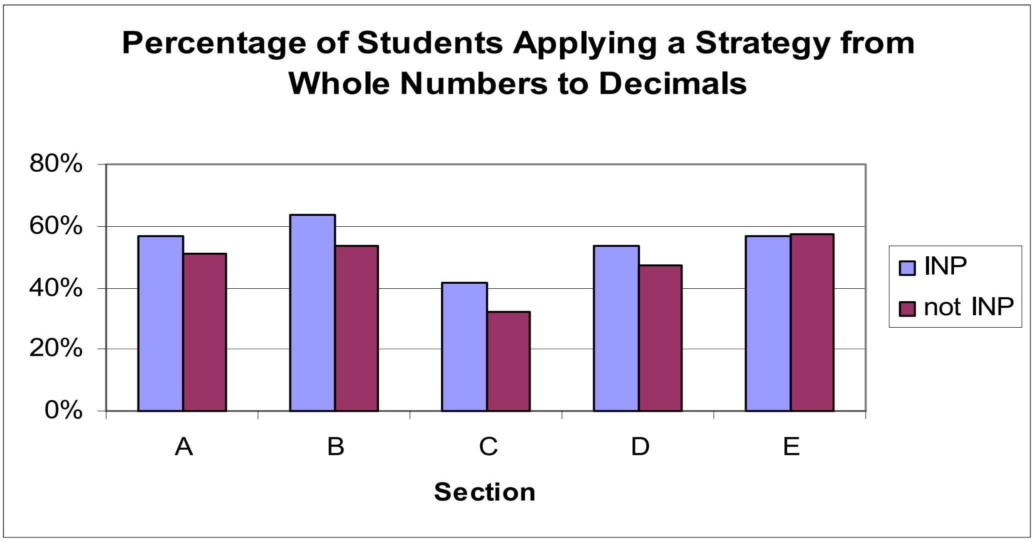

Figure 6.5. Percentage of those students who had been successful on one or more whole number items who were able to transfer that strategy to use with decimals

The percentage of students who transferred a strategy for whole numbers to decimal fractions was superior in INP schools for all sections of this test except for the last, in which the percentage was the same for both groups.

Conclusions

Students in the Numeracy Project were significantly more able to apply a principle involving compensation in all four operations and the distributive property in multiplication than were similar students who were not in the project.

About half of the students who could generalise a principle to whole numbers could also generalise this principle to numbers including a decimal fraction. The mean percentage of Numeracy Project students who could make this generalisation to decimals was 55%. The authors are unaware of any students being taught to use these strategies with decimals, so this may be a measure of independent generalisation. It suggests that the use of decimals in strategy strands should be encouraged.

7. The Numeracy Project for year 9 students

In 2001, ten secondary schools returned numeracy results for between 1 and 12 of their year 9 classes. These schools were of deciles 1, 2, 3, 4, 8, and 9. Of these students, 60% of 1451 students came from low decile schools (deciles 1–3). In 2002, 14 schools of deciles 1, 2, 3, 4, 6, 8, and 9 participated in this project, but in some cases not all classes participated. 56% of 1446 students came from low decile schools. In that year there were also returns for 289 year 10 students.

In 2003 the focus for the Numeracy Project in secondary schools changed, to focus on those students most in need of help in mathematics. Fourteen schools returned results for 762 students. The schools were of deciles 1–9, with the largest number of students from low decile schools. Details are given in Table 7.1 for schools that returned final assessment data. More classes and students were involved than results appear for here.

Table 7.1. Characteristics of the schools that sent in final results for year 9 students for the Numeracy Project in 2003

Decile	Number of schools	Number of classes	Students	% girls	% Māori and Pasifika
1–3	6	21	391 (51%)	46%	73%
4–7	5	18	256 (34%)	47%	24%
8–9	3	7	115 (15%)	50%	14%

One low decile school and one high decile school were girls' schools and one high decile school was a boys' school. All other schools were coeducational. Some schools returned results for only a few students in a class and others reported on up to 27 students in a class.

Initial and final stages on numeracy scales for all target classes

As these students were in the lower classes in the secondary schools concerned, a picture is provided of what their initial and final stage on project scales was. These are given in Figures 7.1–7.6 for the three strategy scales and the three more difficult knowledge scales. These figures show the strategy stage for all students for whom data was available, not just those assessed on the scale on both occasions.

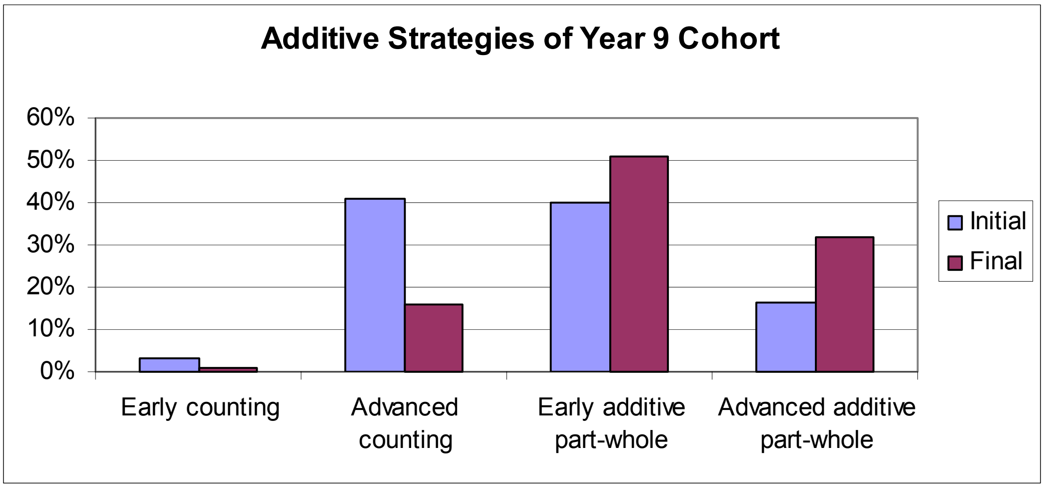

Figure 7.1. Percentage of year 9 students using each additive strategy stage on initial and final assessment in 2003

The impressive aspect of Figure 7.1 is the fall in the percentage of students dependent on counting on for adding. There was an increase in the percentage using one or more part-whole strategies for adding.

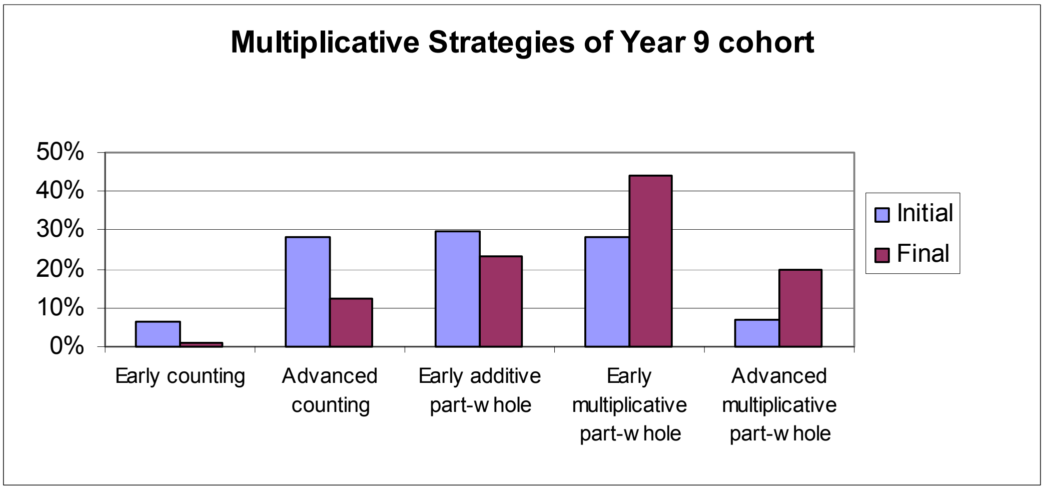

Figure 7.2. Percentage of year 9 students using each multiplicative strategy stage on initial and final assessment in 2003

Multiplication is more difficult than addition, and a higher proportion of students fall back on early counting stages for these tasks. However, Figure 7.2 shows that the percentage of students dependent on counting for multiplication fell from 34% to 13% in the course of the year. The percentage using multiplicative strategies rose from 35% to 64%. This means that 64% of these students are now able to use their multiplication tables in a flexible manner.

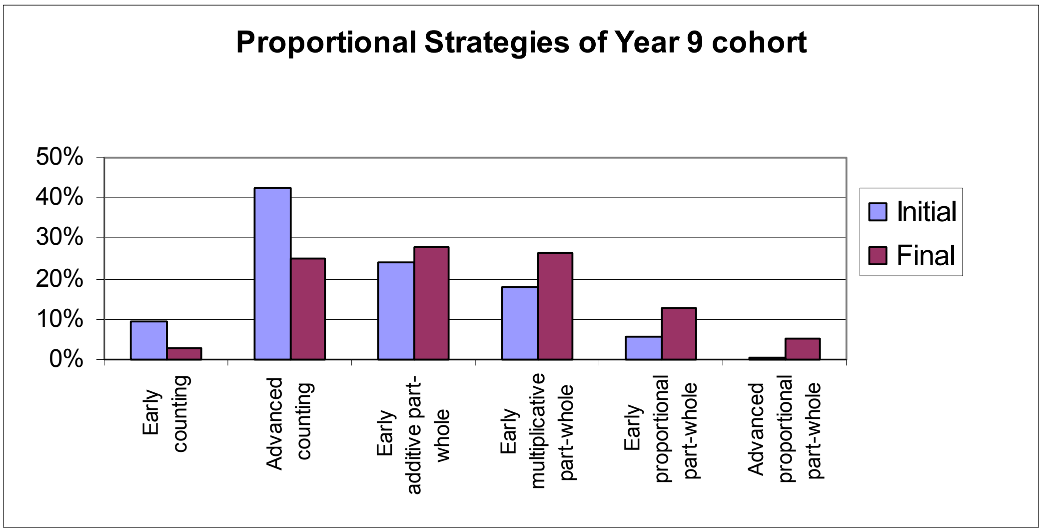

Figure 7.3. Percentage of year 9 students using each proportional strategy stage on initial and final assessment in 2003

The percentage of students represented in Figure 7.3 using counting strategies for proportional reasoning is higher than it was for multiplication. Initially 51% used counting strategies for proportional problems, while in the final analysis 28% used counting strategies. The percentage using proportional strategies rose from 6% to 18%.

The majority of students could produce the number before and after a number up to 1000 or up to 1 000 000 at the start of the year (Forward number sequence: 86%. Backward number word sequence 79%). Gain was made by a reasonable proportion of students on both of these skills, but it is likely that teachers did not emphasise teaching these simpler tasks. Only 41 students were asked about number identification initially and only 13 students were asked on both occasions. It can be assumed that these 13 students were ones whom teachers had reason to be concerned about in this topic.

Figures 7.4–7.6 show the percentage of students at different levels, on initial and final assessment, for the more complex knowledge scales of fractions, decimals, and base 10 grouping. These scales have interconnections and one might expect similarities in the distribution of percentages at similar stages. However, these were not obvious.

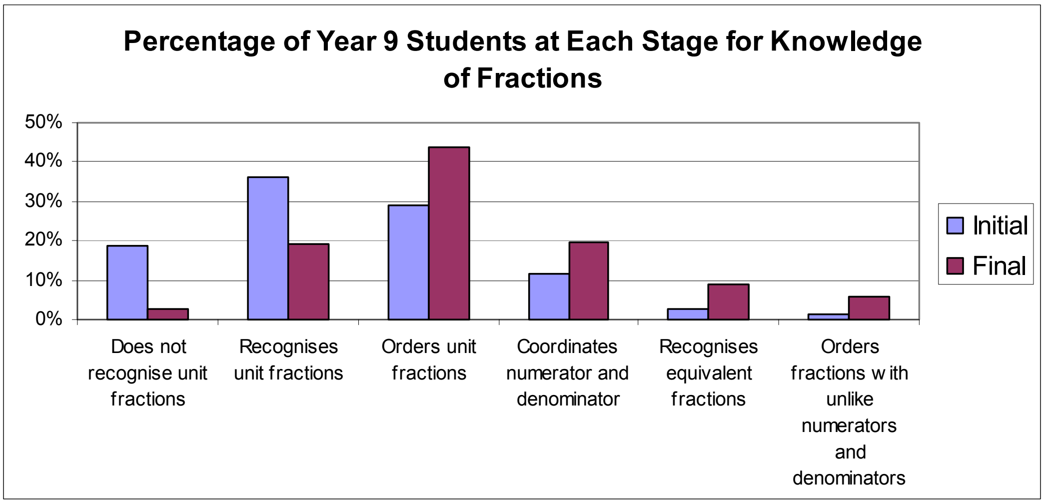

Figure 7.4. Percentage of year 9 students at each stage on knowledge for fractions in 2003

The main feature of this figure is that, while at the start of the year, 45% of the students either could not name fractions or named unit fractions but could not order them (thereby indicating that they had little understanding of their meaning); this dropped to 21% by the end of the year. At the end of the year 78% of the students demonstrated skills that indicated an understanding of the meaning of fractions, while at the start of the year only 55% demonstrated these skills.

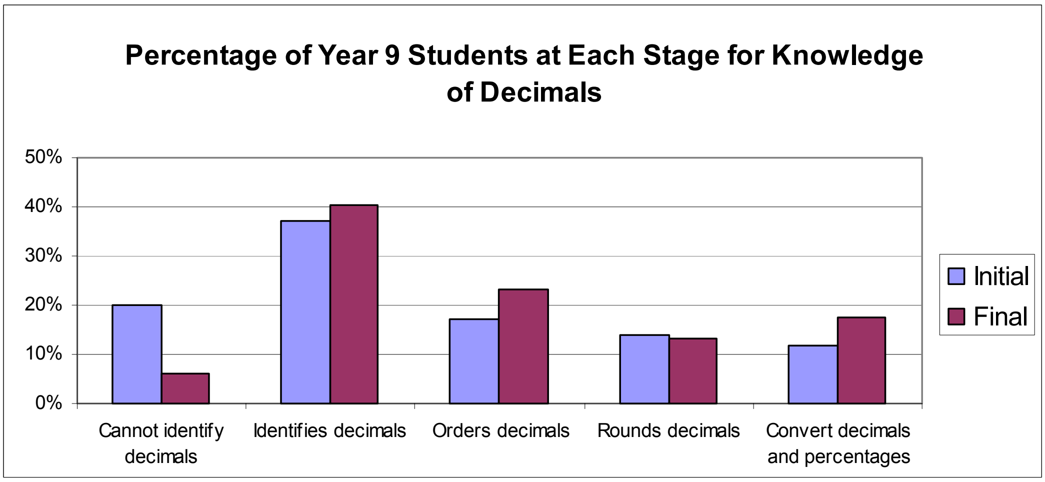

Figure 7.5. Percentage of year 9 students at each stage for knowledge of decimals in 2003

Note that only 273 students were asked about decimals at the start of the year and 135 students were asked about decimals on the final assessment. 117 of the same student were asked on both occasions, although this graph does not distinguish these students.

The percentage of students judged not to be able to identify a decimal decreased from 20% to 6% during the year. I suspect that the nature of the initial question on identifying decimals may have contributed to the percentage of students said to be unable to identify decimals, as in my research most 8-year-old children could identify some decimals and give instances of places in which they had seen them in use (Irwin, 1995). The percentage changes for each of the other stages were minor. Only 67 students in this year 9 cohort could demonstrate gain. This suggests that these students had not gained much in their understanding of decimals.

The students who were asked about decimals were given Form C. It is likely that these were seen as the more competent of this cohort of students.

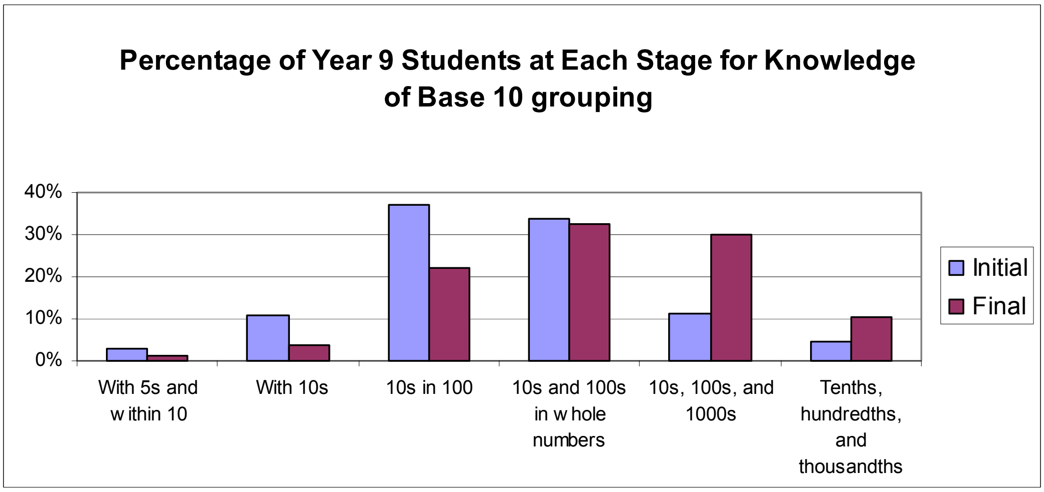

Figure 7.6. Percentage of students at each stage for knowledge of base 10 grouping

This figure shows that the largest proportion of students initially knew the number of 10s in 100 and nearly as many knew the number of 10s and 100s in a number. The major increase was in the number who could also give the number of 1000s in any number by the end of the year. The number who could give decimal portions increased from 4% to 10%. Thus these students appeared to have gained a sound understanding of the relation of the meaning of place value for whole numbers but very few understood the meaning of decimals. Only students assessed on Form C would have had the chance to demonstrate this top level skill. The percentage who demonstrated understanding of 10s, 100s, and 1000s in any whole number either were assessed on Form C on the second occasion or teachers extended Form B on their own to evaluate this.

The poor performance of these students on the final stage is in line with the limited number who demonstrated understanding of decimals on the decimal scale.

Progress on numeracy scales for all target classes

The majority of year 9 students were given most of the knowledge and strategy scales. The exceptions were Number Identification and Decimals, again reflecting the use of only Form B. If Form B was used on both assessments, it would not have been possible for students to show the full extent of their knowledge in proportional strategies, fractions, or decimals. Table 7.2 shows the number of students who were given each of the scales on two occasions, the proportion who failed to gain (were either seen to be at a lower level or at the same level on the final assessment), and those who gained one or more stage. Students who were at the top stage initially and did not change are not included, but those who were at the top stage initially and judged to be at a lower level on the second occasion were counted as making negative progress. Lower scores could be due to students forgetting, or to teachers' errors in judgement. There are not enough of these to cause concern.

Table 7.2. Progress made on numeracy scales by year 9 students (Total N = 762). Students who were at the ceiling initially were only noted if they scored at a lower level on the second occasion

Scale	Number assessed twice	% not gaining	% gaining
Additive strategy	636	50%	50%
Multiplicative strategy	693	41%	59%
Proportional strategy	731	49%	51%
Forward number sequence	566	51%	49%
Backward number sequence	585	77%	23%
Number identification	13	46%	54%
Fraction knowledge	720	40%	60%
Decimal knowledge	117	43%	57%
Base 10 grouping	738	40%	60%

Thus between 50% and 60% of the students assessed made progress on the scales that are most important for their age range: the strategy scales and the more difficult knowledge scales. The smallest percentage gain was on backward number word sequencing, but as 75% of the students assessed were already able to give the number preceding a given number up to 1000 on initial assessment, it is reasonable to assume that teachers did not spend time on teaching this strand.

The data for number identification and decimal knowledge should be treated with caution because of the small number of students given these assessments.

Case study of the Numeracy Project in one year 9 class for lower achievers

The School and Facilitation

One class using the Numeracy Project was studied in detail. This class, like others in this and neighbouring schools, had many additional resources offered to them by their facilitator. These covered measurement, statistics, geometry, and algebra, as well as numeracy. There were several suggested activities for numeracy beyond those offered in the booklets of the Numeracy Project. These resources were kept in a loose-leaf binder and added to from time to time. A workshop was held with the teachers before they used materials in any of the topics. Teachers used these materials in different ways, to suit their classes and teaching preferences.

The sequence for numeracy recommended by this facilitator covered operations with whole numbers first, then a combined section of fractions, decimals and percentages that emphasised the connections, and finally a session on operations with fractions, decimals, and percentages. This last section covered proportional reasoning and was covered after data had been collected from the case study class.

This decile 1 school had been in the Numeracy Project since 2001 (as had some of the other schools in the 2003 year 9 cohort) but in the first year they had not returned final assessments. In 2002 they received more than the usual amount of help from their facilitator, as it was found that the teachers needed ongoing help in the theory behind the project and the use of materials. In 2003, the mathematics department decided that only the teachers who had demonstrated enthusiasm for the project would be asked to use it. There were four teachers in this category out of a department of six teachers. Three teachers taught low stream year 9 classes and one taught a year 9 class for students who were not fluent in English (ESOL). In addition, a teacher aid rotated among the classes to provide extra help with teaching in line with the project. This person was always included in professional development sessions.

An aspect of the project in this facilitator's schools was encouragement for teachers to have students sit two Unit Standards tests in year 9, although these Unit Standards tests are not usually taken until year 11. This was intended both to demonstrate to the students that they could succeed on such tests and to fit the project into schools that tend to emphasise examination success. Students took these tests in the spirit of formative evaluation which allowed them to see how close they were to meeting these standards. Students did not view them as summative evaluation.

Of the classes in this school that participated, two were described by the head of the mathematics department as "reasonable classes, though of low achievement". Another was the ESOL class for students who were relatively new immigrants, and the fourth was "a difficult class, many truants and waggers, very low ability". It was decided to give some Unit Standards tests to three of these classes, but not to the ESOL class. The "difficult class made up of truants and waggers" took Unit Standard 8489 on one occasion and one student passed. It was decided not to give any more Unit Standards tests to this class. The other classes did sit the Unit Standards test with the following results, shown in Table 7.3. Class C in this table is was followed in detail in the following case study.

Table 7.3. Unit standards sat and achieved by students in three classes in the school that is the subject of this study

Unit Standard	Achieved rate Class A	Achieved rate Class B	Achieved rate Class C
8489 (calculation with whole numbers)	1 out of 24	12 out of 29	13 out of 30
8490 (fractions, decimals, and percentages)	Not given	3 out of 29	15 out of 30
8491 (statistics)	Not given	Not given	4 out of 30

The case study class

One class was studied in detail by Angelika Anderson, Research Associate. She was responsible for setting the research questions for evaluation, carrying out the evaluation, and writing the following portion of this section.

The purpose of the evaluation of this part of the initiative was to answer the following research question:

Are the competencies covered in the Numeracy Project appropriate for improving the numerical understanding of at-risk year 9 students, and do these enable them to gain Unit Standards in numeracy in their first year of high school?

To that end data was collected to show:

- The nature of the numeracy teaching for these students
- The number of students passing relevant Unit Standards during the year
- The attitude of a sample of the year 9 students involved
- Some indication of interactions between teachers and students and student attention in the sample of classes.

Method:

Participants

a) One school which met the following criteria:

- in close proximity to the University researcher
- mathematics teachers with a history of implementing the strategies promoted in the Numeracy Project well.

b) Within this school one teacher, who was an exemplary teacher and happy to collaborate with us in this project.

c) Six students within her class who had returned consent forms that were signed by their parents and assent forms signed by themselves.

Procedure

The selected classroom was visited at three different points in time. The purpose of the initial visit was to meet the teacher and gain information that would aid the development of an observational procedure. To this end both the researcher and the author had a conversation with the teacher and observed one mathematics lesson.

Initial school visit:

The purpose of the teacher interview was to:

- identify the teacher's approach to teaching mathematics with this class, specifically, which Numeracy Project strategies that teacher was using, and why

- identify the teacher's attitude towards the initiative, specifically, the teacher's expectations
- identify the specific objectives of the current series of mathematics lessons and the teacher's current specific learning goals (for the class and for individual students).

The purpose of the classroom observation was to:

- develop operational definitions of student and teacher behaviours associated with the teacher's approach and attitudes, and the teacher's goals and objectives as identified above
- identify student behaviours likely to reflect the degree to which the teaching is appropriate and effective
- assess levels of student attention and engagement
- identify teacher–student interaction patterns likely to reflect the degree to which the teaching is appropriate and effective, and the extent to which the teacher is responsive to individual students' needs

Following this initial visit, student and teacher interview schedules were developed, as was an observational system. These tools were used for data collection during the following two school visits. Approval from the Auckland University Human Participants Ethics Committee, for this part of the study, was sought and gained.

1. Direct observations:

From the initial observations and interviews a structured classroom observation system was devised. The focus was on the student and teacher behaviours and interactions likely to be related to the research question, specifically indicative of:

- appropriateness of the instruction (content and mode of delivery, in relation to requirements of the Unit Standards)
- student engagement and attention
- student acquisition of critical concepts and competence (necessary for attainment of Unit Standards).

To this end each lesson was observed in three ways:
1. Timing of events and structure of the lesson: The timing of events and the structure of the lesson were noted.

2. Structured direct observation of student behaviours: During 20 minutes of seatwork the participating students were observed as under A and B below.

A. – partial interval sampling. These behaviours were coded if they were present for most of an observed interval.

1. Working on task: the student was clearly working and/or involved in the activity as instructed, or engaging in other appropriate behaviours as instructed by the teacher.

2. Working off task: the student was clearly not working or engaged in appropriate activities, but rather quite obviously engaged in alternative behaviour (doodling, chatting, day-dreaming).
3. Working: the student was clearly working, involved, or actively engaged in the tasks set for the lesson. Working was also scored as "working on task".
4. Waiting: the student was not actively engaged in a task, but rather waiting for further instruction, or waiting to have work checked. Waiting was also scored as on task.
5. Hand up: the student had his or her hand up and waiting for a turn to speak, to answer or ask a question. This was also coded as on task behaviour.
6. Working collaboratively: the student was interacting with a peer or a group of peers in order to accomplish a task, or cross-check work. This was also coded as on task behaviour.

B. – incidence recording. Every occurrence of the following behaviours was noted if it occurred during the time that the student was observed.

1. Question asking: the student asked the teacher a question, publicly.
2. Correct answer: the student answered one of the teacher's questions correctly, or used a new term correctly.
3. Incorrect answer: the student answered one of the teacher's questions incorrectly.

See Appendix G for a sample observation sheet.

3. Monitoring of teacher behaviours during whole class instruction:
   1. Clear instructions / presentation of task
   2. Feedback
   3. Maintaining order.

During the second visit some slight changes were made to the observation protocol. Instead of using paper and pencil recording methods, an electronic recording software programme (Spectator Go!) was used. The observational categories and behavioural specifications were the same.

2. Teacher interviews:

Using the interview format that was developed (see appendix G) the teacher was interviewed, before and immediately after the lesson that was observed. The main purpose of the interviews was to find out before the lesson what the teacher's plans were for the lesson, and after the lesson to check to what extent the lesson had gone as planned, and if changes were made and why.

3. Student interviews:

The students who were observed in each lesson were also interviewed briefly immediately after that lesson, as a group. The main purpose of the student interview was to assess the students' attitudes to mathematics in general, and more specifically to the observed lesson.

4. Additional information:

For each lesson observed the teacher provided other pertinent information:
1. Copies of worksheets used during the lesson
2. Samples of recent work / tests taken by the participating students.

In addition the researcher copied the instructions from the whiteboard.

5. Monitoring achievement:

The number and level of Unit Standards (mathematics) gained by this group of students was monitored.

Results

Classroom activities

First observation, 5.8.2003;

9:50–10:50 a.m.

Topic: Decimal places
Format:
1. Checking homework: This took about ten minutes. The teacher called the students up one at a time and provided them each with feedback about the correctness of the work. She praised for completion. There was a system whereby each student gets a stamp each time homework is completed (even if incorrect) and signed by parent. Five stickers earn a reward. During the homework checking time the rest of the class engaged in revision exercises which were on the board.

2. Whole class instruction (from 10:06–10:10): A brief revision of decimals using sample problems on the board. Students put their hands up to respond. Students were asked not to put their hands up any more once they had answered a question. Correct answers were always praised. This section lasted about four minutes. During this time the teacher presented at least ten questions. These were followed by prompts, elaborations, or rephrasing of the question if answers were not forthcoming, or students answered incorrectly. There were at least five occurrences of verbal praise. During this time one of the focus students, Garry put his hand up four times and Kylie and Jim (pseudonyms) each used one correct term.

3. Task presentation: Students were instructed to help themselves to three worksheets. The instructions (including that there were worksheets to collect) were on the board: The instructions are given below as they appeared on the board:

- Take three worksheets from the pack – they should all be different (see Appendix G for examples).

- Sheet 1:
  – Cut into prices, carefully and arrange biggest to smallest
  – Get your partner to check you are correct.
  – Ask teacher to check.
- Sheet 2: Bins / decimals
  – Cut and paste under the correct bin
  Careful! How do you compare 0.4 and 0.16?
  – Finished-get your partner to check
- Sheet 3: Matching fractions to %
  – Number cline.
  – Cut and paste.

The teacher went over the main points, checked that all students had the correct material, and told them where to find scissors and glue if needed.

4. Seatwork: From 10:10 to the end of the period, 10:50.

Students sat in groups and there was considerable interaction between members of the group and across groups. During the seatwork the teacher monitored, answered questions, checked results, and organised the collection of paper offcuts. She further elaborated the instructions at times. She looked for students who appeared to need help. Mostly she visited those groups where students had their hand up to check the work. In checking she would provide feedback – praise for correct work, otherwise prompts or questions to help students work out the answer themselves. Some sample comments were:

> "Show me the biggest one."
> "Get out your fraction pieces and turn it back into tens."
> "Well, so far he is right – he hasn't made any mistakes yet, he's very smart."
> "Now, nope you've got one wrong there [pause] nearer to [pause] ten isn't it?"
> "Good – put them away on the back of your book."
> "Did you read the board first? What does it say on the board? Everyone of these goes somewhere on here [pointing to the number cline]."
> "Is this one closer to 0 or 1?"
> "Isn't 4/10 closer to 5/10?"
> "Hey – stop working you lot and pack up."

Structured observation of four students during the seatwork, 10:10–10:30 as described above. On this occasion four of the six students who had consented to participate were present. These were the students observed.

There were 23 x 10 second intervals observed per student. During three intervals for all (2 minutes), the teacher was giving instructions. The rest of the time was all seatwork. Attending to instruction was marked only when during a period of instruction the student attended. This also counted as work on-task. The combined category of "Hand up / waiting" was not marked as working on task if it occurred for an entire interval.

Table 7.4. Percentage of time units in which students were observed to be carrying out different activities on 5.8.2003

Name	Working on task %	Working collaborating	Working off task	Attending to instruction	Hand up and waiting
Garry*	100	34.8	None	-	28.7
Kylie	100	34.8	None	4.3	-
Therese	95.7	60.9	None	-	8.7
Jim	100	4.3	None	-	21.7

* All names are pseudonyms.

<u>Second observation: 20.10.2003</u>

2: 30–3:20 p.m.

The format of the lesson was the same as the first lesson observed: homework was checked, during which time the rest of the class were engaged in revision exercises. This was followed by whole class instruction, and finally seatwork. In this lesson the teacher needed to give fewer instructions about procedures because the class clearly knew the rules and all ran smoothly.

Lesson: 2:30–2:36: Homework check
2:36–2:47 Whole class instruction
2:47–end (3:20) Seatwork in groups as per worksheets

On the board: tonne, kg, g, L, ml, km, m, cm, mm, C^o

Do now:
Choose a unit from the box to fill the gaps.
a) The length of my pencil case is 22 ____.
b) The temperature dropped by 2 ____.
etc.

Today's work:
Objectives
Do now: homework check
Mark geometrical shapes

<u>Worksheet 1</u>
From the bag labelled 1 take a sheet.
Aim: to accurately label the diagrams. Use the sheets you already have to help. N.B. don't just label the shape as "triangle" – write an <u>accurate</u> description. Use your ruler to check your measurements. Use your notes from last week to help.

Worksheet 2
There are 2 worksheets.
Aim: to construct different Polyhedra using the sheets.

Each sheet contains 2 pictures. You have the task of matching the correct net to each model of a solid.

I expect that you will need 2 days to finish this.
Forgotten your card?
Go to Sheet 4
Material and glue

Vocabulary
Polyhedron, Polyhedra (plural)
Nets, faces, model

During this lesson only three students were observed (one participating student was absent) during seatwork as before.

Direct observations this time were carried out using "Spectator Go!" software and the data look a little different. The total observation time was 33 minutes. All subjects were observed on a rotational basis for approximately ten seconds each time.

Table 7.5. Percentage of time units in which students were observed to be carrying out different activities on 20.10.2003

Name	Working on task	Working collaboratively	Working off task	Teacher feedback	Attending to instructions	Hand up and waiting
Garry	89 %	34.5 %	None	3x feedback 1x praise	17%	6.4%
Kylie	87 %	7.9 %	None	1x feedback	3.6 %	8.8 %
(Therese absent)						
Jim	90 %	28.5 %	None	1x feedback	-	4.2 %

Working on task could co-occur with working collaboratively and attending to instructions.

All three subjects were observed concurrently, switching from one to another about every ten seconds. If the duration behaviour ceased during an observation spell, the recording device was turned off, and only turned on again if it occurred once again during the next observation spell. Teacher feedback was an event that was noted every time it occurred with the subject during the spell that person was observed.

Results of interviews

The students were interviewed as a group on two occasions. Asked what they thought or felt about mathematics, initially they all said that they liked it, and on the second occasion added that they were more confident than they used to be. Asked if their views had changed since the first interview, all reported that they had improved in maths. Kylie said that she was more confident now, and not so shy. On the first interview they said that they really appreciated the new maths lessons. They felt that they were getting a lot of help with problem solving. On the second occasion the consensus was that they all loved the classes, they were excellent. On both occasions they were asked how their maths lessons differed. Initially they said that previously they just used to get worksheets with little explanation. Most found it too easy although one had found it too hard. On the second occasion one said that he was learning new ways of getting the answer. Another commented that doing things step-by-step meant that even hard things became easy, and a third commented that it was more fun now. Asked to comment on their progress, three initially commented that they were beginning to enjoy maths while the fourth said that he never had a problem with maths in the first place.

Results of Unit Standards tests

This class had the highest proportion of students passing Unit Standards tests, as indicated in Table 7.3. The preparation for the last Unit Standard in statistics was done by a trainee teacher. The assessment, chosen from the New Zealand Association of Mathematics Teachers' website (http://www.nzamt.org.nz), was seen as difficult.

Numeracy assessment gains for this class

Results were returned for 24 students in this class. Table 7.6 shows the percentage of students who gained from one to three stages. The decimal scale was not given to any students initially, but to all on the final assessment.

Table 7.6 Gain in stages made by students in the case study class, 2003

Scale	Percentage making no gain	Percentage gaining	Main stage(s) of final assessment
Additive strategies	46%	54%	5: Early additive part-whole
Multiplicative strategies	29%	67%	5: Early additive part-whole and 6: Early multiplicative part-whole
Proportional strategies	38%	63%	6: Early multiplicative part-whole
Fractions	12%	88%	6: Coordinates numerator and denominator
Decimals (none assessed initially)			6: Orders decimals
Base 10 Grouping	12%	88%	5: 10s in 100s

While these students are still not doing as well as would be preferred for secondary school students, they appear to have a fairly good understanding of the logic of decimals and fractions and to be able to use their mathematics table flexibly for multiplicative tasks.

Their improvement was not formally assessed for measurement, geometry, statistics, or algebra. Little formal testing was done in class, with most assessment being oral. However, the teacher believed that students were making similar progress in these fields.

Teacher's evaluation

Towards the end of the year the teacher reported that her students now had a positive attitude toward themselves as students of mathematics. If they sat and did not pass a Unit Standards test, their belief was that although they could not do a type of problem now, they would be able to in the future.

She gave her class and the class of "truants and waggers" a self-evaluation sheet. As she had expected, her class responded positively. However, she was surprised to discover that students in the class of "truants and waggers" were now confident about their mathematics ability. Their perception was that their teachers also thought that they had achieved well. Talking about her own class, she said:

> "I was very happy to at least have promoted a positive attitude to the subject … I was after all trying to create basics that could be built on."

Her report was backed up by statements from the students themselves, as given in the section above on the interviews.

She reported that despite this positive attitude her class continued to be daunted by the language of the versions of the Unit Standards tests that she took from the nzamt website. Formal vocabulary was always a part of her lessons, but that did not solve all of these students' difficulties with the language of the selected tests. The Unit Standard that they achieved best in tested skills, not application and understanding.

Conclusions

The results for the whole cohort of these low achieving year 9 students demonstrates that between 50% and 60% increased their level on the Numeracy Project. This is about the same percentage advancing as the students from a full range of classes in previous years (see Irwin & Niederer 2002). In itself, this indicates that the project was worthwhile for these students.

Most students moved to the early part-whole stage in addition, a stage that requires them to demonstrate one method of mentally breaking up numbers to make the problem easier to calculate in their heads. They reached a higher level of achievement in multiplication. This seems to be the appropriate strategy to focus upon for these students. Few students reached the two top stages for proportional reasoning. There might have been more advancement on this scale if the teaching of proportional reasoning and fractions was integrated.

The results for fractions show improvement but only 35% of the students moved beyond the level of fractions that requires simple visualisation or memory of rules. It would be preferable to see more demonstrate the underlying understanding necessary for the top levels on this scale.

The majority of students had an adequate understanding of whole numbers and place value relationships by the end of the year. They could give the number of 10s, 100s, and sometimes 1000s in a whole number.

The records for decimals suffer from lack of information as far too few students were asked these questions. Even among those who were asked, few showed a level of competence that indicated understanding rather than the memory of rules. More about this scale is given in Section 8.

The case study of one class and school showed several positive aspects. These were:

- A full curriculum based on the principles of the Numeracy Project that included the pedagogy of starting at the level of the students' knowledge, working within a framework of advancing concepts, and using appropriate materials for teaching. It had good service from a facilitator who went well beyond the standard project in helping teachers apply the pedagogy of the project to all areas of the curriculum.

- A teacher who was committed to the values of the Numeracy Project and who was skilled in both classroom management and teaching. Her attributes included immediate feedback for the students, organisation, and a positive attitude towards her students' ability to learn mathematics. She included group teaching within a whole class setting, by starting everyone off together and then giving help to different groups as needed. Students were encouraged both to help one another within their group and to ask for help from other groups. In response to her leadership, monitored students were on task for all of the two periods observed. This is unusual for low ability classes in secondary schools like this one.

- Students who responded to being given appropriate materials, praise, and success with the belief that they could do mathematics and by saying that they enjoyed it. Again, this is an unusual response from a low achieving year 9 mathematics class.

- A good proportion of students who were able to demonstrate to themselves and others that they could pass Unit Standards tests and others who knew that if they didn't know something now, they would learn it later. They were not afraid to do mathematics.

8. Understanding of decimals and fractions

In our university course for graduates who want to be teachers, each student takes a competency test in mathematics. Before they can graduate they must demonstrate that they have an understanding of all areas of this test. Every year, the main areas of deficiency for these students with university degrees are an understanding of fractions and of decimals. Some other topics have been forgotten and are easily relearned, but these students appear to have never understood fractions and decimals. If these university graduates display poor understanding of these fields, we can assume that a sizable proportion of the adult population has a poor understanding of these topics.

Years 7 and 8 are the last two years of school in which attention is usually given to these topics. Therefore it is essential that schools do a better job of enabling students to understand them.

Year 9 results are not discussed here because the sample was abnormal, but these are reported in Section 7.

The current practice of using Form B of the Numeracy Assessment for most of these students means that New Zealand educators have an incomplete knowledge of what students know and don't know. The top two levels of fraction knowledge and proportional reasoning (which is largely based on fractional thinking) were not given to most students and the majority of students were not given the test of decimals. Only students who were seen as more capable were given Form C. However, even these more capable students were judged to have poor knowledge in these topics.

The Numeracy Assessment form has been changed for 2004, but it is still the case that the top levels are only on Form C and are unlikely to be given to intermediate school students unless there is a change of policy. The effect of this new assessment form on the assessment of decimals has yet to be seen.

Understanding of fractions

Figure 8.1 gives the stages that students were judged to be at in their understanding of fractions at the end of 2003.

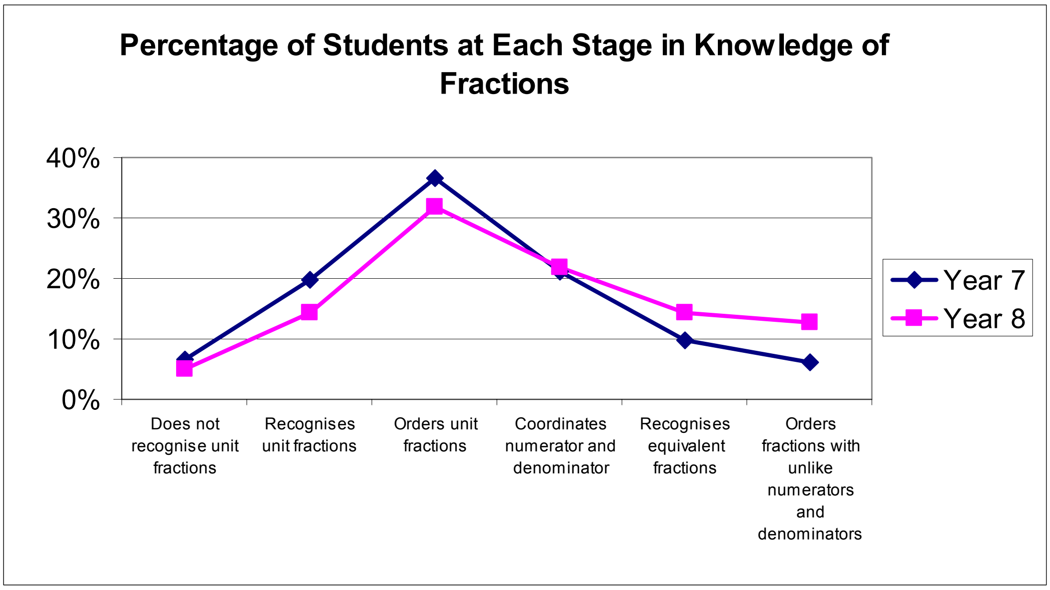

Figure 8.1. Percentage of students in years 7 and 8 judged to be at each stage in their understanding of fractions at the end of 2003

This figure shows the highest percentage of students in both years 7 and 8 to be at the stage of ordering unit fractions. This means that they can order 1/3, 1/4, 1/5, 1/2, and 1/6 (presented to them in this order). This requires either that they associate the fraction with the relative portion of the same diagram or that they have learned a rule about ordering: that the fraction gets smaller as the bottom number increases. In my view a true understanding of fractions starts with the next stage on the scale, that students can explain why 8/6 is the same as $1\frac{2}{6}$, or that 2/3 means that the whole is divided up into three parts and the number refers to two of these parts. By this criterion, 37% of the year 7 students and 49% of the year 8 students demonstrate what I would consider an understanding of the nature of fractions.

While it is pleasing that a higher proportion of year 8 students than year 7 students show this understanding, it means that 51% of the year 8 students still do not understand fractions.

Although students and teachers may not see it as so, the understanding that underlies fractions is similar to that involved in proportional reasoning. Therefore one might expect the proportion of students at various stages on the final assessment to be similar for both topics. On the assessment sheet the stages are given the same stage numbers, although the names that describe the stages are appropriately different. Figure 8.2 gives the distribution for the year 7 and 8 students on the stages in proportional reasoning.

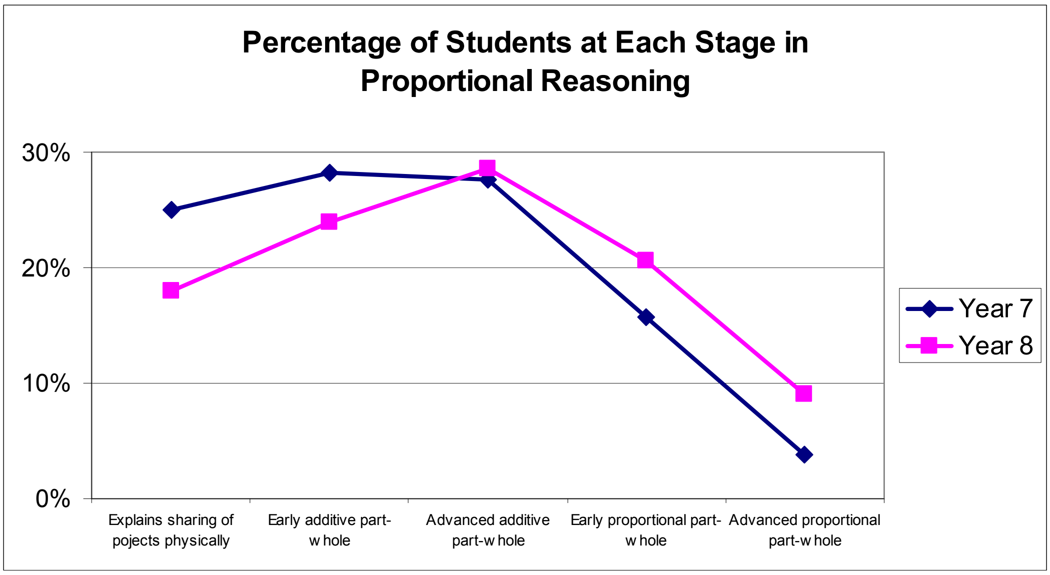

Figure 8.2. Percentage of year 7 and year 8 students at each stage on proportional reasoning tasks at the end of 2003

On the proportional reasoning task 20% of the year 7 students and 30% of the year 8 students demonstrated proportional reasoning (the top two stages).

Note that there are six points on Figure 8.1 for fractions and five points on Figure 8.2. The initial stage on Figure 8.1 indicates no understanding, while this stage is not represented on Figure 8.2. The figures are easiest to compare by counting back, as Stage 8 is the final stage on each figure.

The largest proportion of students was at stage 5 for fraction knowledge and at stage 6 in proportional reasoning.

This suggests that these topics are not being taught or understood as requiring similar concepts, or that the stages are of different difficulty. One important difference between the tasks is that all of the proportional reasoning tasks expect students to work with at least one whole number, while the fraction knowledge test deals mostly with recognizing, interpreting, and ordering fractions. For example, the proportional reasoning assessment asks students to know what 3/4 of 28 is, while the fraction knowledge test asks students about ordering fractional numbers, for example "How do you know that 1/4 is less than 1/3?"

Understanding of decimals

There is information on year 7 and 8 students' understanding of decimals from two sources: results from the students who were given the decimal test (less than half of the students) and from the final item on each section of the generalisation test given to the year 8 students. Figure 8.3 gives the percentage of the year 7 and 8 students who were assessed on Form C at the end of 2003. Only 38% of the year 7 students and 48% of the year 8 students were given this assessment.

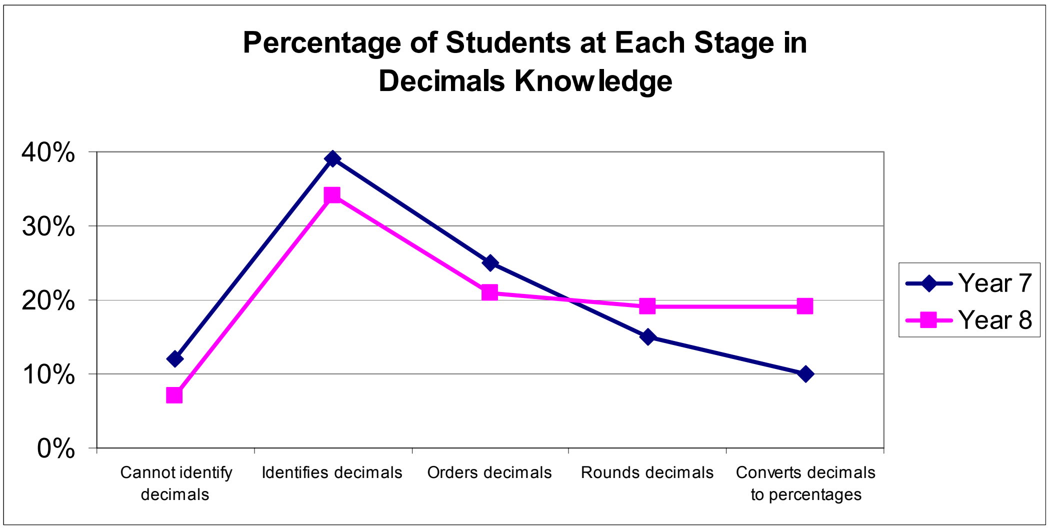

Figure 8.3. Percentage of year 7 and 8 students at different stages in decimal knowledge at the end of 2003

I have some concern about the items in this assessment in that all of the concepts assessed can be taught by rules which students are likely to forget. For example, identification and ordering of decimals can be done by learning the names of the columns, and they can be ordered like whole numbers if zeros are added after the last numeral. Both rounding and converting to percentages can be done by learning the rules for these exercises. In my experience, both children and adults forget these rules unless they have an underlying understanding of the division inherent in dividing a unit into 10, 100, or 1000 parts. There are other assessments of decimals that require students to demonstrate understanding in a variety of ways. In my view the best of these is the Chelsea Diagnostic Test (Hart et al., 1985). The writers of the Numeracy Assessment might like to look at the different ways in which the Chelsea test assesses understanding and see which of these could be adapted to the Numeracy Project. Such ways might involve asking "why" questions for all stages.

Figure 8.3 shows that only 10% of the more competent 38% of year 7 students and 17% of the more competent 48% of year 8 students could translate decimals to percentages. This is perhaps the most useful task on this scale in adult life.

Because even higher percentages of students (77% of year 7 and 63% of year 8) were not given this test at the start of the year, teachers do not have a way of knowing about their students' deficiencies.

While initial assessment is essential to show teachers whether decimal concepts need to be taught, the following data suggest that they can be taught and reinforced along with additive and multiplicative strategies. Section 6 of this report gave the proportion of students, both in the Numeracy Project and not in the project, who were able to use a strategy for adding, subtracting, multiplying, and dividing whole numbers and could also use that strategy on decimals. This test required understanding of decimals to one decimal place, and could not have been done on the basis of memorised rules. Figure 8.4 repeats the data given in Figure 6.5.

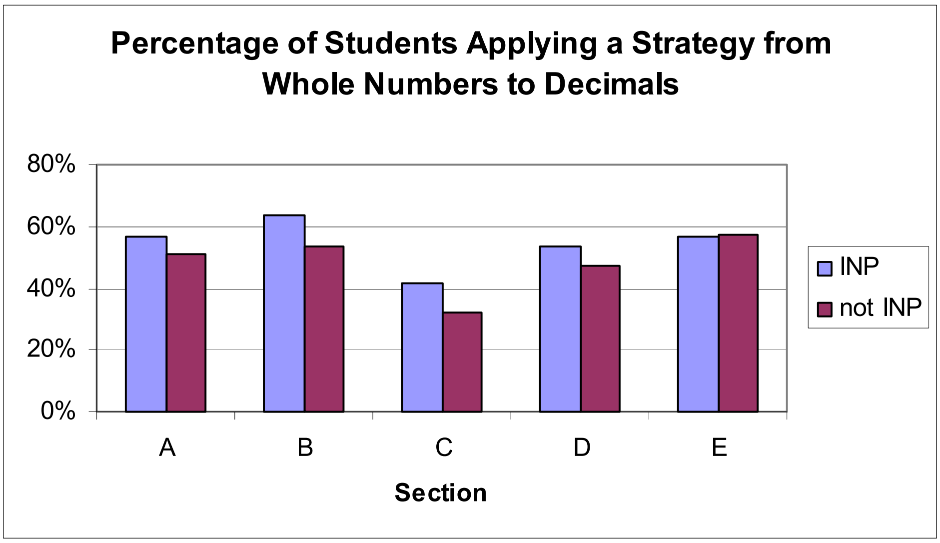

Figure 8.4. Percentage of year 8 students who used a strategy on at least one whole number problem and could also use that strategy on a number that included a decimal

The decimal fractions in this test were all tenths. The test required them to realise that adding or subtracting a few tenths from a number to make it a whole number, and applying the appropriate compensation, made the problem easier to do mentally. For example, they needed to know that when 0.2 was added to 35.8 it became 36 and when 0.5 was doubled it equalled 1.

The only prerequisite experience and knowledge of decimals resulting from a division that students would need before they could use tenths in the strategy strands would be physical division of a unit, using contexts such as chocolate, cutting carrots, decimats, or divisions of a number line (currently in the numeracy equipment). The decimal pipes would be more useful for an initial introduction if students had to physically cut the unit pipe into tenths. A study by Irwin (1999) showed that the best teaching of students at the early stages of learning about decimals was limited to learning about tenths only, with a few rather than many physical models. Further divisions could be taught as students showed competence with tenths, and could accompany more complex strategy lessons.

Another useful way for teaching decimals is described in Moss and Case (1999) which starts with percentages and moves from that to fractions and decimals.

Conclusions

The performance of year 7 and 8 students on fractions and decimals is well below what would be wished. Integration of fractions with proportional reasoning would aid understanding of those topics.

I am particularly concerned about decimals, as most students were not even assessed on this scale in 2003, and those who were assessed did not do well. Teachers need information on their students' strengths and weaknesses in this area so that they can address them. Decimals need to be taught using the same principles used throughout the Numeracy Project: using materials and using imaging before students are asked to work with number properties. One of the sources of difficulty with decimals is that many students are so proficient with whole numbers when decimals are introduced that they do not see the need to go back through the stages of working with materials and imaging. This issue needs to be emphasised. In my view decimals should be introduced at about year 5, in the context of fractions.

I also am concerned that all of the assessment items involving decimals that appear in the assessment can be learned by following rules without underlying understanding. I suggest that the assessment of decimals needs to be rethought, and separated from an assessment of understanding of the place value of whole numbers. These are logically connected for competent mathematicians who understand the number system, but this logic is not apparent to learners. Learners tend to think of place value initially as a way of recording counting and then as multiplication, as used for moving from 10s to 100s, etc. This is different to them from the division necessary for understanding decimal fractions.

A positive finding from the evaluation of year 8 students in 2003 is that many of the students who could understand various ways of adjusting whole numbers to make them easier to operate with could also do this with decimals. Once teachers have ensured that students have a basic knowledge of tenths then they can include problems that involve tenths and other decimal divisions in their discussion of part-whole strategies. This would assure understanding of decimals rather than just the knowledge of forgettable rules.

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Appendices

Appendix A:	Assessment forms B and C used in judging students' level of competence
Appendix B:	Percentage of students in schools in each decile range at different stages in additive and multiplicative thinking at the end of 2001, 2002, and 2003
Appendix C:	Number (and percentage) of students from full primary schools on different deciles in 2003
Appendix D:	asTTle test used in assessment of year 7 students
Appendix E:	Test of generalisation in part-whole relationships
Appendix F:	Number and percentage of year 8 students in each school correct on each task in the test of generalisation
Appendix G:	Sample of materials used in one year 9 class

Appendix A: Assessment Forms B and C used in judging students' level of competence

NumPA Form B Individual Assessment Sheet

* denotes cards needed # question booklet needed

Child's Name: Date: Teacher:

Operational Strategy Questions

Questions	Stage 4 Advanced Counting		Stage 5 Early Additive Part-Whole	
Addition and Subtraction (Strategy Windows) # (2) I have 8 counters under here and I am putting some more counters under here. Altogether there are 13 counters now. How many are under here? (3) You have 37 lollies and you eat 9 of them. How many have you got left? (4) There are 53 people on the bus. 26 people get off. How many people are left on the bus?				
	Comments			

Questions	Stage 2-3 Count From One Counts all the objects		Stage 4 Advanced Counting Uses skip counting		Stage 5 Early Additive Part-Whole Uses repeated addition		Stage 6 Advanced Additive Part-Whole Derives multiplication facts	
Multiplication and Division # (1) Here is a forest of trees. There are 5 trees in each row and there are 8 rows. How many trees are there in the forest altogether? If I planted 15 more trees, how many rows of 5 would I have then? (2) What is $3 \times 20 = ?$ If $3 \times 20 = 60$ what is $3 \times 18 = ?$ (3) What is $5 \times 8 = ?$ If $5 \times 8 = 40$ what is $5 \times 16 = ?$								
	Comments							

Questions	Stage 1 Unequal sharing of objects		Stage 2 - 4 Equal Sharing of objects physically or by imaging		Stage 5 Early Additive Part-Whole Using addition facts		Stage 6 Advanced Additive Part-Whole Using multiplication and division facts	
Proportions and Ratios # (4) Which of these cakes have been cut into thirds? Here are twelve jellybeans to spread out evenly on top of the cake. You eat one third of the cake. How many jellybeans do you get? (5) Find $\frac{3}{4}$ of 28.								
	Comments							

Knowledge Questions

Forward Number Word Sequence (FNWS) *

(6) Start counting from 10. I will tell you when to stop (*at 32*).

For each number I show you, tell me the number that comes just after it, the number that is one more. Like, if I show you 4 you say 5.

(7) 12 (8) 17 (9) 29

(10) 99 (11) 209

(12) 999 (13) 3049 (14) 989 999

Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
FNWS to 10	FNWS to 20	FNWS to 100	FNWS to 1000	FNWS to 1000 000

Comments

Backward Number Word Sequence (BNWS) *

(15) Start counting backwards from 23. I will tell you when to stop (*at 10*).

For each number I show you, read the number, and tell me the number that comes just before it, that is the number that is one less. For example, if I show you 4 you say 3.

(16) 13 (17) 19 (18) 30

(19) 100 (20) 680

(21) 900 (22) 2 400 (23) 603 000

Stage 3	Stage 3	Stage 4	Stage 5	Stage 6
BNWS to 10	BNWS to 20	BNWS to 100	BNWS to 1000	BNWS to 1000 000

Comments

Fractional Numbers # *

(24) Here are some fractions ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$). Say each fraction as I show it.

(25) Put these fractions *(from question 24)* in order from smallest over here to largest over here. *(If correct ask)* Why do you think one-quarter is less than one-third?

(26) Which of these numbers is the same as eight-sixths *(pointing to* $\frac{8}{6}$ *)*? (Show the numbers, $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{2}{3}$, 1, $\frac{2}{14}$, in the test booklet). Explain how you know this.

Stage 2 - 3	Stage 4	Stage 5	Stage 6
Unit fractions not recognised	Unit fractions recognised	Ordered unit fractions	Co-ordinated numerators and denominators

Comments

Grouping (including place value) #

Tell me the answer to?

(27) 5 + 2 (28) 5 + 4 (29) 6 and how many more makes 10?

(30) 10 + 4 (31) 7 + 10 (32) 6 tens

(33) How many tens in eighty?

At the Ten Bank they only have ten-dollar notes. How many ten-dollar notes would they need to make these amounts of money?

(34) $230 (35) $6 074 (36) $78 900

How many one-hundred-dollar notes would you need to make these amounts?

(37) $7 900 (38) $151 000

Stage 0-1	Stage 2-3	Stage 4	Stage 5	Stage 6
Non-grouping with fives and within ten	With fives and within ten	With tens	Tens in 100	Tens and hundreds in whole numbers

Comments

NumPA Form C Individual Assessment Sheet

* denotes cards needed # question booklet needed

Child's Name: ________ Date: ________ Teacher: ________

Operational Strategy Questions

Addition and Subtraction (Strategy Windows)

(5) There are 53 people on the bus. 26 people get off. How many people are left on the bus?
(6) Sandra has 394 stamps. She gets 79 stamps from her brother. How many stamps does she have then?
(7) Hone has $403 in his bank account. He takes out $97 to buy a new skateboard. How much money is left in his account?

Stage 6	Advanced Additive Part-Whole Uses at least two different mental part-whole strategies

Comments

Multiplication and Division

(1) Here is a forest of trees. There are 5 trees in each row and there are 8 rows. How many trees are there in the forest altogether?
If I planted 15 more trees how many rows of 5 would I have then?
(2) What is $3 \times 20 = ?$ If $3 \times 20 = 60$ what is $3 \times 18 = ?$
(3) What is $5 \times 8 = ?$ If $5 \times 8 = 40$ what is $5 \times 16 = ?$
(4) There are 24 muffins in each basket. How many muffins are there altogether?
(5) At the car factory they need 4 wheels to make each car. How many cars can they make with 72 wheels?

Stage 4 Advanced Counting Uses skip counting	**Stage 5** Early Additive Part-Whole Uses repeated addition	**Stage 6** Advanced Additive Part-Whole Derives multiplication facts	**Stage 7** Advanced Multiplicative Part-Whole Uses at least two different mental strategies

Comments

Proportions and Ratios

(6) Which of these cakes have been cut into thirds? Here are twelve jellybeans to spread out evenly on top of the cake. You eat one third of the cake. How many jellybeans do you get?
(7) Find $\frac{3}{4}$ of 28.
(8) $\frac{2}{3}$ of a number is 12. What is the number?
(9) It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies?
(10) There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class are boys?

Stage 2 - 4 Equal Sharing of objects physically or by imaging	**Stage 5** Early Additive Part-Whole Using addition facts	**Stage 6** Advanced Additive Part-Whole Using addition with multiplication and division facts	**Stage 7** Advanced Multiplicative Part-Whole Finds fractions of numbers using multiplication and division	**Stage 8** Advanced Proportional Part-Whole Uses at least two different mental strategies

Comments

Knowledge Questions

Forward and Backward Whole Number Word Sequence *

For each number I show you, tell me the number that comes just after it, the number that is one more. Also tell me the number that comes just before it, the number that is one less.

(11) 499 (12) 840 (13) 2400 (14) 3 049
(15) 603 000 (16) 989 999

Stage 4	Stage 5	Stage 6
FNWS/BNWS to 100	FNWS/BNWS to 1000	FNWS/BNWS to 1000 000

Comments

Fractional Numbers # *

(17) Here are some fractions ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{6}$). Say each fraction as I show it.

(18) Put these fractions *(from question 24)* in order from smallest over here to largest over here. *(If correct ask)* Why do you think one-quarter is less than one-third?

(19) Which of these numbers is the same as eight-sixths *(pointing to* $\frac{8}{6}$ *)*? (Show the numbers, $\frac{6}{8}$, $1\frac{2}{6}$, $1\frac{2}{3}$, 1, $\frac{2}{14}$, in the test booklet). Explain how you know this.

(20) Here are some fractions ($\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{6}{9}$, $\frac{7}{16}$). Put them in order from smallest over here to largest over here.

Stage 2 - 3	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Unit fractions not recognised	Unit fractions recognised	Ordered unit fractions	Co-ordinated numerators and denominators	Equivalent fractions recognised	Ordered fractions with unlike denominators and numerators

Comments

Decimals and Percentages # *

Say each decimal as I show it to you.

(21) 0.8 (22) 0.39 (23) 0.478

(24) Put these decimals (0.8, 0.39, 0.478) in order from smallest over here to largest over here.

(25) Round 7.649 to the nearest tenth. (26) Round 2.38501 to the nearest hundredth.

(27) Round 234.99 to the nearest ten. (28) What is 1.25 as a percentage?

(29) Name 37.5% as a decimal.

Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Emergent decimal identification	Decimal identification	Ordered decimals	Rounded decimals	Decimal conversions

Comments

Grouping and Place Value

At a bank they only have ten-dollar notes and one-dollar coins. How many ten-dollar notes would they need to make these amounts of money?

(30) \$230 (31) \$6 070 (32) \$78 900

How many one-hundred-dollar notes would you need to make these amounts?

(33) \$78 900 (34) \$151 000

(35) How many thousands are in all of this number (408 000)?

(36) How many tenths are in all of this number? *(4.67)*

(37) How many hundredths are in all of this number? *(2.592)*

Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
With Tens	Tens in 100	Tens and hundreds in whole numbers	Tens, hundreds, thousands in whole numbers	Tenths, hundredths, thousandths in decimals

Comments

Appendix B: Percentage of students in schools in each decile range at different stages in additive and multiplicative thinking at the end of 2001, 2002, and 2003

Additive Strategies

Year 7

Low decile

	2001	2002	2003
Counting stages	19%	22%	20%
Early additive part-whole	51%	50%	47%
Advanced additive part-whole	30%	28%	33%

Middle decile

	2001	2002	2003
Counting stages	16%	15%	16%
Early additive part-whole	45%	45%	42%
Advanced additive part-whole	39%	40%	42%

High decile

	2001	2002	2003
Counting stages	6%	7%	9%
Early additive part-whole	30%	33%	37%
Advanced additive part-whole	64%	60%	55%

Year 8

Low decile

	2001	2002	2003
Counting stages	14%	15%	14%
Early additive part-whole	46%	44%	40%
Advanced additive part-whole	40%	41%	46%

Middle decile

	2001	2002	2003
Counting stages	6%	9%	10%
Early additive part-whole	38%	36%	39%
Advanced additive part-whole	55%	54%	51%

High decile

	2001	2002	2003
Counting stages	7%	3%	3%
Early additive part-whole	29%	26%	26%
Advanced additive part-whole	65%	71%	71%

Multiplicative Stages

Year 7

Low decile

	2001	2002	2003
Counting stages	24%	24%	21%
Early additive part-whole	26%	32%	27%
Early multiplicative part-whole	35%	29%	37%
Advanced multiplicative part-whole	14%	14%	15%

Middle decile

	2001	2002	2003
Counting stages	9%	17%	14%
Early additive part-whole	23%	25%	23%
Early multiplicative part-whole	38%	34%	41%
Advanced multiplicative part-whole	30%	24%	23%

High decile

	2001	2002	2003
Counting stages	5%	8%	1%
Early additive part-whole	16%	19%	18%
Early multiplicative part-whole	32%	35%	45%
Advanced multiplicative part-whole	47%	39%	36%

Year 8

Low decile	2001	2002	2003
Counting stages	20%	16%	14%
Early additive part-whole	27%	26%	24%
Early multiplicative part-whole	33%	33%	35%
Advanced multiplicative part-whole	20%	25%	26%

Middle decile

	2001	2002	2003
Counting stages	6%	10%	8%
Early additive part-whole	20%	18%	19%
Early multiplicative part-whole	38%	33%	40%
Advanced multiplicative part-whole	36%	38%	33%

High decile

	2001	2002	2003
Counting stages	2%	3%	4%
Early additive part-whole	14%	14%	10%
Early multiplicative part-whole	36%	33%	36%
Advanced multiplicative part-whole	48%	49%	49%

Appendix C: Number (and percentage) of students from full primary schools on different deciles in 2003

	Full Primary Schools			Intermediate Schools		
	Year 7	Year 8	Total	Year 7	Year 8	Total
Low decile	1284 (50%)	1098 (47%)	2382 (49%)	1918 (50%)	1825 (52%)	3743 (51%)
Medium decile	770 (30%)	721 (31%)	1491 (31%)	1788 (47%)	1540 (44%)	3328 (45%)
High decile	492 (19%)	497 (21%)	989 (20%)	135 (4%)	140 (4%)	275 (4%)

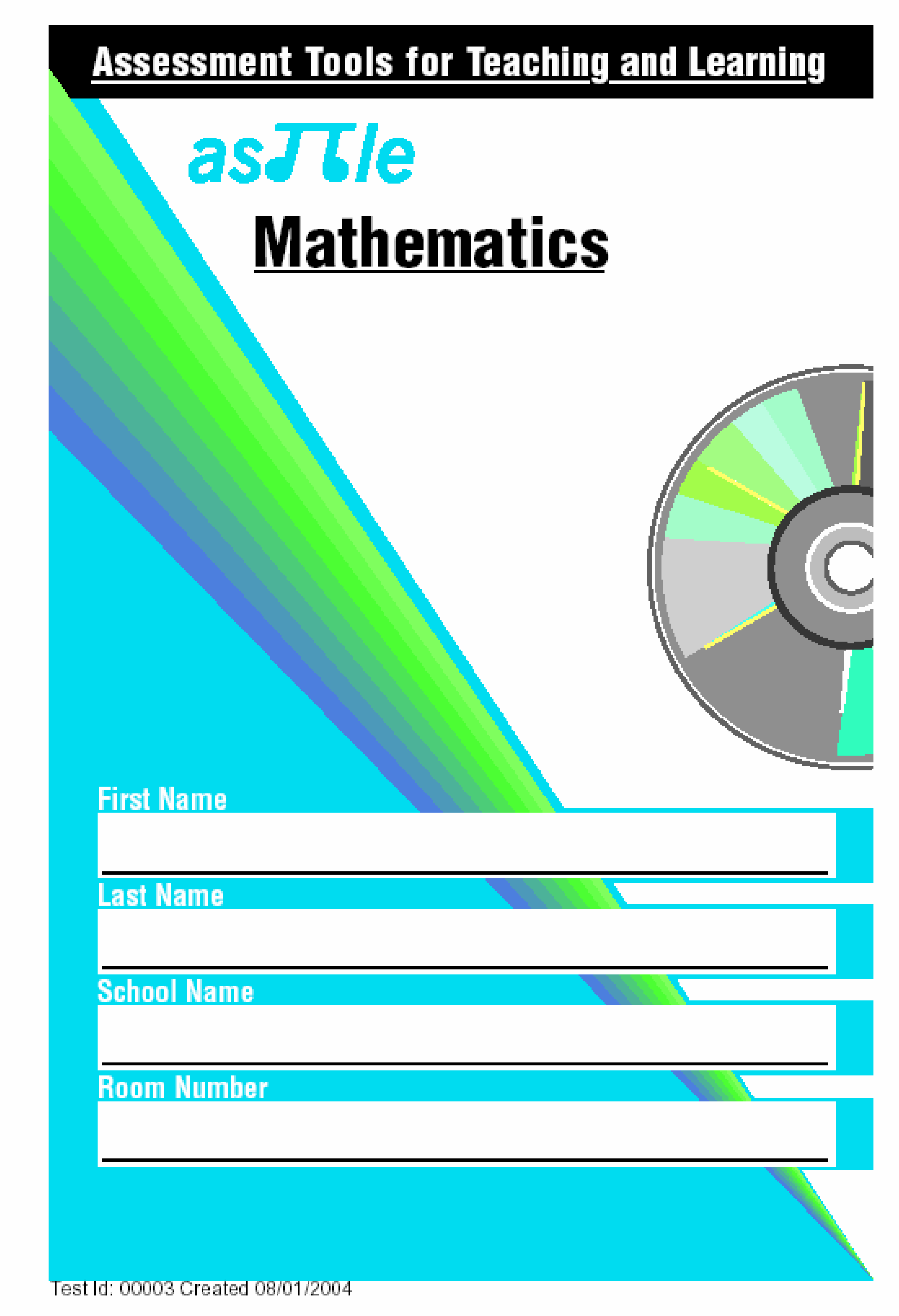
Percentage of year 8 students in the first and second year of the project at different stages for multiplicative problems at the end of the year

Decile 2	1st year N=104	2nd year N=114
Counting	14%	20%
Early additive part-whole	27%	16%
Early multiplicative part-whole	33%	40%
Advanced multiplicative part-whole	26%	24%

Decile 3	1st year N=234	2nd year N=166
Counting	8%	13%
Early additive part-whole	26%	28%
Early multiplicative part-whole	33%	33%
Advanced multiplicative part-whole	34%	26%

Decile 5	1st year N=573	2nd year N=324
Counting strategies	8%	9%
Early additive part-whole	19%	21%
Early multiplicative part-whole	46%	38%
Advanced multiplicative part-whole	28%	32%

Appendix D: asTTle test used in assessment of year 7 students



To be completed by the student:

Are you a boy or a girl? Boy Girl

What year are you in? 4 5 6 7 8 9 10

Which people group do you **most** belong to? (*Choose one)*

NZ European/ Pakeha
NZ Maori
Pacific Nation
Other

How often do **you** speak English at home?

Always or Usually
Sometimes or Never

Attitude Questions

*Tick the box that is **closest** to how **you** feel about each question*	1	2	3	4
A1. How much do you like doing maths at school?				
A2. How good do you think you are at maths?				
A3. How good does your teacher think you are at maths?				
A4. How good does your Mum or Dad think you are at maths?				
A5. How much do you like doing maths in your own time (not at school)?				
A6. How do you feel about doing things in maths you haven't tried before?				

Practice Questions

P1.

How many books are there?

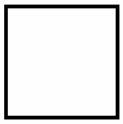
◯ 4 ◯ 5 ◯ 6 ◯ 7

P2. 17 + 2 = _____________

P3. Draw a circle around the picture that has exactly 3 doughnuts.

P4. Draw a line to match the sentence with the correct shape.

I have 3 sides.

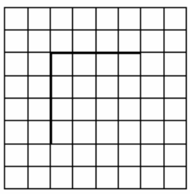


I have 4 sides.

P5. Write the answer in the box.

7 – 5 = ☐

P6. Complete the drawing of the square on the grid below.



Draw lines to match the answer to each problem.

1. $5 + \square = 20$ • 5

• 10

2. $\square - 20 = 5$

• 15

3. $4 \times \square = 20$

• 20

4. $20 \div 2 = \frac{1}{2} \times \square$ • 25

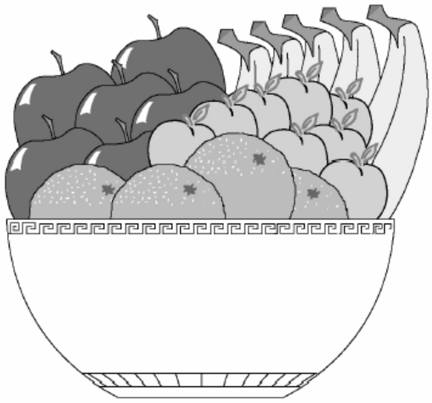
Complete this number pattern and write the rule.

5. 36, $\square$, 50, 57, 64, $\square$

6. Rule: ______________________________

Read about the fruit bowl and answer questions 7 to 9.

In a fruit bowl there were 5 bananas, 6 apples, 4 oranges and 8 plums.



7. Campbell ate half of the plums. How many plums did he eat?

____

8. Josh ate 2 apples. What fraction of the apples did he eat?

____

9. One banana was rotten and has to be thrown out. What fraction of the bananas was left?

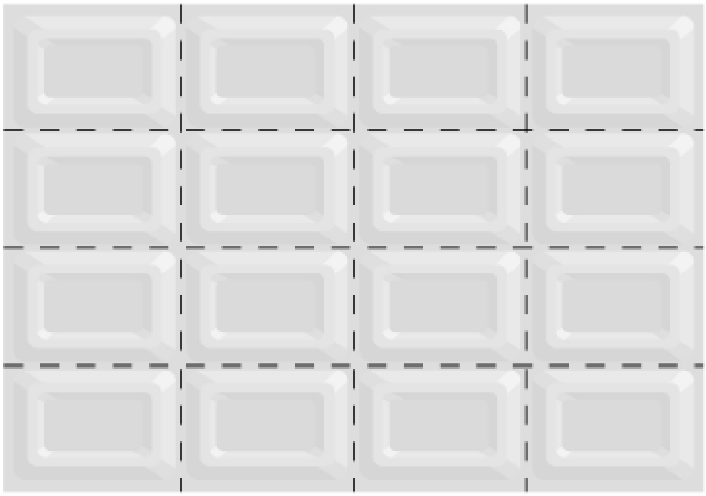
____

Write the answers to each of the following.

10. $4 \times 3 =$ __________

11. $11 - 2 \times 3 =$ __________

This chocolate bar has sixteen small pieces.



2. John is given half. Shade in his half.

3. Mary had half of what was left.

   What fraction of the **whole bar** did Mary get?

   ____

Read about Harold, Liu and Simima and answer questions 14 to 15.

Harold, Liu and Simima collect marbles.

Finish these sentences about Liu, Simima, Harold and their marbles.

14. There are ______________ marbles altogether.

15. ______________ has more marbles than ________________ but

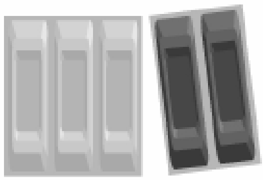
fewer than ______________.

Complete this number pattern and write the rule.

16. 30, 45, ☐, ☐, 90

17. Rule: ____

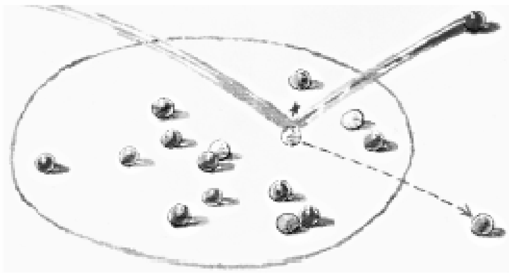
18. This block of chocolate is made up of five smaller pieces.



What fraction of the block has been removed?

____

Matthew had 28 marbles and Robert had 14 marbles.



19. How many marbles does Mathew have to give Robert so that they both have the same number of marbles?

____

20. 30 children visited the beach.
One third of the class could not swim because they forgot their 'togs'.
The rest went swimming.

How many children went swimming? _______________

21. Write the answer to this number sentence.

5024.6 – 2975.8 = _______________

Use the table to answer questions 22 to 23.

The table shows the number of people living in New Zealand's main urban regions in 1996.

REGION	POPULATION
Auckland	991 797
Christchurch	325 251
Dunedin	110 802
Hamilton	158 046
Hastings	58 494
Napier	52 953
Palmerston North	73 860
Rotorua	54 297
Tauranga	82 287
Wellington	334 050

22. The total population in the three smallest regions is closest to which one of these figures?

- ◯ 107 000
- ◯ 166 000
- ◯ 471 000
- ◯ 1 651 000

23. How many **more** people live in Auckland than in Christchurch and Wellington combined?

____

24. Mr and Mrs Maku and their two children are going to Disneyland. Wiremu is five and Huia is eight.

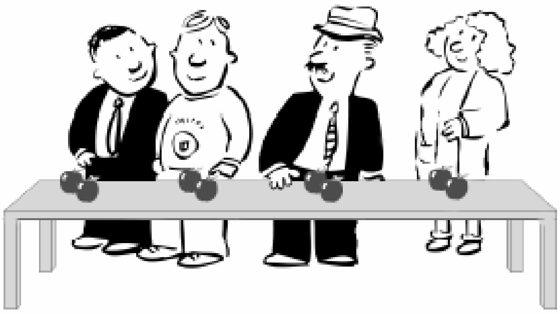
Which is the correct way to work out the cost for the Maku family?

- ⬭ 2 + 1999 + 1316
- ⬭ 2 × 1999 + 1316
- ⬭ 2(1999 + 1316)
- ⬭ 2 × 1316 + 1316

25. The temperature was –5°.
It rose 9°.

What is the temperature now? ____________________

26. Using the diagram, write a story involving division that gives an answer of 2.



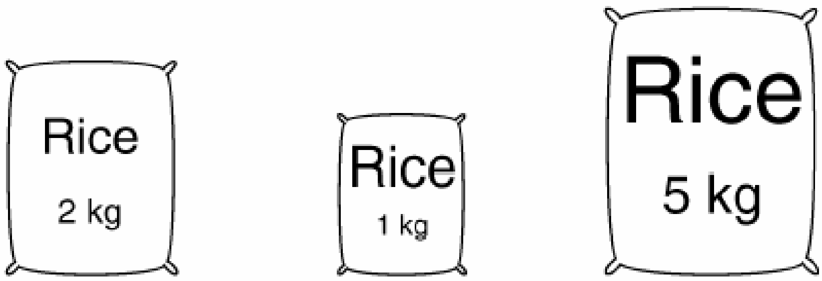
____

____

____

Read about Kiri and answer questions 27 to 28.

Kiri needed 10 kg of rice.



$3.95	$2.15	$9.95

27. Which size represents the best value?____________________________

28. What is the least amount she can pay for 10 kg of rice? ____________________

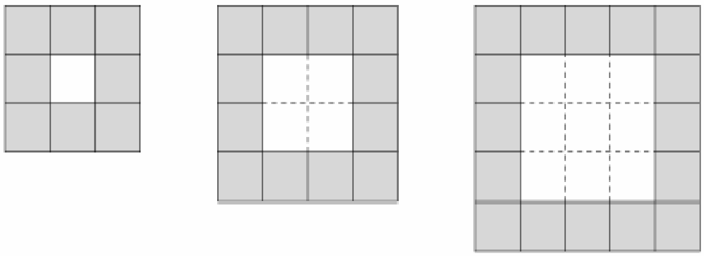
29. What is the value of $\square$ that makes this number sentence true?

$7 + 2 \times \square = 13$

$\square$ = ___________

Read about Jenny and answer questions 30 to 32.

Jenny was making a pattern using white squares and grey squares.



30. How many white squares would be in the tenth pattern?

____

31. Write the rule in words for finding the number of grey squares.

____

____

32. Write the rule in words for finding the pattern for the number of squares in the middle. ____

____

Summary

Name	Kay #3
Date Created	19/12/2003 12:49
Last Modified	19/12/2003 12:49
Area	Mathematics
Status	Accepted
Sequence Number	00003

Content	
Number Knowledge	9
Number Operations	12
Patterns in Number	11

Difficulty	
2B	5
2P	3
2A	2
3B	3
3P	8
3A	6
4B	2
4P	2
4A	1

Cognitive Processing	
Surface	16
Deep	16

Appendix E: Test of generalisation in part-whole relationships

Section A

Jason uses a simple method to work out problems like 47 + 25 and 67 + 19 in his head.

Problem	Jason's calculation
47 + 25	50 + 22 = 72
67 + 19	66 + 20 = 86

1) Show how to use Jason's method to work out 36 + 49

Problem	*Show all your working in the space below*
36 + 49	

2) Show how to use Jason's method to work out 268 + 96

Problem	*Show all your working in the space below*
268 + 96	

3) Show how to use Jason's method to work out 35.8 + 4.6

Problem	*Show all your working in the space below*
35.8 + 4.6	

Section B

Kate uses a simple method to work out problems like 87 – 48 and 183 – 97 in her head.

Problem	Kate's calculation
87 - 48	89 - 50 = 39
183 – 97	186 – 100 = 86

1) Show how to use Kate's method to work out 54 – 26

Problem	*Show all your working in the space below*
54 – 26	

2) Show how to use Kate's method to work out 262 – 96

Problem	*Show all your working in the space below*
262 – 96	

3) Show how to use Kate's method to work out 47.2 – 6.7

Problem	*Show all your working in the space below*
47.2 – 6.7	

Section C

Josh uses a simple method to work out problems like 3 x 58 and
4 x 97 in his head.

Problem	Josh's calculation
3 x 58	(3 x 60) – 6 = 174
4 x 97	(4 x 100) – 12 = 388

1) Show how to use Josh's method to work out 8 x 79

Problem	*Show all your working in the space below*
8 x 79	

2) Show how to use Josh's method to work out 3 x 298

Problem	*Show all your working in the space below*
3 x 298	

3) Show how to use Josh's method to work out 4 x 7.8

Problem	*Show all your working in the space below*
4 x 7.8	

Section D

Witi uses a simple method to work out problems like 48 x 5 and
36 x 25 in his head.

Problem	Witi's calculation
48 x 5	24 x 10 = 240
36 x 25	9 x 100 = 900

1) Show how to use Witi's method to work out 78 x 5

Problem	*Show all your working in the space below*
68 x 5	

2) Show how to use Witi's method to work out 25 x 88

Problem	*Show all your working in the space below*
25 x 88	

3) Show how to use Witi's method to work out 48 x 0.5

Problem	*Show all your working in the space below*
48 x 0.5	

Section E

Kiri uses a simple method to work out problems like 160 ÷ 5 and 300 ÷ 25 in her head.

Problem	Kiri's calculation
160 ÷ 5	320 ÷ 10 = 32
300 ÷ 25	1200 ÷ 100 = 12

1) Show how to use Kiri's method to work out 900 ÷ 5

Problem	*Show all your working in the space below*
900 ÷ 5	

2) Show how to use Kiri's method to work out 2100 ÷ 25

Problem	*Show all your working in the space below*
2100 ÷ 25	

3) Show how to use Kiri's method to work out 31 ÷ 0.5

Problem	*Show all your working in the space below*
31 ÷ 0.5	

Algebraic Rationale for sections in assessment of generalisation

Section A – *Additive Compensation*

$x + y = (x + a) + (y - a)$

Section B – *Subtractive Compensation*

$x - y = (x + a) - (y + a)$

Section C – *Distributive Law*

$xy = xm - xn$ where $y = m - n$

Section D – *Multiplicative Compensation*

$xy = (ax)\left(\frac{y}{a}\right)$

Section E – *Division Compensation*

$x \div y = ax \div ay$

Appendix F: Number and percentage of year 8 students in each school correct on each task in the test of generalisation

School	Test section	% correct on at least one whole number problem	% correct on decimal problem	% correct on at least one whole number problem	% correct on decimal problem
Decile 1	A	92 (55%)	46 (28%)	204 (53%)	81 (21%)
Decile 3	A	192 (85%)	119 (53%)	174 (69%)	96 (38%)
Decile 5	A	145 (91%)	79 (50%)	137 (82%)	86 (51%)
Decile 1	B	40 (24%)	27 (16%)	79 (21%)	38 (10%)
Decile 3	B	97 (43%)	65 (29%)	84 (33%)	50 (20%)
Decile 5	B	67 (42%)	38 (24%)	60 (36%)	32 (19%)
Decile 1	C	49 (29%)	16 (17%)	99 (26%)	22 (6%)
Decile 3	C	125 (56%)	60 (27%)	124 (49%)	51 (20%)
Decile 5	C	96 (60%)	36 (23%)	89 (53%)	30 (18%)
Decile 1	D	61 (37%)	28 (16%)	107 (28%)	39 (10%)
Decile 3	D	132 (58%)	80 (36%)	125 (49%)	58 (23%)
Decile 5	D	112 (52%)	56 (35%)	88 (52%)	50 (20%)
Decile 1	E	38 (23%)	18 (11%)	76 (20%)	28 (7%)
Decile 3	E	116 (52%)	67 (30%)	110 (43%)	73 (29%)
Decile 5	E	82 (52%)	49 (31%)	71 (45%)	46 (27%)

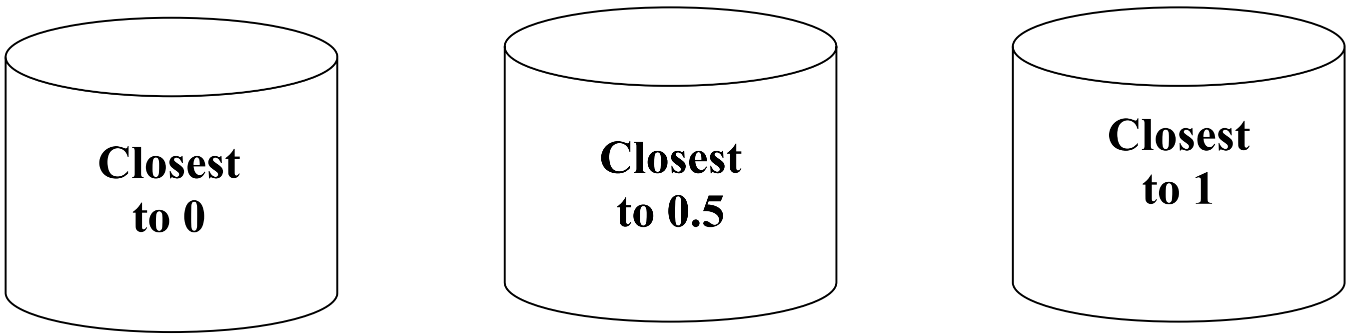
Appendix G: Sample of materials used in one year 9 class

Worksheet 1 for decimals

0.75	**Four tenths and three hundredths**	**Twenty-three hundredths**
0.60	**Six hundredths**	**Sixty-eight hundredths**
0.7	**Two tenths and three hundredths**	**Seventy-five hundredths**
0.23	**Seven tenths and five hundredths**	**Sixty hundredths**
0.06	**One tenth**	**Forty-three hundredths**
0.68	**Six tenths and no hundredths**	**Seventy hundredths**
0.43	**Six tenths and eight hundredths**	**Ten hundredths**
0.1	**Seven Tenths**	**Sixty hundredths**

Worksheet 2 for decimals

0.1	**0.2**	**0.3**	**0.4**	**0.5**
0.6	**0.7**	**0.8**	**0.9**	**1.0**



0.1	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
0.1	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
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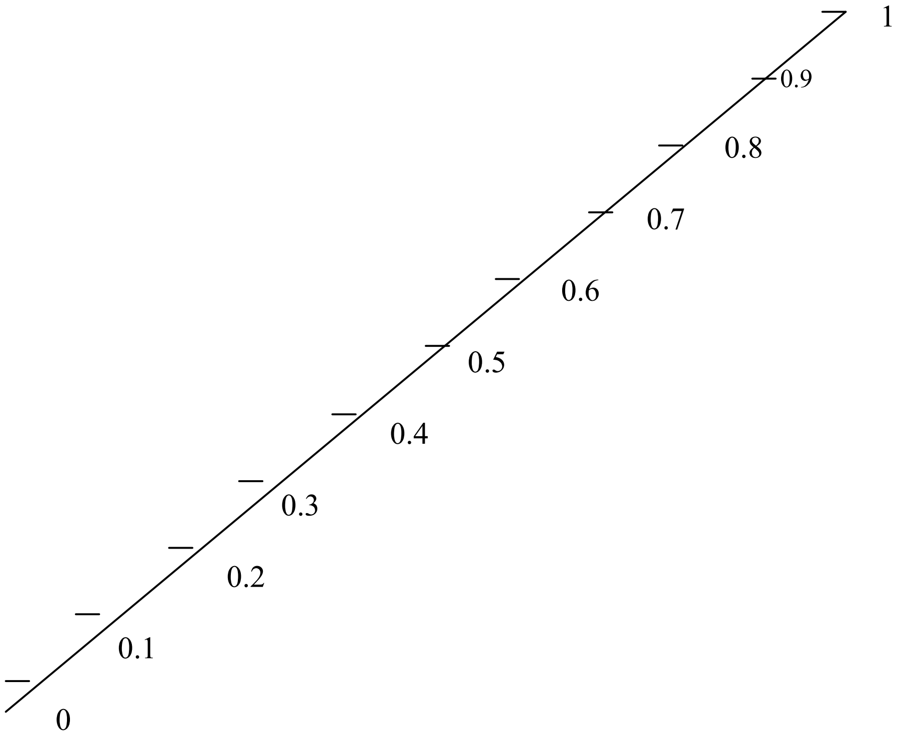
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	
	0. 01	

PERCENTAGES AND DECIMALS

Write each fraction on the number cline,
matching it with the correct decimal.

1. 70%
2. 45%
3. 25%
4. 10%
5. $33\frac{1}{3}$%
6. 30%
7. 75%
8. $66\frac{2}{3}$%
9. 20%
10. 45%
11. 100%
12. 5%



Initial observation categories

Child	Beh	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Corr ans																														
	Incorr ans																														
	Attend																														
	Collaborate																														
	Off-task																														
	Working																														
	Question																														
	Hand up																														
	Answers inc																														
	Answers C																														
	Information																														
	Concept																														
	Application																														
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	Answers C																														
	Information																														
	Concept																														
	Application																														

Interview with students

1. What do you think / feel about mathematics?
   Like
   Dislike
   Indifferent
   Something I am good at
   Something I am not good at

2. Have you always felt like this about math? Or has this changed recently?
   changed / not changed

3. How do you like this math class?

   Love it
   It’s ok
   Hate it

4. Is it different?

   If yes – how?

5. Do you feel you are making progress?

   Yes
   Math makes more sense to me.
   I am getting the hang of it.
   I am beginning to enjoy it.

   No
   I am just useless at math.
   Who needs math anyway.
   It’s obviously not my thing.

   I never had a problem with math anyway.

Interview with teacher

Before the lesson:

1. What is the focus / objective of this lesson?

2. How will you teach this?

……………………………………………………….

After the lesson

4. How did the lesson go?

Did all the students I observed complete the exercise successfully?

5. How did this reflect the numeracy initiative?

6. If you changed your plan
   - why?

7. If you did this again – is there anything you would do differently?

   - Why?