Exploring Issues in
Mathematics Education

An Evaluation of the Numeracy Exploratory Study (NEST) and the Associated Numeracy Exploratory Study Assessment (NESTA) Years 7–10, 2001

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The Project

This document reports on the evaluation of the Numeracy Exploratory Study (NEST) and the Numeracy Exploratory Study Assessment (NESTA) for years 7 through 10. This project, aimed at improving students' knowledge and strategies for solving numerical problems mentally, took place in twelve secondary schools and six intermediate schools located in six New Zealand centres during the second half of 2001. It is an upward extension of a project designed initially for students between years 1 and 6 (the Numeracy Project).

Key Findings

Enough data were available for meaningful analysis of students' numeracy in years 7, 8, and 9, as follows:

1. Initial data show that there was little difference between these year groups in the numeracy tasks assessed.
2. Initial assessment showed these year groups to be unexpectedly weak in knowledge of fractions, knowledge of the base 10 nature of our number system, and finding a fraction of a whole number.
3. With only one term of teaching, all age levels advanced on all six scales of NESTA. Approximately 55% of all students gained one or more stages in knowledge of the reading and ordering of whole numbers and unit fractions and in knowing how many 10s or tenths were in larger numbers.
4. Approximately 45% of all students gained at least one stage in the use of mental strategies for operating with numbers. As some schools spent little time on this project, it is reasonable to assume that these strategies are readily taught once teachers are aware of gaps in students' knowledge.
5. The profile of students from lower decile schools at different stages at the end of one term of this project looked like the profile of students from the upper decile schools at different stages at the beginning of the project. This indicates that the programme enables students from lower decile schools to "catch up" with those from upper deciles.
6. Students in upper decile schools also advanced. This shows that there is room for growth for the vast majority of students, even those who start off at relatively higher stages. There are noticeable ceiling effects only on the scales for Identification of Whole Numbers and Additive Strategies, both of which only go to stage 6.
7. More students from upper decile schools than from lower decile schools gained on most of the subscales. This suggests that they had a better grounding in numerical concepts.
8. Differences in ethnicity were so closely linked to differences in the decile ranking of schools that this factor was not analysed separately. For example, 88% of the students in one secondary decile 1 school were from Pacific nations and 34% of the students in one decile 10 intermediate school were Asian.

9. On average, the students who started at lower levels advanced more than did those who started at higher levels. This was true for students who scored at the lowest scored stage on this scale, but not for the group who failed to score at this lowest stage. A Rasch analysis of a subset of the data also showed that the top two stages were further apart than the lower stages.
10. Teachers and administrators were enthusiastic about the assessment and the project.
11. Teachers' enthusiasm was not necessarily matched by the general advancement of their students. This may have been due to the fact that, because of the size of the steps in this scale, the students' progress was not reflected in altered stages. The enthusiasm may also have been a response to new insights given by the novelty of the programme.
12. Students were not always at the same levels across the additive, multiplicative, and proportional reasoning scales. There was an interesting pattern here. The students who were at higher stages in additive reasoning than in multiplicative or proportional reasoning were generally those who had low scores on all scales and had started to use "part-whole" strategies in adding but had not generalised these strategies to other domains. In contrast, those who scored higher on multiplicative and proportional reasoning strategies than on Additive Strategies were the students who understood mathematical concepts and generalised easily. They did not necessarily start with advanced strategies but quickly understood the benefit of strategies and generalised across domains.
13. The Numeracy Project was implemented differently in secondary schools and in intermediate schools. There was considerable variation in how, and how often, the approach was used in secondary schools. One factor in the adoption of the approach was that this project was introduced toward the end of the year while numeracy had already been taught in the first term.
14. The project made all schools rethink their teaching programme. With next year in mind, considerable thought has gone into what students should be taught and at which stage of the school year. This includes changes to the year 10 programme and changes for those year 11 students who will not be taking the National Certificate of Educational Achievement (NCEA).
15. Case studies demonstrate that different implementation patterns can be effective. Teachers' thoughtful use of the project and how they integrated it into their existing mathematics programmes were important factors in success.

Recommendations

1. The Numeracy Project is worth continuing at this level, having been shown to benefit both students and teachers.
2. Special attention should be paid to the conceptual underpinnings of students from lower decile schools. These students made good progress, but did not progress to the highest stages as frequently as students from upper decile schools.
3. In 2002, attention needs to be paid to students who have received the Numeracy Project in their previous schools. This will provide insight into how their performance differs from those in this cohort who are all new to this type of numerical thinking.
4. Secondary schools need additional help to develop ways of adapting the project to their needs.
5. These secondary school adaptations should be carefully documented for the benefit of other secondary schools.
6. Transfer from numeracy to other mathematics curriculum areas, especially algebra, should be fostered.
7. Initiatives made by schools to benefit from the individual assessment, when they will not have full government support for this, should be watched; in particular to ensure that the assessment continues to be seen as the main focus of the Numeracy Project.

1. Introduction

In an attempt to improve the numeracy skills of all New Zealand school students, the Ministry of Education has funded a major project, hereafter referred to as the Numeracy Project. This project focuses on professional development for teachers within a framework that helps explain how students come to understand numerical relationships and to calculate efficiently. Initially this project focused on the first three years of school, years 1 to 3, and its major focus is still at this level. In 2000 it was extended to include years 4 to 6 in a limited number of schools. In 2001 it was also offered to some teachers of students in years 7, 8, 9, and 10. This report evaluates the last of these three programmes: that for years 7 through 10.

The project is based on a systematic approach known as The Number Framework. Although this framework has roots in work in other countries, it has been written by New Zealanders. In 2001 the project for the first three years of school was called the Early Number Project. It has been evaluated by Thomas and Ward (2001, 2002). The project for years 4 though 6 was called the Advanced Numeracy Project and has been evaluated by Higgins (2001, 2002). The programme for years 7 through 10 has been called the Numeracy Exploratory Study (NEST). An inherent part of the project is the individual assessment of each child using an interview based on the Numeracy Exploratory Study Assessment (NESTA). Years 7 through 10 are the last two years of primary school or intermediate school and the first two years of secondary school. In practice the project has been used much more in the first year of secondary school (year 9) than in year 10.

The assessment and the programme recommended for years 7 and 8 and for the secondary school levels are the same. The results for these two groups are given separately in most sections of this report. Note that because of rounding errors, not all totals in this report equal 100%.

At all levels, the Numeracy Project focuses on enabling teachers to assess and improve the knowledge and mental strategies that students can apply to numerical problems. The progression of these numeracy strategies covers early and advanced counting strategies, early and advanced Additive Strategies, early and advanced Multiplicative Strategies, and early and advanced strategies for proportional reasoning, as well as the knowledge related to all these strategies.

The Number Framework and its associated teaching programmes are still being developed. The programme taught in 2002 has been changed slightly on the basis of previous years' findings. This evaluation serves both to show what the effect of its introduction has been for students and teachers, and to identify areas in which more focused teaching or professional development may be needed.

Eight facilitators in five New Zealand centres introduced NEST to teachers in 2001. They worked with about 160 teachers, who in turn worked with over 4,000 students. Twelve secondary schools and six schools for year 7 and 8 students participated. Details of these schools are given in Chapter 4. These facilitators led several staff workshops in which they introduced the logic of the programme and the assessment tool to staff, and trained them to do individual assessments of all their students. The facilitators visited each classroom three times, either demonstrating teaching procedures, co-teaching, or commenting on the teachers' procedures. They also responded to additional questions and requests during the period of the project. The teachers then reassessed all children individually to document the progress that they had made.

The programme, including the facilitation and assessment, was not available to participating schools until the second term of 2001. After the training workshops in Term 2, the teachers interviewed each of their students individually and recorded the results on the form that appears in Appendix B. Teachers were then expected to adapt this teaching in light of what they had found. This teaching took place in Term 3 in most schools. Appropriate teaching methods were suggested and modelled by the facilitators. Teachers were expected to teach students in groups that were at the same stage in understanding and learning needs. Each teacher in the project was provided with a large folder of activities that could be used in teaching, with paper resources, and with money to buy equipment. The activities provided were aimed at helping each student develop from one stage of The Number Framework to the next. Many of these activities have been made available on the website (http://www.nzmaths.co.nz) in 2002. Each teacher was assigned a facilitator who made at least three visits to their classroom to help with the teaching. In this initial year, teachers were required to use the same assessment but were given leeway to adapt their teaching as appropriate. In particular, teachers of students in secondary schools used their judgement to decide on teaching issues. This was fostered in order to see what methods were best for teaching this older age group.

Because this was an exploratory study, teachers were given leeway in how they used the resources. They were encouraged to use their initiative in teaching to advance their students' knowledge and use of strategies. They were all to give the same assessment early in Term 4 (some did not give the second assessment and their data has not been analysed).

Brief Overview of The Number Framework Stages and Scales

The Number Framework provides a model of stages that learners go through in developing their understanding of numeracy. These stages, given below and in Appendix A, provide the framework for all of the Numeracy Projects. It was expected that students would use objects for completing tasks at the early stages. From stage 5 on, they would be expected to work out problems mentally, using known addition and multiplication facts, including the understanding that numbers can be broken up to help solve problems. Strategies for doing this are called "mental strategies".

Stage 0
Pre-counting. Students at this level cannot count a small group of objects.

Stage 1
Counting from one with materials. Students at this stage can count and can form a set of up to 10 objects by counting each one. They cannot solve simple addition problems by joining these sets.

Stage 2
Adding by counting from one with materials. These students can add four counters and two counters by counting all of them.

Stage 3
Counting from one by imagining the objects to be counted. These students use counting but do not need to see objects in order to add.

Stage 4

Advanced counting. Students at this stage solve addition problems by "counting on". For example, for 8 + 3 they say "Eight, nine, ten, eleven." to get the answer 11. They can also count by 10s.

Stage 5

Early additive part-whole thinking. At this stage students recognise that addition problems can be solved efficiently by breaking up numbers into their component parts. They may do this by breaking up a number into parts. For example, 9 + 7 is the same as 10 + 6.

Stage 6

Advanced additive / early multiplicative part-whole thinking. Students at this stage use a variety of methods to break up numbers for addition problems and may solve multiplication problems by using these part-whole addition strategies. For example, they may mentally work out that 63 – 29 can be solved mentally by thinking that 63 – 30 = 33, and adding one (perhaps using a visualised number line) to give 34.

Stage 7

Advanced multiplicative / early proportional part-whole thinking. At this stage students can use their understanding of multiplication to break up numbers. For example, they may realise that 50 x 124 is the same as 100 x 62, so it equals 6,200.

Stage 8

Advanced proportional thinking. Students at this stage can use a range of multiplication and division strategies to solve proportional problems. This includes finding a percentage of a whole number. Students who can do this might find 15% of 240 by first finding 10% of 240 (24) and then adding half of this (12). When these two percentages are added together they would give the correct answer (36).

The lowest stage scored on the assessment used for years 7 through 10 in 2001 was stage 4. Teachers were instructed to label all students below this stage as being at stage 3. However, these students could have been at stages 0, 1, 2, or 3. This means that it was not possible to differentiate students who scored below stage 4. In this report we have chosen to label these students as scoring "nil". As is discussed in Chapter 8, these stages do not represent a succession of steps of equal increase in difficulty. Students take longer to develop understanding of the upper stages than of the lower stages.

The assessment produced stage scores for each of six areas for each student: three scores for numerical knowledge and three for problem-solving mental strategies.

Knowledge scales were:

Identification of Whole Numbers. In this scale students were asked to identify printed numerals and then to name the numbers that would come after and come before each number. The numbers ranged from two-digit to six-digit figures.

Identification of Fractions. In this scale students were required to read unit fractions (1/2, 1/4, 1/3), to put these fractions (as well as 2/5, 2/3 and 5/5) in order of increasing size, and to order decimals of between one and three decimal places and percentages.

Knowledge of Base 10 Grouping. In this scale students were required to name the number of 10s in numbers between 60 and 82,600, to give the number of 100s in four and six-digit figures, and to name the number of tenths in 3.2 and 506.9.

Strategy scales dealing with computation were:

Strategies for Addition and Subtraction. These operations are called "Additive Strategies" in the figures in this report, as well as in general writing about this field. In this scale students were given addition and subtraction problems to do. The teacher noted whether the student counted objects to obtain an answer; counted on or counted back from one of the numbers; used a part-whole strategy in which they broke up one of the numbers being added or subtracted to make the problem easier; or used a range of such part-whole strategies.

Strategies for Multiplication and Division. Together, these are referred to as "Multiplicative Strategies". In this scale teachers noted whether students completed a problem that could have been solved using multiplication by using a counting strategy or repeated addition (for example, 13 x 7 is equal to 10 x 7, plus three more sevens: "Seventy-seven, eighty-four, ninety-one."), whether they derived the answers to unknown multiplication questions from known facts in addition and multiplication (for example, 32 x 7 is the same as 30 x 7 and 2 x 7), or whether they used a range of part-whole strategies.

Strategies for Solving Ratio and Proportional Problems. These are referred to in this report as "Ratio and Proportional Strategies". This scale took students into fraction, ratio and proportional problems. At lower stages the student is asked to find a fraction of a whole number, like 1/3 of 24. At the more advanced stages students are required to find a relationship between two numbers and then apply this relationship to a third number (for example, if it takes 10 boxes to pack 16 books, how many books are in 24 boxes?).

Stages as Defined for Each Scale

The general framework was applied to each scale. Examples from each of the relevant stages are given below to help the reader understand what these stages mean in practice.

Note that it is our convention to score as "nil" those students who did not achieve the first stage on the scales. On teachers' scoring sheets this was called stage 3 or, in the case of Identification of Fractions, stage 4.

Knowledge Scales

Nil

On the scale for Identification of Whole Numbers of whole numbers, nil indicates failure to read numbers under 100 or give the numbers before or after these. Nil, on Knowledge of Base 10 Grouping, indicates a student's inability give the number of 10s in 60 or 230. On Identification of Fractions, nil indicates inability to identify unit fractions at stage 5.

Stage 4

In Identification of Whole Numbers this stage indicates that students can identify numbers to 100 and give the preceding and succeeding numbers. In Knowledge of Base 10 Grouping it indicates that students can give the number of 10s in 60 and 230.

Stage 5

In Identification of Whole Numbers this stage indicates the ability to read all numbers to 1,000 and give the preceding and succeeding numbers. In Identification of Fractions it indicates the ability to read 1/2, 1/4, and 1/3. In Knowledge of Base 10 Grouping it indicates knowing the number of 10s in three-digit numbers.

Stage 6
In Identification of Whole Numbers this stage indicates the ability to read all numbers to 1,000,000 and give the preceding and succeeding numbers. In Identification of Fractions it indicates the ability to order 1/2, 1/4, and 1/3 and read decimals to two decimal places. In Knowledge of Base 10 Grouping it indicates knowing the number of 10s in five-digit numbers.

Stage 7
There is no stage 7 in Identification of Whole Numbers. In Identification of Fractions it indicates the ability to order the above fractions as well as 2/5, 2/3, and 3/3 and to order decimals of different lengths. In Knowledge of Base 10 Grouping it indicates knowing the number of 10s and 100s in any number and the number of tenths in one.

Stage 8
In Identification of Fractions, this stage indicates the ability to order fractions, decimals, and percentages. In Knowledge of Base 10 Grouping it indicates that a student knows how many tenths, hundredths, or thousandths there are in numbers that include decimal fractions and can multiply and divide any number by powers of 10.

Strategy Stages for Each Scale

The other NESTA scales relate to strategies for addition and subtraction, multiplication and division, and working with proportions or ratios.

Nil
For all strategy scales, this indicates that students were unable to use advanced counting (stage 4) or more advanced strategies for additive, multiplicative, or proportional problems.

Stage 4
For all strategy scales this stage indicates using advanced counting to solve additive, multiplicative or proportional problems, either through counting on, counting back, or skip counting.

Stage 5
This stage indicates using early part-whole Additive Strategies for both addition and multiplication (such as breaking numbers into components that make 10 or multiplying by repeated addition) or applying addition facts to ratio and proportion problems.

Stage 6
This stage indicates using a range of part-whole strategies for addition, deriving answers to unknown multiplicative problems by using known multiplication facts, and combining addition and multiplication facts to solve ratio problems.

Stage 7
This stage does not apply to additive problems. For multiplicative problems students use a range of part-whole multiplication strategies for solving multi-digit problems, and multiplication and division for solving proportional problems.

Stage 8

This applies only to strategies for solving ratio and proportional problems, or advanced Ratio and Proportional Strategies. It involves using a wide range of multiplication and division strategies to solve ratio and proportion problems.

There is evidence that shows there are larger moves between stages 6 and 7, and 7 and 8, than between stages 4 and 5, and 5 and 6. See Chapter 8 and Appendix C.

Outline of This Report

Chapter 2 presents the research questions addressed in this report. Chapter 3 discusses the history of the Numeracy Project and its relation to other developments in New Zealand and in the United Kingdom and the United States. Chapter 4 describes the schools and students that participated in this trial. Chapters 5 through 9 describe the initial and final scores gained by the students involved, looking at this information in different ways. Chapter 10 presents information gained from teachers, heads of mathematics departments, and the principals of the schools involved by means of interviews and questionnaires. Chapter 11 presents case studies of three of the schools, based on interviews with teachers and senior staff as well as assessed student gains. Chapter 12 presents a brief summary and some implications for further research. A copy of the student assessment form, The Number Framework, the Rasch analysis, a summary of the data, and outlines of the format of interviews with teachers and administrators are given in the appendices.

2. Research Questions

This numeracy initiative for years 7 through 10 was an exploratory study in 2001. Questions are therefore limited to those about development students made in the areas the project focused upon, about the scale itself, and about characteristics of successful interventions. The main research questions are given below.

1. Does the assessment tool developed for these year groups, called the Numeracy Exploratory Study Assessment (NESTA), differentiate students in a meaningful way?

This question required examination of both the spread of scores obtained and the teachers' views on the accuracy of the ranking of results of the assessment. It would be answered in the negative if, for example, it was found that a majority of students scored at the top stages. This is related to the theoretical question of whether or not an approach developed for young learners is appropriate for the more complex mathematics required of older learners. Chapters 5 and 6 present the answers in relation to students' scores, and teachers' comments are given in Chapters 10 and 11.

2. Did students of different year groups, different initial achievement, and different economic backgrounds make different advances in stages?

This question is addressed by a comparison of initial and final assessment stages for all six scales. It should be remembered that there was only one term of teaching between the initial and final assessments. Chapters 5, 6, 7, and 8 address this question.

3. Is this framework appropriate for all learners in years 7 through 10, or should the assessment and teaching programme be seen as only appropriate for less competent students?

Some teachers have seen a need for a remedial tool in numeracy and hoped that this project would fill that need. However, the authors of the programme expected it to be of use for students at all levels of achievement. This was assessed by seeing if many students started at, or reached, the top stages of the assessment (a ceiling effect) and if teachers felt that NEST was appropriate for all students. Years 9 and 10 were the focus of this question, and a meaningfully large number of results was only available for year 9. This question is addressed in Chapters 6 and 11.

4. Does a student score at the same stages on strategies for different mathematical operations?

The stages assigned in the assessment are designed to be consistent across scales. It should be unlikely for a student who scored at stage 7 on Multiplicative Strategies (advanced multiplicative part-whole thinking) to score at 4 on Additive Strategies (advanced counting without part-whole thinking). However, students' scores on any measurement tool are subject to variability for several reasons. This question is addressed by looking for cases like the example above, where students were credited with either higher or lower stages on multiplicative thinking or proportional thinking than on Additive Strategies. This question is addressed in Chapter 9.

5. Are there differences between the way the programme is implemented in years 7 and 8 and the way it is implemented in secondary schools, and, if so, are there factors that could be related to any differences in progress as measured by NESTA?

Without observing a great number of classes, the only ways of addressing this question were through interviews with teachers and senior staff in a selection of schools, and through comparing teachers' statements with the progress that students made. The answers to this question should be seen as tentative for this reason. See Chapters 10 and 11.

6. Are there indicators that this is a successful intervention?

At this exploratory stage of the Numeracy Project, all those involved have been asked to see it as an evolving framework and to use their professional judgment in planning and teaching. Teachers' views on the project's strengths and on areas for improvement were sought. These views were then synthesised to specify what led to particularly useful interventions, as judged both by gain in students' scores and in teachers' responses. This question is addressed in Chapters 5 through 11.

Cautions to Be Borne in Mind When Reading This Report

There are several factors that could indicate that the results given in this report may not hold true for all students or all teachers of this age range. A limited number of schools were included in the study. These schools were not selected randomly and do not represent all the economic levels of New Zealand schools. The facilitators were new to the job of helping teachers of this age group work with The Number Framework. The teachers were new to the assessment, the framework behind it, and the teaching programme. They were learning what the assessment items called for and were just beginning to learn how to teach the programme. Some teachers commented that they understood the assessment much better the second time they gave it. In addition, assessment items were chosen to represent certain types of mathematical thinking, but like all sets of assessment items, they are only a small sample of the items that could be developed for such analysis. There has been no item analysis of the scale, nor would such an analysis be appropriate at this stage. It may be that some items have prompted certain types of response in a manner unintended by those who wrote the items.

One issue that is frequently raised with data such as this is whether students would have made the same amount of progress with their present teaching or as they grew older and more experienced. If that were the case, year 8 students would be expected to score at higher stages initially than year 7 students, and year 9 students to start at higher stages than year 8 students. There is only one instance for which this was true, and then for only one stage of one scale. This was the proportion of students in each year group at stage 6 in Identification of Fractions. Otherwise, the patterns of achievement of students in years 7, 8 and 9 before the intervention is remarkably similar. This makes it clear that students would not have made this progress without the teaching. See Chapters 5 and 7. The main reason for this similarity is likely to be that teachers were unaware of students' abilities in the aspects of numeracy addressed in The Number Framework and had not taught to weaknesses in these domains previously. Once the assessment provided this awareness, they addressed these issues and students improved.

The purpose of this exploratory study was to get an initial picture of whether introduction of the Numeracy Project in this age range was useful and to find examples of what happened when it appeared to be working well.

3. Relation of the Numeracy Exploratory Study (NEST) to Other Numeracy Programmes

Background to the New Zealand Numeracy Project

The Numeracy Project arose out of the American studies of Steffe and Cobb (e.g., Steffe, 1991), as developed into "Count Me in Too" by Bob Wright and colleagues in Australia (e.g., Wright et al., 1994) and then modified and extended in major ways for use in New Zealand by Peter Hughes and Vince Wright for the Ministry of Education. See Thomas and Ward (2001) for a history and evaluation of these programmes.

The overall framework for the development of essential numeracy understanding moves from counting, to understanding of additive properties, to multiplicative reasoning, and on to proportional reasoning. Counting and additive skills are covered in the Early Numeracy Project, used in years 1 to 3 in 2000 and 2001. Multiplicative and proportional reasoning were introduced in the Advanced Numeracy Project (ANP), introduced for years 4 to 6 as an exploratory study in 2000 and in a larger number of schools in 2001. See Higgins (2001, 2002) for details and evaluation of this project.

This Numeracy Exploratory Study was based upon the Advanced Numeracy Project (ANP) but included some different items, particularly in proportional reasoning. It explored the development of some skills beyond the level of the ANP. In the main, however, the assessment items and recommended teaching methods are similar for both the ANP and NEST.

The recommended teaching procedure for all of the projects builds on a simplification of the theoretical model of the growth of student's understanding developed by Susan Pirie and Tom Kieren (1989). This emphasises the development from primitive knowing through having mental images to noticing properties of the numbers involved. At any stage, learners move back and forth between approaches to meet their needs. The teaching model in this project reflects this developmental model, especially in encouraging the forming of mental images and then noticing number properties when moving on to part-whole thinking.

Relation to *Mathematics in the New Zealand Curriculum*

The curriculum for teaching mathematics in New Zealand (Learning Media, 1992) is a legal document that teachers are required to implement. It has a section called "Number" which covers some, but not all, of the topics in The Number Framework. It has six levels, but these are not the same as the six to eight stages on the scales of The Number Framework. It is worth pointing out some of the similarities and differences, as these are relevant for the current Curriculum Stocktake.

- Although this curriculum document emphasises outcomes and includes some sections on how to teach, suggestions on how to teach are given in much less detail than in the Numeracy Project.

- The steps in the early levels of the curriculum are less well defined than those in The Number Framework.

- Development from additive to multiplicative reasoning is not clearly spelled out in the curriculum document, while possible steps in this transition are spelled out in the Numeracy Project.

- The basis of proportional reasoning lies in an understanding of fractions. Although recognition of an area model of fractions starts at stage 1 of *Mathematics in the New Zealand Curriculum*, it is poorly developed. Most secondary school teachers interviewed remarked upon lack of students' knowledge in this domain. This could be related to lack of emphasis in the curriculum.

- The Number strand of *Mathematics in the New Zealand Curriculum* covers several topics not in the Numeracy Project. These include negative integers and standard form. It would be possible to include both of these in the scales for whole number knowledge, Base 10 knowledge, and/or Additive and Multiplicative Strategies.

Relation to NCEA Criteria

The new National Certificate of Educational Achievement (NCEA), which will be used for the first time in 2002, has standards with direct relevance to the New Zealand Numeracy Project. This numeracy programme provides a good preparation to passing these standards. The most obvious link is with standard 1.7 although there are also links with standard 1.1.

Mathematics, Level One, Achievement Standard 1.7

- Solve straightforward number problems in context.

Mathematics, Level One, Achievement Standard 1.1

- Use straightforward algebraic methods and solve equations. Some of these problems can be solved with numerical methods.

The sample examples for both of these standards cover addition, multiplication, and proportional problems. In each case, students are required to write down their reasoning.

Relation to the British Numeracy Standards

In 1998 the British set up a national numeracy programme. In 2001 they added a third stage to this programme, which covers the same years as does NEST: 11 to 14-year-olds. This has some similarities to the New Zealand Numeracy Project and some important differences. For example, the British programme uses the terms “stage” and “level” with different meanings from those of the New Zealand programme. For further information on this British programme see the website (http://www.standards.dfee.gov.uk/keystage3/strands/mathematics/).

The British numeracy standards are mandated for all schools. They include details of the amount of time to be spent on each phase of a mathematics class, starting with 5–10 minutes of an oral or mental “starter”, a main teaching activity (25–40 minutes) which may be worked on as a whole class, individually, or in groups, and a final plenary period (5–15 minutes) in which the class comes together to discuss what they have done.

It sets expectations for 14-year-old students (year 10 in New Zealand) that are more extensive than the expectations of the Numeracy Project, in that they cover all areas of the mathematics curriculum. Several of the goals given below fit well with those of the New Zealand Numeracy Project. The English goals are given below, each followed by a comment in square brackets on its relationship to the New Zealand numeracy programme.

- Have a sense of the size of a number and where it fits into the number system [Identification of Whole Numbers knowledge scale].
- Recall mathematical facts confidently [needed for Additive, Multiplicative, and Ratio and Proportional Strategies].
- Calculate accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies [mental strategies emphasised in the New Zealand Numeracy Project].
- Use proportional reasoning to simplify and solve problems [Ratio and Proportional Strategies].
- Use calculators and other ICT resources appropriately and effectively to solve mathematical problems, and select from the display the number of figures appropriate to the context of a calculation [not covered].
- Use simple formulae and substitute numbers in them [not covered].
- Measure and estimate measurements, choosing suitable units, and reading numbers correctly from a range of meters, dials and scales [not covered].
- Calculate simple perimeters, areas and volumes, recognising the degree of accuracy that can be achieved [not covered].
- Understand and use measures of time and speed, and rates such as £ per hour or miles per litre [not covered].

- Draw plane figures to given specifications and appreciate the concept of scale in geometrical drawings and maps [not covered].

- Understand the difference between the mean, median and mode and the purpose for which each is used [not covered].

- Collect data, discrete and continuous, and draw, interpret and predict from graphs, diagrams, charts and tables [not covered].

- Have some understanding of the measurement of probability and risk; explain methods and justify reasoning and conclusions, using correct mathematical terms [not covered].

- Judge the reasonableness of solutions and check them when necessary [implicit in the numeracy strategies].

- Give results to a degree of accuracy appropriate to the context [implicit in the numeracy strategies].

The British numeracy programme pays some attention to individual differences by offering activities for able pupils and for those who have fallen behind so that they may keep "in step with the majority of their year group" (management strategy from website), but the emphasis is on whole class teaching and the underlying assumption that all students within a particular year group will share similar understandings. A study by Gray, Howat, and Pitta-Pantazi (in press) suggests that low achieving students do not benefit from this programme as much as high achieving students do, and may actually be disadvantaged.

Relation to the Standards Promoted in the United States

A document called *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) has been developed by the National Council of Teachers of Mathematics in the United States. It provides, in their words, an ambitious vision for school education (2000). It is not a mandated curriculum, like the British standards, nor are specific goals set for different ages. However, the goals of this document are not unlike those of New Zealand and Great Britain. There are standards for number and operation, algebra, geometry, measurement, data analysis and probability, reasoning and proof, communication, connections, and representation. Below is the list of the goals for number and operation for grades 6 through 8 (years 7 through 9 in New Zealand). Relationships to relevant scales from the New Zealand Numeracy Project are given in square brackets.

Standards are presented for students to understand numbers, ways of representing numbers, relationships among numbers and for understanding the number systems are given. For Grades 6 - 8 these includes the ability to:

- Work flexibly with fractions, decimals and percents to solve problems [Proportional reasoning, all stages].

- Compare and order fractions, decimals and percents [Identification of Fractions, upper stages].

- Develop meaning for percents greater than one [similar to Base 10 grouping, upper stages].

- Understand and use ratios and proportions to represent quantitative relationships [Proportional reasoning, all stages].

- Develop an understanding of large numbers, using exponential, standard form [not covered].

- Use factors and multiples, prime factorisation … to solve problems [Multiplicative and Ratio Strategies].

- Develop meaning for integers and represent and compare quantities with them [not covered].

- Understand meanings of operations and how they relate to one another [Multiplicative and Proportions strategies].

- Understand the meaning and effects of arithmetic operations with fractions, decimals and integers [Proportional reasoning and Identification of Fractions for fractions and decimals but not for integers].

- Use associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify operations with integers, fractions, and decimals [all strategy scales].

- Understand and use the inverse relationship of addition/subtraction, multiplication/division and squaring/finding the square root to simplify computations and solve problems [all strategy scales, except that squares and square roots are not covered].

- Compute fluently and make reasonable estimates [all strategy scales, although estimation is not assessed separately].

- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and apply the selected methods [only mental computation is covered in the New Zealand Numeracy Project].

- Develop and analyse algorithms for computing with fractions, decimals and integers, and develop fluency in their use [algorithms are not part of The Number Framework, but are mentioned in the teaching suggestions].

- Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of results [Multiplicative and Proportional strategies].

- Develop, analyse and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios [Proportional reasoning strategies].

- The American principles and standards also give suggestions on ways of teaching each of these standards.

Summary

The Number Framework is built upon research done in the United States and in Australia. The teaching recommendations for this framework are backed by a theoretical learning model developed by Canadian researchers. It fits fairly well with the number strand in *Mathematics in the New Zealand Curriculum* (Learning Media, 1992) but provides a theoretical continuum not present in that document. It also includes work on fractions and ratios that needs to be covered more systematically in the national curriculum document. It provides background for the numeracy standard of the National Certificate of Educational Achievement and could be expanded to provide a good basis for the algebra standard.

It is different from similar projects being developed in the countries with which New Zealand educators most frequently compare our programmes and progress, in that it focuses on only some aspects of numeracy, not all of mathematics. In particular, it leaves out written records of students' work, the concept of negative numbers, and scientific notation, all of which could be included if desired. It is in line with the intent of the programmes in these countries in that it intends to raise the level of numeracy of its students. The programmes in these other countries, as well as others like the Dutch Realistic Mathematics (e.g., M. van den Heuvel-Panhuizen, ed, 2001), provide resources that can be of use to New Zealand teachers.

What is distinct about the New Zealand programme is its attention to the individual understandings of students from age 5 through age 14; the attention to mental strategies; the integration of multiplicative and proportional reasoning as developing out of additive thinking; and the emphasis on teaching in groups based on students' initial levels of understanding. By individualising assessment and recommending teaching at levels appropriate for the students concerned, it is likely to avoid some of the British difficulties discussed by Gray, Howat, and Pitta-Pantazi (in press), who suggest that the bottom quartile of students are not having their needs catered for by the British National Numeracy Strategy.

We should bear in mind that the longer-term efficacy of all of these programmes has yet to be proven.

4. Characteristics of Participants in NEST in 2001

Eighteen schools, eight facilitators, about 160 teachers, and over 4,000 students participated in NEST in 2001. There were twelve secondary schools and six 'schools' that were either intermediate schools or the top two classes of full primary schools. Two of the secondary schools did not return final results for reasons beyond the control of this research and a few classes returned only a small number of results. Initial results were given for 4,034 students, but final assessment results were given for only 3,329 students. The secondary schools that did not return their final data were a decile 1 school and a decile 3 school. Initial and final assessments were returned for 1,458 secondary and 1,871 year 7 and 8 students. This represents 82% of those initially listed as in the project.

Nature of the Sample

The sample was not intended to be random and should not been seen as typical of New Zealand. However, the nature of the sample does allow for some interesting comparisons to be made in students' achievements.

As shown below, 70% of the year 7 and 8 students came from decile 3 and 4 schools and 30% came from one decile 10 school. Other studies (e.g., Flockton and Crooks, 1997) show clearly that achievement in mathematics is associated with economic status, and it should be no surprise that the children from more affluent backgrounds performed at more advanced stages than those from less affluent homes. However, this unusual sample allows us to see where the differences in achievement between these two main groups lie. It allows us to look at the comparative gains of the different groups, answering questions about whether or not the project is of use to students in both groups.

Eighty-three percent of the secondary school sample were students from decile 1, 2, 3, and 4 schools, and 17% of the sample were students from decile 8 and 9 schools. This covers more of the economic spectrum than the primary school sample, but is not a distribution typical of the students of New Zealand. Again this allows us to compare the achievement of students in upper and lower decile schools.

Students in Years 7 and 8

The six schools for year 7 and 8 students were in Auckland, Gisborne, Wellington, and Christchurch. One facilitator worked with School A and another worked with Schools B, C, and E. A third facilitator worked with School D, with the help of a colleague who was facilitator for a neighbouring secondary school. A fourth facilitator worked with School F (see table 4.1).

Characteristics of the schools with year 7 and 8 students are given in Table 4.1. Some of the schools, such as School E, had combined year 7 and 8 classes.

Table 4.1 Number of students in years 7 and 8 in the six schools for this age level that took part in NEST

School	Decile level	Year level	Number of classes	Number of students
School A	3	7	10	123
		8	10	121
School B	3	7	5	124
		8	4	115
School C	3	7	6	148
		8	3	67
School D	4	7	13	267
		8	11	286
School E	4	7	2	28
		8	2	34
School F	10	7	9	232
		8	11	326
Total				1,871

It can be seen that the sample was made up of two large schools, Schools D and F; three medium sized schools, Schools A, B, and C; and one small school that, for the purposes of this study, consisted of the top two classes of a full primary, School E.

The three decile 3 schools made up 37% of the sample, the two decile 4 schools made up 33% of the sample, and the one decile 10 school accounted for the remaining 30% of the students.

Sex and Ethnicity of the Year 7 and 8 Students in NEST

There were 889 girls (48%) and 982 boys (52%) in the year 7 and 8 sample. Ethnicity is shown in Table 4.2, with the schools grouped by decile. The geographical location of these schools has some effect on ethnicity of the students in the schools.

Table 4.2 Ethnicity of students in years 7 and 8 by school decile level

Decile	Asian	European	Māori	Other	Pacific Islander
3	2%	60%	29%	3%	6%
4	1%	67%	28%	2%	2%
10	34%	52%	1%	10%	3%
Total	11%	60%	20%	5%	4%

European students were relatively equally represented in schools from all three deciles. Māori students were primarily present in the decile 3 and 4 schools. Asian students and those classified as "Other" were disproportionally represented in the decile 10 school. There were more students from Pacific nations in decile 3 schools than in the other two deciles represented, although percentages of these students were low.

Secondary Schools

The ten secondary schools that completed both the initial and final assessments in 2001 were in Auckland, Gisborne, Wellington, Christchurch, and Dunedin.

The main characteristics of the secondary school classes in NEST are given in Table 4.3.

Table 4.3 Schools in the secondary portion of NEST by decile, year level of students, number of classes, and number of students

School	Decile level	Single sex or co-ed	Year level	Number of classes	Number of students
Secondary M	1	Girls	9	6	142
Secondary N	2	Co-ed	9	12	287
Secondary V	2	Co-ed	9	8	136
Secondary O	3	Co-ed	9	10	177
Secondary P	3	Co-ed	9	7	131
Secondary Q	4	Girls	9	8	176
Secondary R	4	Co-ed	9	4	27
Secondary R	4	Co-ed	10	1	7
Secondary S	4	Co-ed	9	8	129
Secondary T	8	Boys	9	5	117
Secondary U	9	Co-ed	9	1	129
Total					1,458

The vast majority of students (99.5%) were in year 9. The scores of the year 10 students for whom results were returned did not differ from those of the year 9 students. As data from initial and final assessment were available for only seven year 10 students, these students were dropped from all analyses where class level was relevant. They have been included only in Chapter 8, which looks at advances from initial level regardless of class.

All schools were roughly similar in size, although few results were returned from School R for unknown reasons. The school says that they were entered in the data base, but they did not appear in the data for analysis. However, there is no reason to believe that these were atypical results. The scores returned centred around 4 and 5 as they did for students from other schools in similar deciles.

One facilitator worked with Schools M and N, one worked with School O, one worked with Schools P, R, and S, one worked with Schools Q and V, and one worked with Schools T and U. In some cases the same facilitator worked with both year 7 and 8 classes and with secondary schools in the same region.

Distribution of students by decile is given in Table 4.4. Ethnic distribution was closely related to the decile ranking of the school, and for this small sample, has not been analysed separately. For example, in one decile 1 secondary school 88% of the students were from Pacific Island backgrounds, and in one decile 9 secondary school 88% of the students were European.

Table 4.4 Distribution of secondary school students by school decile level

Decile	Number of students	Percentage
1	142	10%
2	423	29%
3	308	21%
4	339	23%
8	117	8%
9	129	9%

In summary, 39% of the students came from decile 1 and 2 schools, 44% came from decile 3 and 4 schools, and 17% came from decile 8 and 9 schools.

Sex and Ethnicity of the Secondary School Sample

There were 799 girls (55%) and 658 boys (45%) in the secondary school sample. Two of the schools were single sex schools for girls, and one of the schools was a single sex school for boys.

Ethnicity, ordered by school decile level, is presented in Table 4.5.

Table 4.5 Ethnicity of secondary school sample by school decile level

Decile	Asian	European	Māori	Other	Pacific Islander
1	1%	2%	4%	6%	88%
2	2%	36%	53%	2%	7%
3	4%	53%	21%	6%	17%
4	2%	65%	25%	3%	4%
8	6%	85%	3%	3%	3%
9	2%	88%	7%	1%	2%
Total	3%	52%	27%	3%	15%

Decile ranking is itself influenced by ethnicity, and in this case was accentuated by the locations of the schools. The high percentage of students from Pacific nations reflects the area that this decile 1 school was in. Similarly, the high proportion of Māori students in the decile 2 schools reflects the parts of the country that these schools were in, and the high proportion of European students in the decile 8 and 9 schools reflects where these schools were.

5. Gains Made by Students in Years 7 and 8

A good proportion of both the year 7 and 8 and the secondary students made noticeable gains over the one term they were involved in the Numeracy Exploratory Study. The results for year 7 and 8 students are given in this chapter, and the results for secondary students in Chapter 6.

When noting these gains, it should be remembered that the structure of the assessment and the teaching programme were novel to all these teachers. These teachers had many things to learn at the same time, firstly in understanding The Number Framework, and secondly in learning how to help students advance to using more complex strategies. This one term and the preceding workshops can be seen as a period of intense professional development for the teachers. In addition, most of the students were new to the idea of discussing mental strategies in school. Having been taught to use written algorithms for addition and multiplication for years, they needed to alter the way in which they thought about solving numerical problems. Both the students' starting points and the results of working on the Numeracy Project might be different in another year when teachers and students were more familiar with The Number Framework and with the benefits of using these strategies.

The data for year 7 and year 8 students are presented together. The relatively small differences between these age groups are presented in Chapter 7. Graphs show the percentages of students at each stage on each of the six scales. Then the graphs for the lower and the upper decile schools are presented separately for each scale. Following this, tables present the percentage of students gaining on each scale, for lower decile schools, for the upper decile school, and for the whole group.

Identification of Whole Numbers

Over 60% of year 7 and 8 school students succeeded on all of these items on their initial assessment in identifying whole numbers and knowing what came directly before and after them. This proportion was increased to 80% after one term of teaching. Most of the students who were not at this top stage could read numbers in the thousands, but not larger than that.

This proved to be the easiest of the six scales for this age group. It is important to ensure that all students understand the meaning of all numbers in the range presented, as this is necessary for dealing with Knowledge of Base 10 Grouping and all of the strategies. The high proportion of students reaching the top level does not necessarily suggest that more advanced levels need to be added, although this could be done by adding higher stages, including such topics as the understanding of powers and scientific notation.

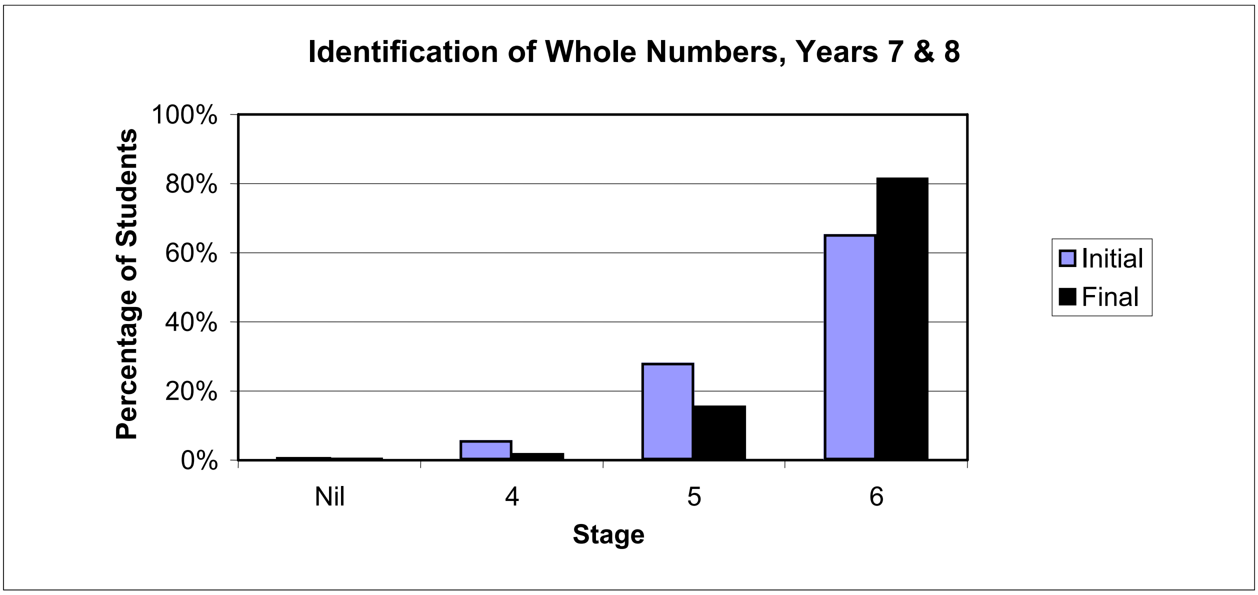
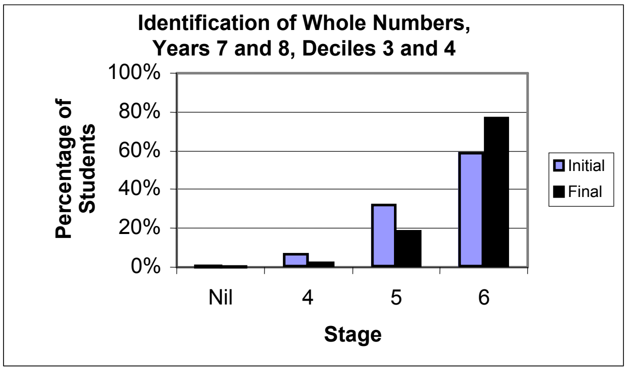


Figure 5.1 Percentage of year 7 and 8 students at each stage on initial and final assessment of Identification of Whole Numbers

Of the students who were not already at the ceiling of this scale, nearly half progressed. The biggest gain was from stage 5 to stage 6. Identification of whole numbers and the numbers preceding and following them appears to be relatively easy for this age group to learn in a short period. The failure of more students to be at this ceiling initially may have been related to their teachers being unaware of this weakness beforehand.

Identification of Whole Numbers by Students from Schools at Different Decile Levels

In the decile 3 and 4 schools, 60% of students were at the ceiling on this scale at the initial assessment, and in the decile 10 school, 80% of the students were at the ceiling at the start of the project. Of those for whom progress was possible, more than half of the students from both of these groups gained either one or two stages. The percentage of students from lower decile schools at the end of one term at each stage was similar to that of students from the upper decile school at the start of the project. This is the one scale on which most students reached the ceiling. This necessary skill appears to be achievable by nearly all students.



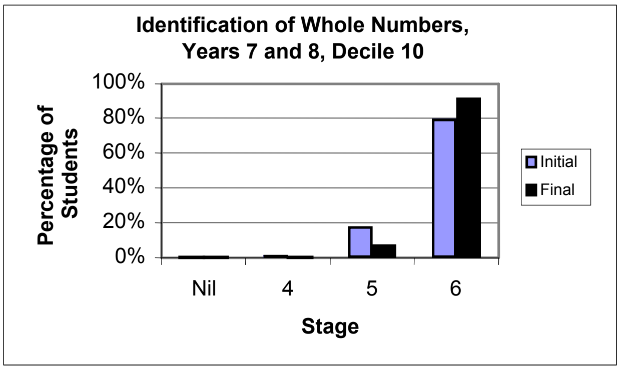


Figure 5.2 **Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Identification of Whole Numbers**

Percentage of Students Gaining One or More Stages

Overall, 65% of students in these years were at the top level initially. Fifty-nine percent of students from lower decile schools and 80% of students from higher decile schools were initially at the top level. Table 5.1 shows the proportion of those who were not already at this top level who gained one or more stages.

Table 5.1 **Percentage of year 7 and 8 students, from lower and upper decile schools making gains on Identification of Whole Numbers**

	Decile 3 and 4	Decile 10	Total
Gain 0*	48%	40%	46%
Gain 1	48%	54%	49%
Gain 2	5%	5%	6%

* Does not include those at ceiling

Overall, 53% of the students from lower decile schools who could gain made progress, 60% of the students from upper schools who could gain made progress, and 54% of the total group who could progress, did so.

Identification of Fractions

The scale for identification and ordering of fractions, decimals, and percentages was considerably more difficult than the scale for Identification of Whole Numbers for these students. There was ample room for progress from initial performance. Note that the vertical axis on this and on most of the later graphs is 50%, while the vertical axis for Identification of Whole Numbers was 100%.

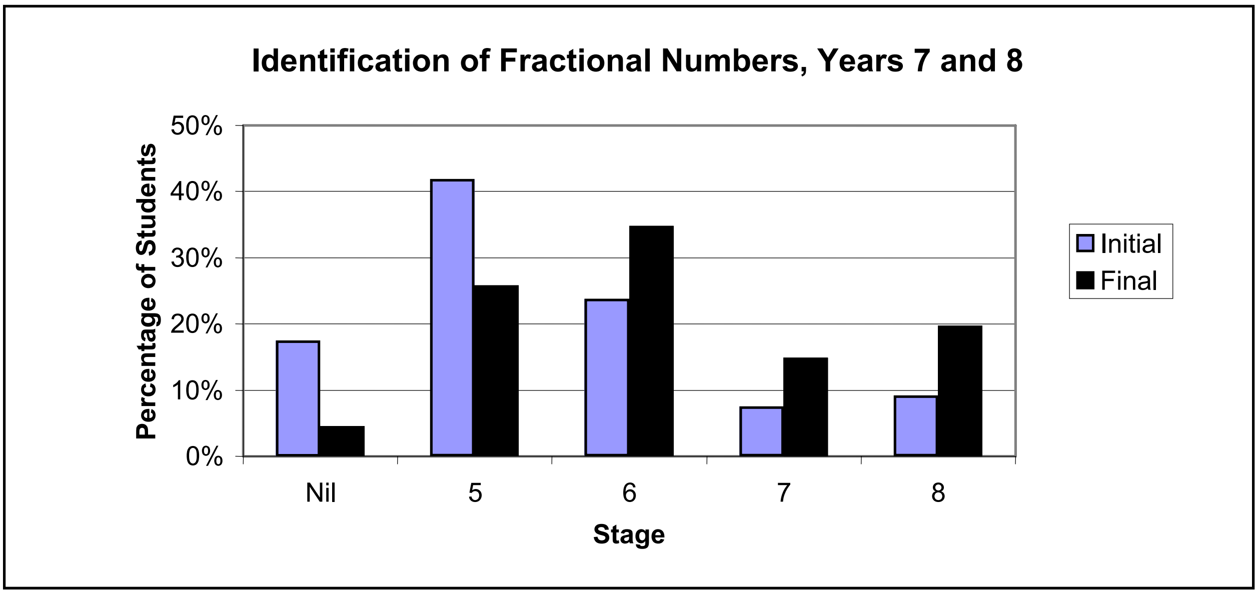
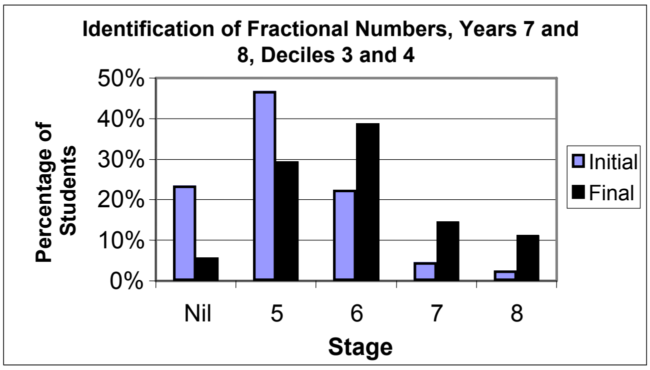


Figure 5.3 **Percentage of year 7 and 8 students at each stage on initial and final assessment of Identification of Fractions**

Teachers and administrators were disturbed at the percentage of students who were unsuccessful on the first item on the initial assessment: the ability to read 1/3 and 1/4 as well as 1/2. However, very few students failed to identify these at the end of the project. The modal stage moved from stage 5 (ability to name these unit fractions) to stage 6 (the ability to order these fractions and to identify decimal fractions to two places).

Identification of Fractions by Students from Schools at Different Decile Levels

Although the profiles for the lower and the upper decile schools look different initially, the profile for lower decile schools at the end of the project was not markedly different from the profile of the students from upper decile schools at the start of the project.



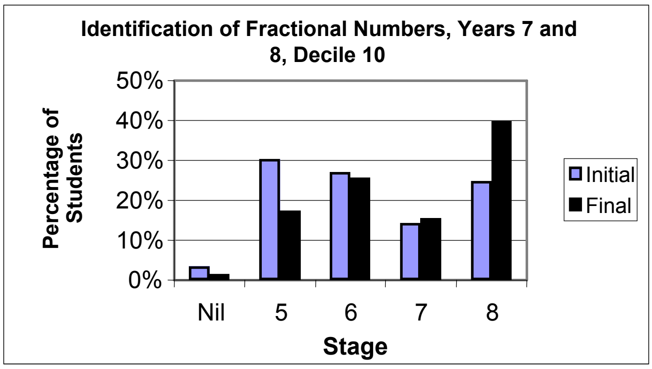


Figure 5.4 **Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Identification of Fractions**

The main difference in this comparison was the percentage of students from the upper decile school who were already at stages 7 and 8 before the project. These students already had a good understanding of decimals. The proportion of students from the upper decile school at the top stages after one term in the project (56%) was markedly different from the proportion of students from the lower decile schools at these stages (26%).

Percentage of Students Gaining One or More Stages on Identification of Fractions

A noticeably higher percentage of students from the decile 10 school were at ceiling on this scale initially, as stated above. Initially, 25% of students from the upper decile school were at the top level, and 3% of students from the lower decile schools were at this level. The percentage of students making gains is given in Table 5.2. (Not all totals equal 100% because numbers have been rounded to the nearest percent.)

Table 5.2 Percentage of year 7 and 8 students from lower and upper decile schools making gains on Identification of Fractions

	Decile 3 and 4	Decile 10	Total
Gain 0*	41%	53%	44%
Gain 1	37%	33%	36%
Gain 2	16%	11%	15%
Gain 3	4%	3%	4%
Gain 4	1%	0%	1%
Gain 5	0%	0%	0%

* Does not include those at ceiling

Overall, 59% of the students from lower decile schools who could gain made progress, 47% of the students from upper schools who could gain made progress, and 56% of the total group who could progress did so. Sixty-eight students (5%) from the lower decile schools made gains of three, four, or five stages, while 11 students (3%) from the upper decile school made gains of this size. This gain is likely to be related both to a low starting stage (see Chapter 8) and to the fact that these students could grasp the concepts relatively quickly once their attention had been focused on them. The percentage gaining five stages did not reach 1%.

Knowledge of Base 10 Grouping

Knowledge of Base 10 Grouping gets at the heart of our place value system, enabling students to gain a solid understanding of decimals and why we can multiply and divide by powers of 10 easily. Many teachers see place value as based on a straight-forward concept. However, young children who work with whole numbers often see it as an additive exercise in which they add another column to the left after they reach nine in the column that they have been working on. It is not until the introduction of decimals, which require division by 10, that students require a better understanding of the base 10 nature of place value.

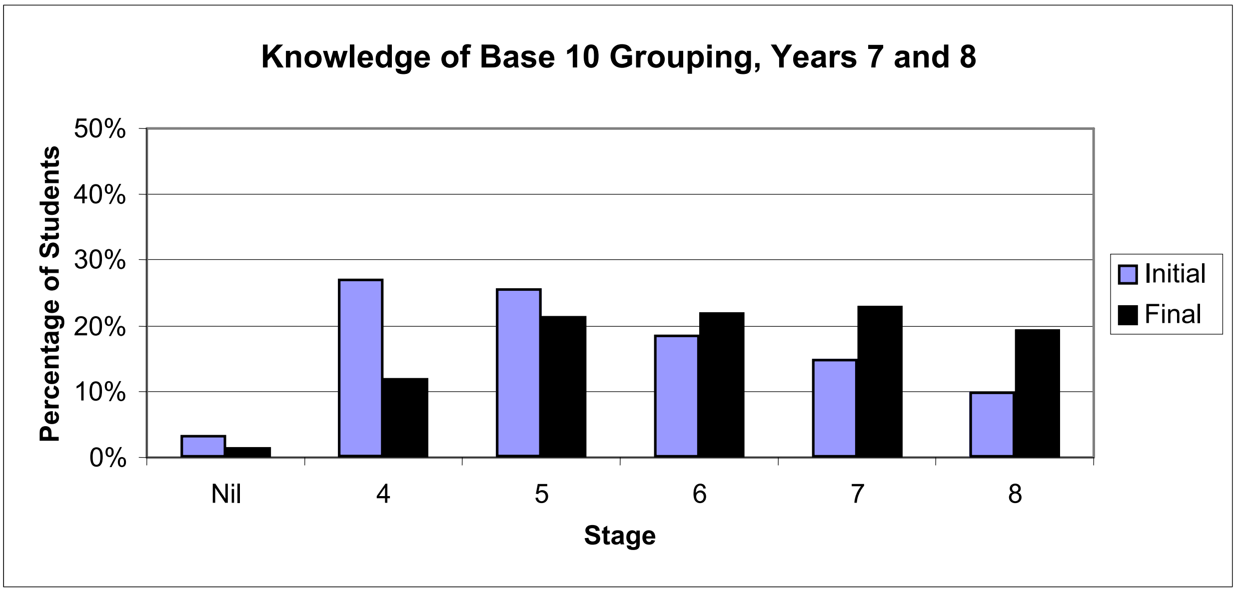
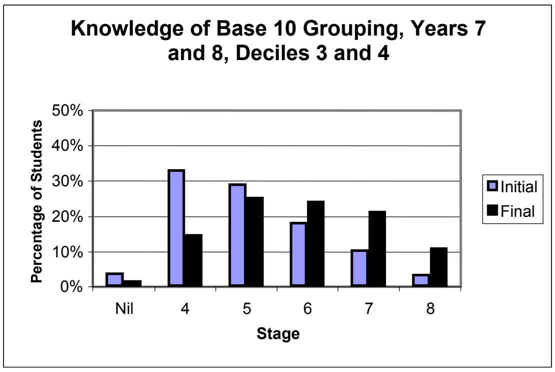


Figure 5.5 Percentage of year 7 and 8 students at each stage on initial and final assessment of Knowledge of Base 10 Grouping

The majority of students on the initial assessment were assessed as being at stage 4 or stage 5, indicating the ability to give the number of 10s in hundreds and thousands respectively. By the end of one term, there was very little difference among the percentage of students who were at stages 5, 6, and 7. Stage 6 reflects the ability to give the number of 10s in any whole number and stage 7 reflects the ability to give the number of 100s as well as 10s in whole numbers and the ability to give the number of tenths in numbers like 2.4.

Knowledge of Base 10 Grouping Stages of Students from Schools at Different Decile Levels

When schools were divided into upper and lower decile groups, quite different patterns of achievement were shown (see Figure 5.6). The modal group for lower decile schools moved from stage 4 to stages 5 and 6. These students could give the number of 10s in thousands or tens of thousands. The students from the upper decile school moved from a modal group at stage 7 (which involves giving the number of 10s or hundreds in any whole number and the number of tenths in one), to having 39% of students at the top stage (being able to give the number of tenths, hundredths, and thousandths in any number and to multiply or divide by 10), which reflects the essence of our place value system.



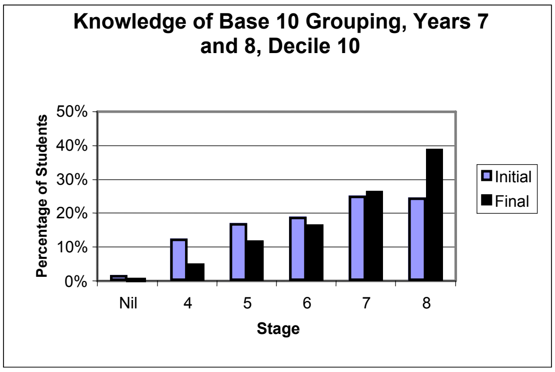


Figure 5.6 Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Knowledge of Base 10 Grouping

Percentage of Students Gaining One or More Stages in Knowledge of Base 10 Grouping

Analysing groups from upper and lower decile schools separately shows that 25% of the upper decile group were at the top stage at the beginning of the project while only 10% of students from lower decile schools were at that stage initially.

Table 5.3 Percentage of year 7 and 8 students from lower and upper decile schools making gains on Knowledge of Base 10 grouping

	Decile 3 and 4	Decile 10	Total
Gain 0*	44%	52%	46%
Gain 1	39%	33%	36%
Gain 2	13%	12%	13%
Gain 3	3%	4%	4%
Gain 4	1%	0%	1%
Gain 5	0%	0%	0%

* Does not include those at ceiling

Overall, 56% of the students from lower decile schools who could gain made progress, 48% of the students from upper schools made progress, and 54% of the total group who could progress did so.

Additive Strategies

This scale looked at the strategies that students could use to add or subtract numbers mentally. Most year 7 and 8 students had at least one mental strategy that involved part-whole thinking at the initial assessment (stage 5). At the end of one term, 48% of students could use a variety of strategies for addition or subtraction (stage 6). The thinking that is involved in using these strategies is called part-whole thinking and is a major goal of the Numeracy Project at the earlier levels. The results for older students of this brief introduction to using these strategies suggest that these students learned to use strategies with relative ease. However, teachers credited mental use of the written algorithm as a part-whole strategy, so the early part-whole stage has a different meaning for this group than it has for young children who have not yet learned to use the vertical algorithm. Mental use of the written algorithm does involve dividing a number into its component parts, usually dealing with the units first. After this students deal with the 10s, and if they continue to think of them as 10s, then this would be considered legitimate part-whole thinking. If they then treat the 10s as units, then they have lost the essential part-whole component that this project concentrates upon.

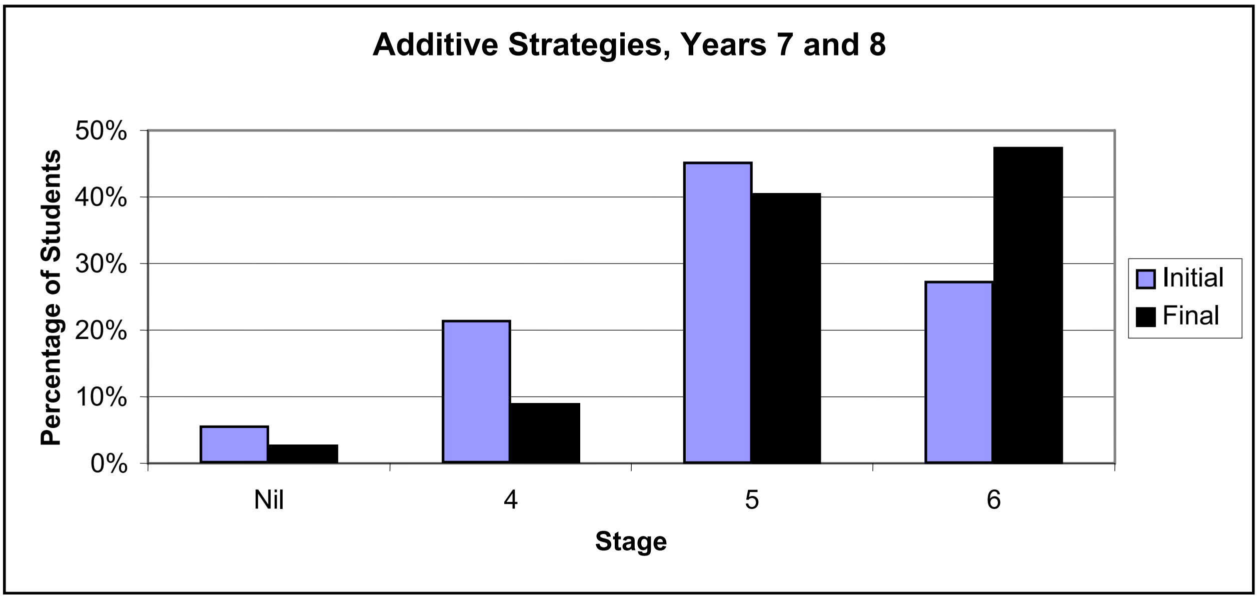


Figure 5.7 Percentage of year 7 and 8 students at each stage on initial and final assessment of Additive Strategies

Additive Strategies of Students in Schools at Different Decile Levels

When students' levels were examined by the deciles of the schools that they attended, the most noticeable difference was the percentage of the students from the upper decile school who were able to use a variety of strategies by the end of the project. In the upper decile school 65% of the students were credited with reaching this top stage in the final assessment, while 40% of students in the lower decile schools reached this stage.

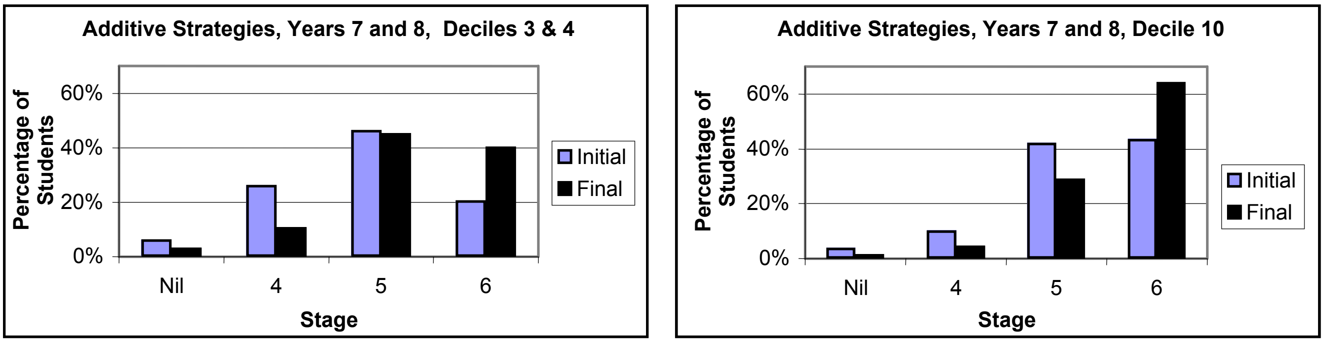


Figure 5.8 Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Additive Strategies

Note that the vertical axis of these graphs goes to 70%, to accommodate the students in the upper decile school at the top stage, while the graph for all year 7 and 8 students (Figure 5.7) has a vertical axis that goes to 50%.

The profile of stages for the students from lower decile schools at the end of the project was similar to the profile of stages for students in the upper deciles before the project.

Gains Made by Students on Additive Strategies

On the initial assessment 18% of the students from the lower decile schools were at the top stage, 44% of students from the upper decile school were at the top stage, and 26% percent of the total group were at the top stage. Table 5.4 gives the percentage of students who were not already at this top stage who gained.

Table 5.4 Percentage of year 7 and 8 students, from lower and upper decile schools, making gains on Additive Strategies

	Decile 3 and 4	Decile 10	Total
Gain 0*	54%	54%	54%
Gain 1	41%	42%	41%
Gain 2	5%	6%	5%
Gain 3	0%	0%	0%

* Does not include those already at ceiling

Overall, the percentage of the students from both groups and from the total who could make gains, and did so was 46%.

Multiplicative Strategies

This scale looked at the strategies that students could use to multiply or divide numbers mentally. This scale includes stages in which students use counting or adding strategies in order to do a multiplication problem. If a student uses skip counting for a multiplication problem they are assessed as being at stage 4 and if they use repeated addition they are assessed as being at stage 5. True multiplicative thinking, which involves conceptualising and manipulating groups of numbers in the same way that adding involves conceptualising and working with single numbers, is credited as stage 6 or stage 7, depending on whether the student can mentally use only a few Multiplicative Strategies or a variety of such strategies. Students must know their multiplication tables to work at these levels.

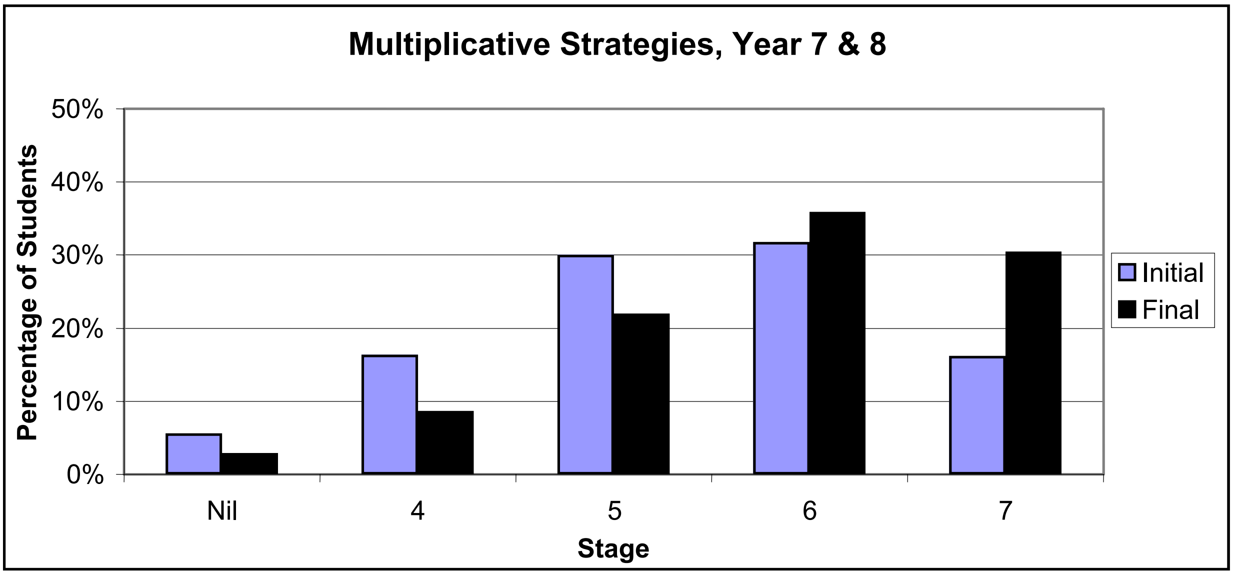


Figure 5.9 Percentage of year 7 and 8 students at each stage on initial and final assessment of Multiplicative Strategies

This graph demonstrates that initially 52% of the students used addition strategies while 48% used multiplicative part-whole strategies. By the end of the project, 66% of the students used multiplicative part-whole strategies mentally for these problems.

Multiplicative Strategies of Students in Schools at Different Decile Levels

As the graphs in Figure 5.10 show, at the end of the project the students in lower decile schools were at stages similar to those of students in the upper decile school at the start of the project. Both groups gained. On the final assessment nearly half (47%) of the year 7 and 8 students from the upper decile school were at the top stage, able to use a range of part-whole strategies. This was at the end of one term of teaching on the Numeracy Project. Of the two groups, 60% of students from lower decile schools could use some genuinely Multiplicative Strategies by the end of the project (stages 6 and 7) and 81% of the students from the upper decile school were able to use these Multiplicative Strategies. This is a good outcome.

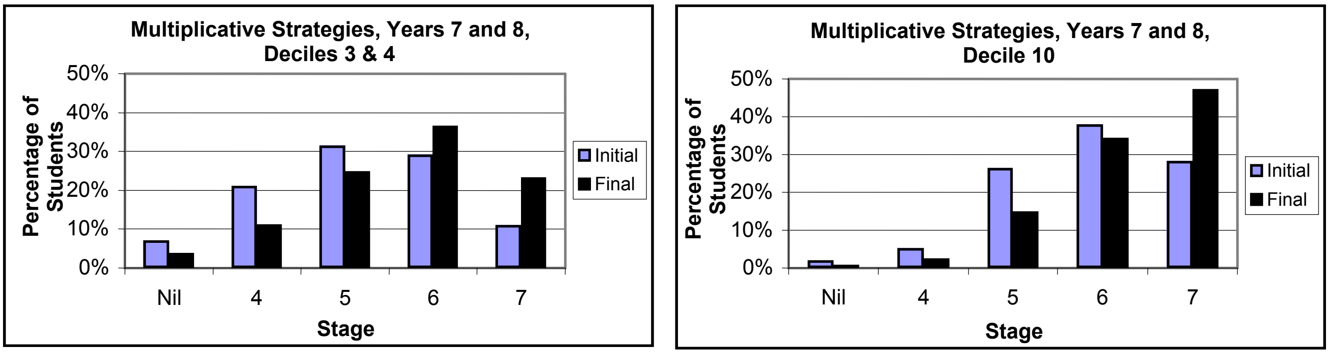


Figure 5.10 Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Multiplicative Strategies

Gains Made by Students on Multiplicative Strategies

What may be a more important question than who achieved the top stage is the percentage of students that learned about using strategies. On this scale, 12% of students from lower decile schools and 28% of students from the upper decile school were at the top stage initially, while 17% of the entire group of students was at the top stage initially. Percentages in Table 5.5 relate to the students for whom gain was possible.

Table 5.5 Percentage of year 7 and 8 students, from lower and upper decile schools, making gains on Multiplicative Strategies

	Decile 3 and 4	Decile 10	Total
Gain 0*	56%	53%	55%
Gain 1	35%	43%	37%
Gain 2	7%	4%	6%
Gain 3	2%	1%	1%
Gain 4	0%	1%	0%

* Does not include those at ceiling.

Overall, 44% of the students from lower decile schools who could gain made progress, 47% of the students from upper schools who could gain made progress, and 45% of the total group who could progress did so. There were more students from lower decile schools than from the upper decile school making gains of three or four stages (22 versus 5) although the percentage of students was similar.

Ratio and Proportional Strategies

The most difficult scale in The Number Framework proved to be that which requires mentally working out fractional parts, ratios, and proportions. Note that the vertical axis on these graphs is 40% rather than 50% as in most of the previous graphs.

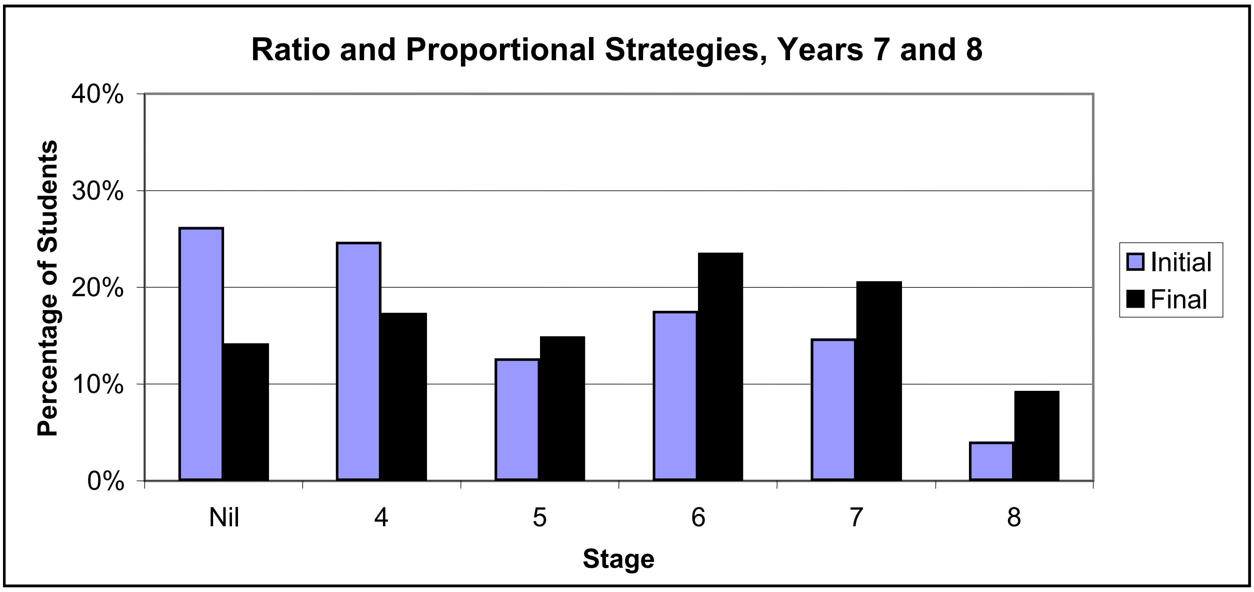
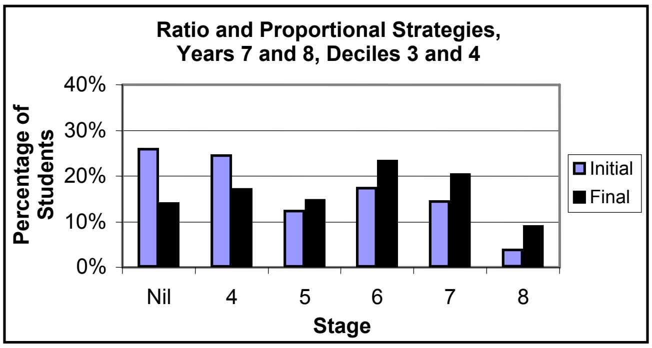


Figure 5.11 Percentage of year 7 and 8 students at each stage on initial and final assessment of Ratio and Proportional Strategies

Most teachers were surprised at the high percentage of students who could not find a fraction of a whole number, in this case 1/3 of 24 (nil – 26%). They appeared not to know how to do the problem. This alerted the teachers to an aspect that needed teaching. In Chapter 6 we show that the proportion of year 9 students unable to do this was even higher than the proportion of year 7 and 8 students. Students credited with the next stage, stage 4, were able to do this and similar problems only with concrete objects. Thus 51% were at these bottom two stages initially. By the end of one term, the largest group (45%) was at stage 6 or stage 7 for Ratio and Proportional Strategies (able to do more complex problems mentally with additive and multiplicative part-whole strategies). This is a great credit to the teaching done in this project.

Ratio and Proportional Strategies of Students in Schools at Different Decile Levels

The profile of final assessments for students from lower decile schools was similar to that of students from higher decile schools before the intervention, except that a higher proportion of students from the upper decile school were at stages 6 and 7 initially.



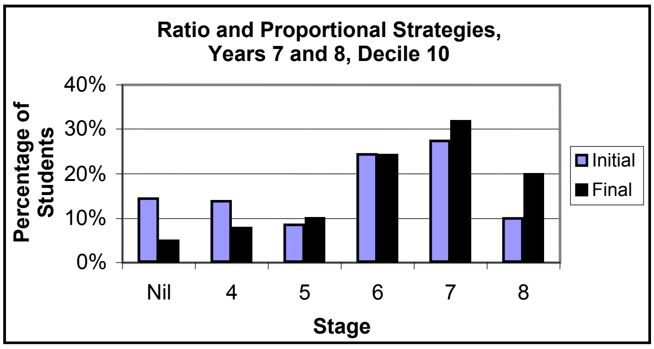


Figure 5.12 Percentage of year 7 and 8 students from lower and upper decile schools at each stage on initial and final assessment of Ratio and Proportional Strategies

These graphs demonstrate that considerably higher proportions of students from the upper decile school were at the top three stages after the project. As the most difficult concept in The Number Framework, proportional thinking using part-whole strategies could be seen as the goal for this age group, as part-whole thinking for addition is seen as the goal for young children. Forty-four percent of the students from lower decile schools reached these stages while 76% of the students from upper decile schools reached them.

Gains Made by Students on Ratio and Proportional Strategies

While 1% of students from lower decile schools were at the top stage initially, 10% of students from the upper decile school were at the top stage on initial assessment. Table 5.6 shows the percentage of students who were not already at the top stage who gained one or more stages.

Table 5.6 Percentage of year 7 and 8 students, from lower and upper decile schools making gains on Ratio and Proportional Strategies

	Decile 3 and 4	Decile 10	Total
Gain 0*	59%	53%	56%
Gain 1	26%	31%	28%
Gain 2	10%	10%	10%
Gain 3	5%	6%	5%
Gain 4	1%	1%	1%
Gain 5	0%	0%	0%

* Does not include those at ceiling

Overall, 41% of the students from lower decile schools that could gain made progress, 47% of the students from upper schools that could gain made progress, and 44% of the total group who could progress did so. One student from a lower decile school gained five stages, but this does not show up as a percentage. As on other scales there were more students from lower decile schools than from the upper decile school making gains of three, four, or five stages, but the proportion of students was similar. It may be that none of these students had specifically been taught how to do problems of this type, so there was marked room for growth for most students.

Comparison of Initial Stages of Year 9 Students and Final Stages of Year 7 and 8 Students

A way of showing whether these students could be expected to have made the same amount of progress without intervention is to compare the initial scores of year 9 students with the initial and final scores of year 7 and 8 students. Table 5.7 compares the initial percentages of year 9 students at each stage with the initial and final year 7 and 8 scores for the three strategy scales.

Additive Strategies

Table 5.7 Comparison of initial year 9 scores with initial and final year 7 and 8 scores for Additive Strategies

Stage	Year 9 initial	Year 7 and 8 initial	Year 7 and 8 final
Nil	6%	6%	3%
4	19%	22%	9%
5	47%	45%	41%
6	27%	27%	48%

If students developed strategies without this particular programme, then the percentage of year 9 students should be similar to that of year 7 and 8 students at the end of the project. Instead, the percentage of year 9 students at each stage is remarkably similar to that of the year 7 and 8 students at the start of the project. Students do not appear to develop these skills within our educational system without this intervention.

Multiplicative Strategies

The same comparison of initial year 9 scores and initial and final year 7 and 8 scores has been done for Multiplicative Strategies, as shown in Table 5.8.

Stage	Year 9 initial	Year 7 and 8 initial	Year 7 and 8 final
Nil	4%	2%	1%
4	13%	5%	3%
5	33%	27%	15%
6	34%	38%	34%
7	16%	28%	47%

Table 5.8 Comparison of initial year 9 scores with initial and final year 7 and 8 scores for Multiplicative Strategies

In this comparison, year 9 students did less well initially than did year 7 and 8 students on either their initial or final assessment. There was a higher percentage of year 9 than year 7 and 8 students at the two lowest stages (17% versus 7%) and a lower percentage of year 9 students than year 7 and 8 students at the highest stage (16% versus 28%). If these data are representative of our wider population, they suggest that students lose skills in this domain between intermediate and secondary schools.

Ratio and Proportional Strategies

Table 5.9 presents the same comparison of initial year 9 and initial and final year 7 and 8 percentages for Ratio and Proportional Strategies.

Table 5.9 Comparison of initial year 9 scores with initial and final year 7 and 8 scores for Ratio and Proportional Strategies

	Year 9 initial	Year 7 and 8 initial	Year 7 and 8 final
Nil	37%	26%	14%
4	20%	25%	17%
5	7%	13%	15%
6	19%	18%	24%
7	14%	15%	21%
8	2%	4%	9%

Year 7 and 8 students did better initially and finally than year 9 students did at the start of the project. Initially, 37% of year 9 students did not know how to find 1/3 of 24 while only 26% of year 7 and 8 students did not know how to do this. The percentage of year 9 students at the top three stages at the beginning of the project was similar to those percentages at the start of years 7 and 8, and below the percentages at the end of the project for year 7 and 8 students. As with the results for Multiplicative Strategies, this may have been due to sampling error, and results can be compared with those achieved in 2002.

Summary

The data presented in this chapter show that the following percentages of year 7 and 8 students made progress on the six scales. Only those students who were not at the top stage initially were considered.

Table 5.10 Percentage of year 7 and 8 students gaining at least one stage on each of the six scales in NEST

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
Percentage of students gaining at least one stage	54%	56%	54%	46%	45%	44%

About 55% of all students made progress on the knowledge scales, and about 45% of students made progress on the strategy scales.

A higher percentage of students from the upper decile schools made progress on the scales for Identification of Whole Numbers, Multiplicative Strategies, and Ratio and Proportional Strategies. A higher percentage of students from the lower decile schools made progress on Identification of Fractions and Knowledge of Base 10 Grouping. The same proportion of each group made progress on Additive Strategies.

The two strategy scales on which a higher proportion of students from the higher decile schools progressed were those that assessed the more advanced strategies of multiplicative reasoning and reasoning for ratio and proportional problems.

These figures can be looked at from two points of view: emphasising either the percentage that made progress or the percentage that did not make progress (which would be 100%, less the figures above). For an initial study, in which teaching took place for only one term, it seems more appropriate to look at the figures as positive, while being aware that not all students appeared to gain from the programme. The steps between stages are large, especially at the upper levels, and for students to make this much gain is impressive. This data is explored further in Chapter 7.

6. Gains Made by Secondary School Students

Many of the secondary students involved in NEST made marked gains in each scale in this project. There were some differences between the pattern of their advancements and that of the year 7 and 8 students, but in general the patterns of improvement were similar.

The results for the seven year 10 students are omitted from this chapter.

Graphs show stages attained by students both before and after a term's involvement in NEST. Additional graphs compare students from lower decile schools with those from upper decile schools. Finally, a table in each chapter details the gains made by students between the initial and final assessments.

Only one secondary school appeared to have based the entire term's programme on the Numeracy Project. Teachers' and facilitators' reports from other schools indicated that the project had been focused on for variable lengths of time, from a few days to a few weeks. In some cases the Numeracy Project was the focus only for "starters" to the lessons. The decisions on how much time to spend on numeracy might have been the right decision for the schools involved. However, this variation does not allow us to say that gains made were the result of the same amount of instruction for all students.

Identification of Whole Numbers

Initially, 67% of the year 9 students were at the ceiling of this portion of the assessment, being able to read all the numbers they were shown. After one term of teaching, the percentage at the top stage had increased to 83%. Of the 27% of students who did not reach stage 6 in the final assessment, most reached stage 5. Therefore, after one term's instruction 98% of the students were able to reach the top two stages of the assessment. This appears to be an area that is well taught and understood by the majority of year 9 students (see Figure 6.1).

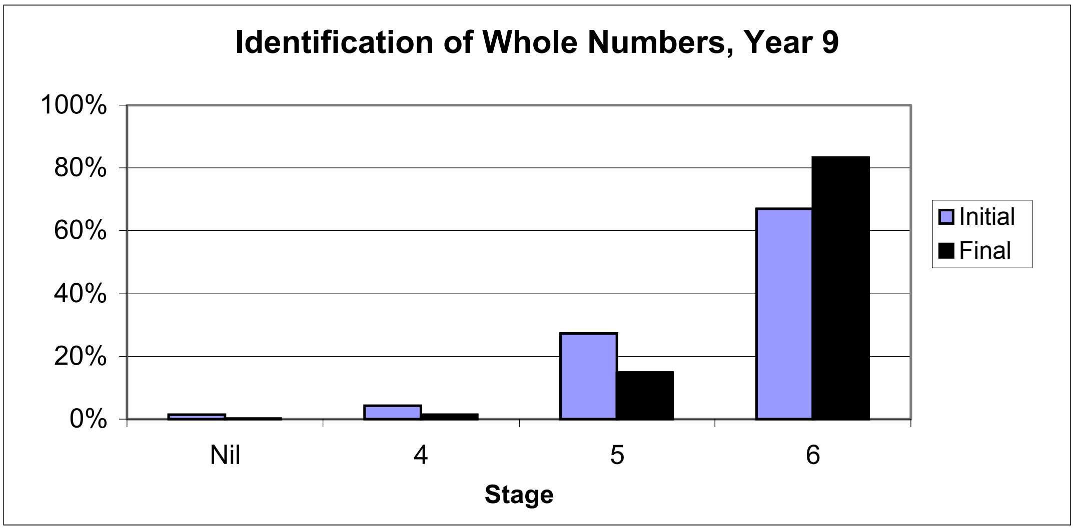
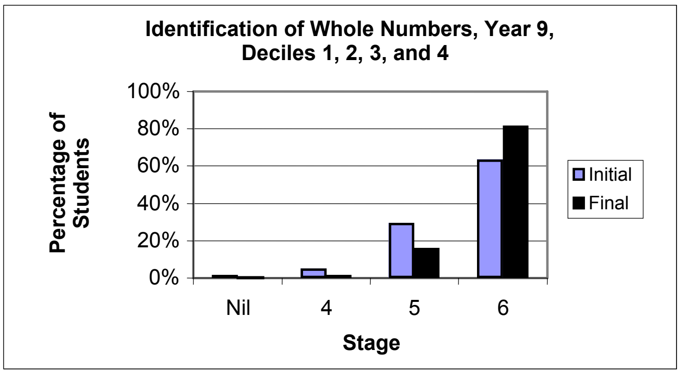


Figure 6.1 **Percentage of year 9 students at each stage on initial and final assessment of Identification of Whole Numbers**

Identification of Whole Numbers by Students from Schools at Different Decile Levels

Results were separated to enable comparison between the assessment results of students from lower and upper decile schools. In the initial assessment, 64% of the students from lower decile schools attained stage 6. However, after one term of teaching, 82% of these students had already attained stage 6. This figure compared favourably with the initial assessment result for the upper decile schools. Within one term, students from the lower decile schools had caught up with where the students from the upper decile schools were prior to receiving instruction. In the upper decile schools the proportion at the top stage had increased from 83% to 91%.



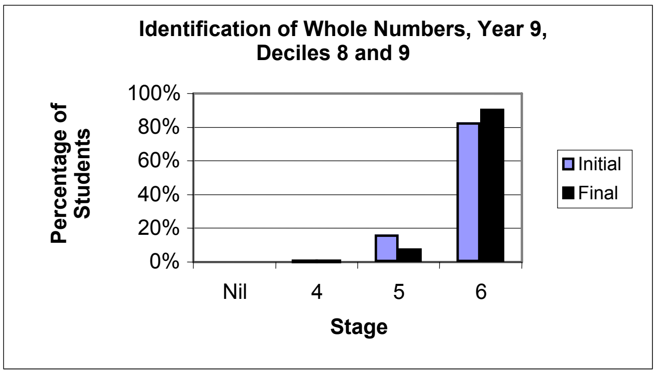


Figure 6.2 **Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Identification of Whole Numbers**

Percentage of Students Gaining One or More Stages on Identification of Whole Numbers

In the lower decile schools, 64% of the students were at the top stage of this scale at the first assessment. Of the students who were not at the ceiling, 58% made gains between the initial and final assessment and 42% made no gain. Most of the students who did not gain were assessed as being at stage 5 on the initial assessment. These students were able to identify numbers in the range 1 to 1,000 and give numbers before or after a number in this range, but were unable to identify numbers in the 1 to 1,000,000 range in a similar fashion.

In the upper decile schools, 83% of the students were at the top stage at the initial ssessment. Of the students who were not at the ceiling, 48% gained at least one stage and 52% made no gain. Most of the students who did not gain were on stage 5.

Overall, 67% of students were at the ceiling in the initial assessment.

Table 6.1 Percentage of year 9 students gaining on Identification of Whole Numbers

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	42%	52%	43%
Gain 1	51%	48%	51%
Gain 2	6%	0%	5%
Gain 3	1%	0%	1%

* Does not include those at ceiling

Identification of Fractions

There was more room for movement of students in the section of the assessment on Identification of Fractions, with 76% of all students at stage 5 or stage 6 when they were initially assessed. Many teachers were concerned at the number of their students (10% of all students) who could not identify unit fractions such as 1/4 or 1/3. A further 40% of year 9 students were initially assessed as being at stage 5. These students could identify unit fractions but could not successfully complete stage 6 tasks, which involve ordering unit fractions and identifying decimals to two places.

After one term of teaching there was a marked improvement, with 85% of students assessed as being at stage 6 or higher. As with the year 7 and 8 students, the overall modal stage moved from stage 5 (the ability to name unit fractions) to stage 6 (the ability to order unit fractions and identify decimals to two places).

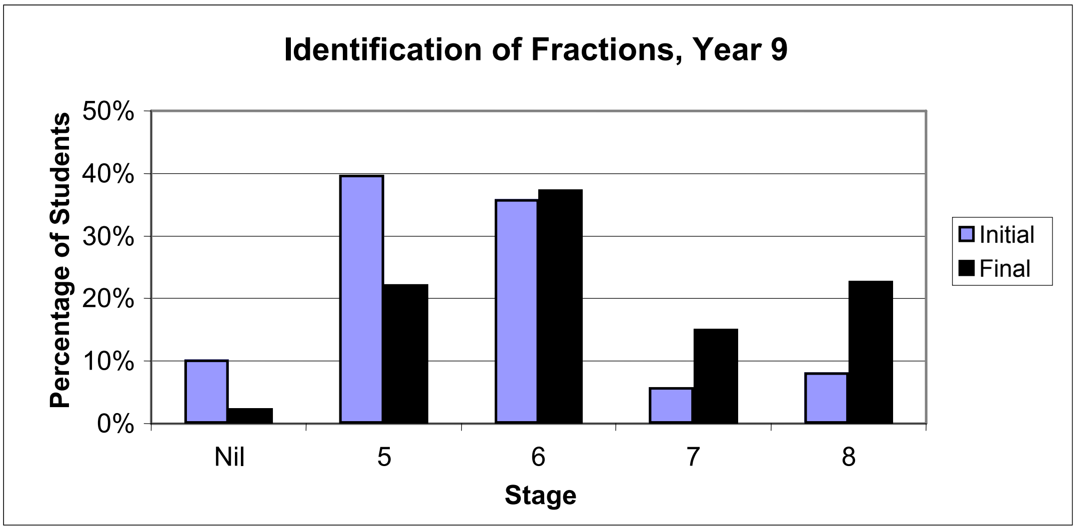


Figure 6.3 **Percentage of year 9 students at each stage on initial and final assessment of Identification of Fractions**

Identification of Fractions by Students from Schools at Different Decile Levels

Comparing students' results according to their schools' decile rating showed that students from the lower decile schools moved from an initial modal level of stage 5 to a final modal level of stage 6. Within the space of a term, students from the lower decile schools were able to attain slightly better results than the initial assessment results for the students from the higher decile schools.

Students from the higher decile schools moved from an initial modal level of stage 6 to a final modal level of stage 8.

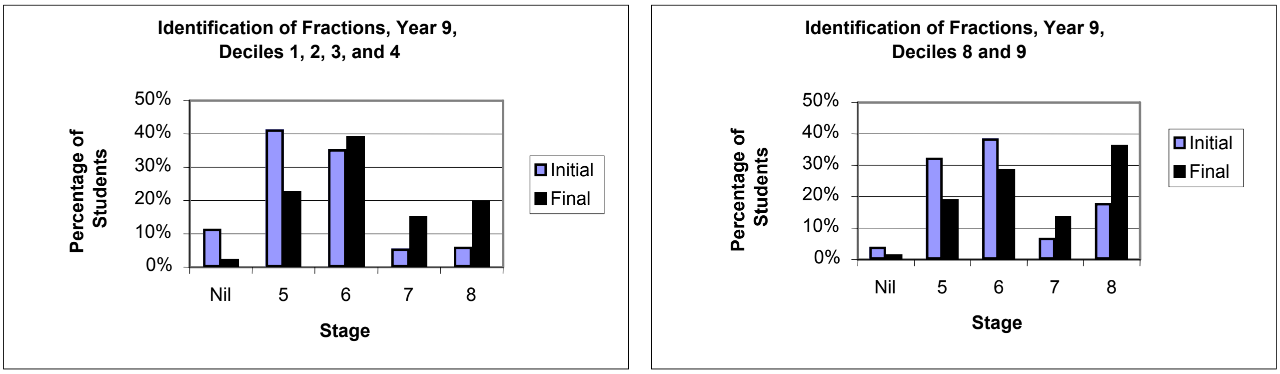


Figure 6.4 **Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Identification of Fractions**

Percentage of Students Gaining One or More Stages on Identification of Fractions

Overall, 8% of the students were at the ceiling on this scale at the initial assessment. This figure includes 6% of the students from the lower decile schools and 18% of the students from the upper decile schools.

Table 6.2 Percentage of year 9 students gaining on Identification of Fractions

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	48%	47%	48%
Gain 1	26%	34%	27%
Gain 2	18%	16%	18%
Gain 3	7%	3%	6%
Gain 4	1%	0%	0%
Gain 5	0%	0%	0%

* Does not include those at ceiling

Therefore 52% of the students from lower decile schools and 53% of the students from upper decile schools who could gain, did so. Nine students from the lower decile schools made major shifts in their understanding on this scale in the course of one term.

Knowledge of Base 10 Grouping

When students were initially tested, 55% of students were not able to state the number of 10s in any whole number, which is stage 6 on this part of the assessment. After one term's instruction, 69% of students had reached this stage or higher. Students' Knowledge of Knowledge of Base 10 Grouping responded well to teaching, although there is certainly still room for growth in this area. See Figure 6.5.

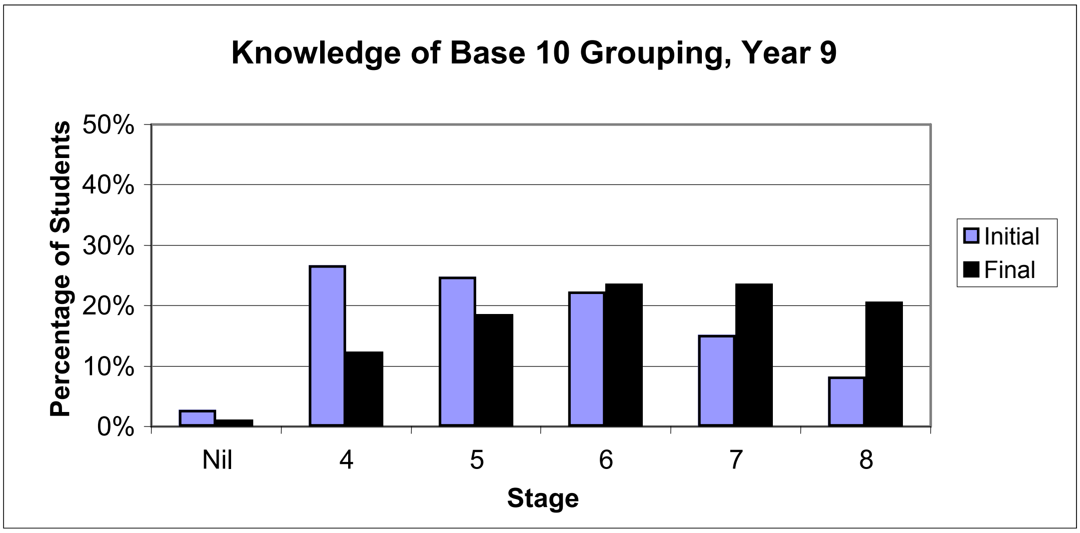
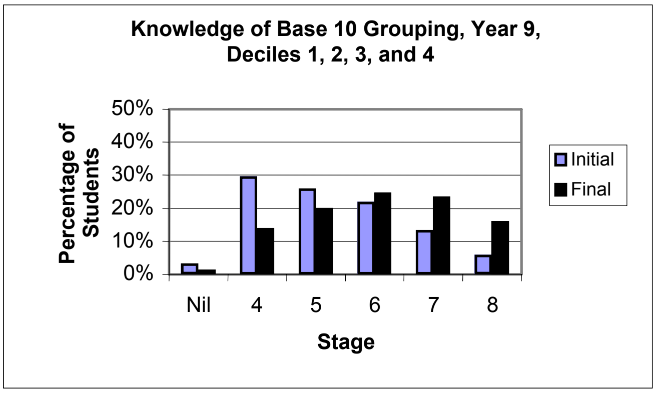


Figure 6.5 **Percentage of year 9 students at each stage on initial and final assessment of Knowledge of Base 10 Grouping**

There was a marked difference between the assessment results for the lower and higher decile schools. In the lower decile schools, 45% of students attained stage 6 or higher in the initial assessment. After one term's teaching this number had risen to 69%. In comparison, 68% of students in the higher decile schools attained stage 6 or higher in their initial assessment. This figure rose to 84% in the final assessment. In the final assessment, 43% of students from the higher decile schools were at the ceiling, compared with 21% of students from the lower decile schools.



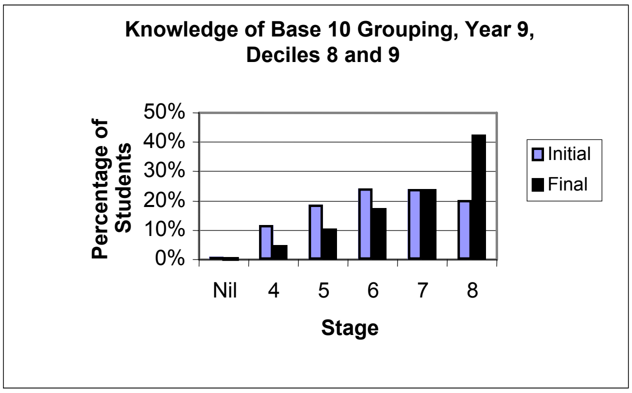


Figure 6.6 **Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Knowledge of Base 10 Grouping**

If we consider the overall shapes of the graphs, the final results for students from the lower decile schools tend to match the initial assessment results for students from the higher decile schools.

Percentage of Students Gaining One or More Stages on Knowledge of Base 10 Grouping

Overall, 8% of students were at the top stage of the scale for the initial assessment. However, only 6% of the students from the lower decile schools were at the ceiling, compared with 20% of the students from the upper decile schools.

Of the students not at the top stage initially, 51% made gains of one or more stages between the initial and final assessments. The most impressive gains were made by 14 students from the lower decile schools, who started the programme with quite a low level of understanding but made gains of four or five stages in the space of one term.

Table 6.3 Percentage of year 9 students gaining on Knowledge of Base 10 Grouping

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	51%	39%	49%
Gain 1	29%	40%	30%
Gain 2	14%	18%	15%
Gain 3	5%	3%	5%
Gain 4	1%	0%	1%

* Does not include those at ceiling

Of the students not already at the top stage, 49% of the students from lower decile schools and 61% of the students from upper decile schools gained.

Additive Strategies

Students made sound progress, over the course of the term's teaching, in their ability to use a range of strategies for addition and subtraction problems. In the initial assessment, 75% of students were assessed as being at stage 5 or 6 (being able to use a limited or wider range of strategies for addition and subtraction problems). In the final assessment, 89% of students reached this level. Initially, 19% of students were identified as being at stage 4, where students solved addition or subtraction problems by counting on or counting back. This had dropped to 9% of students after a term's teaching (see Figure 6.7).

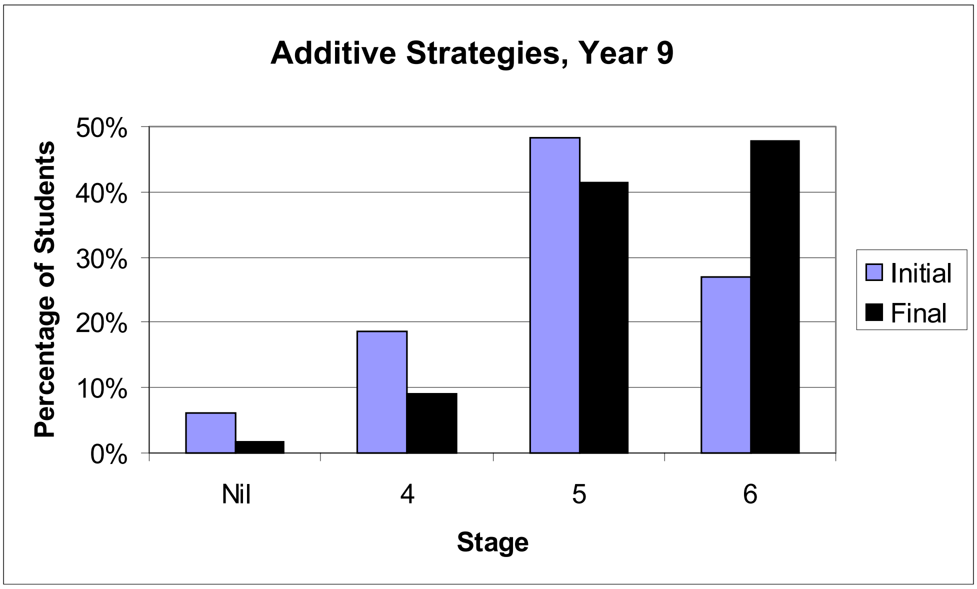


Figure 6.7 Percentage of year 9 students at each stage on initial and final assessment of Additive Strategies

Additive Strategies of Students from Schools at Different Decile Levels

The modal stage for both groups of students moved from stage 5 to stage 6. At the end of a term's teaching, 43% of students from the lower decile schools were at stage 5 and 45% were at stage 6. In the higher decile schools, the number of students with a range of strategies for solving addition or subtraction problems (i.e., students at stage 6) increased from 33% to 62%. In the lower decile schools, the percentage of students at stage 6 increased from 26% to 45%. Once again, it would appear that the teaching focus in this programme enabled the students from lower decile schools to catch up with the initial results for students from higher decile schools, with just one term of teaching.

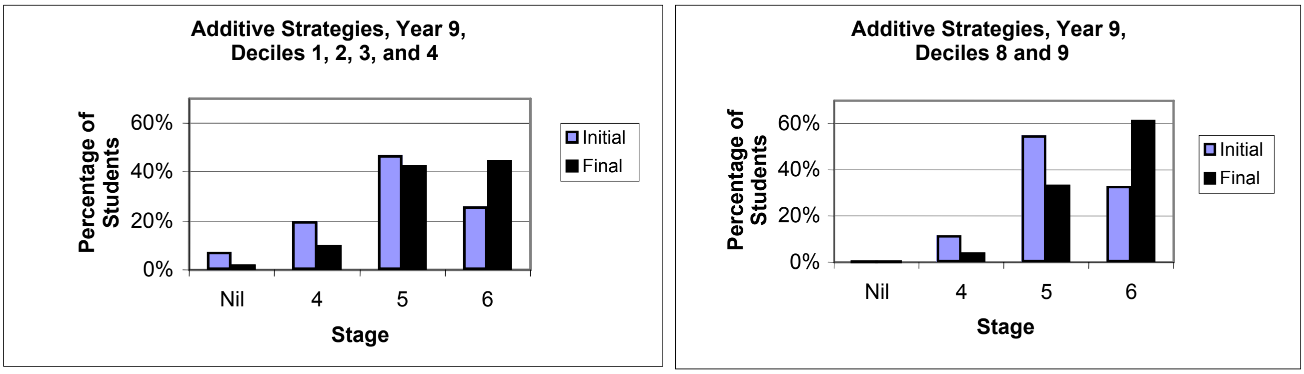


Figure 6.8 Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Additive Strategies

Percentage of Students Gaining One or More Stages on Additive Strategies

In the initial assessment, 27% of the students were at the ceiling on this scale. This figure includes 26% of the students from the lower decile schools and 33% of the students from the upper decile schools.

In the final assessment, 55% of students remained at the same stage in their use of strategies for solving addition or subtraction problems, even though these students were not at the ceiling of the assessment. This could be because, at this age, more students are entrenched in their use of the standard algorithms to solve these problems and find it difficult to change their approach. It could also be that inadequate time was spent in teaching these strategies.

Table 6.4 Percentage of year 9 students gaining on Additive Strategies

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	56%	48%	55%
Gain 1	37%	48%	39%
Gain 2	7%	5%	6%
Gain 3	1%	0%	1%

* Does not include those at ceiling

Of the students who could gain, 45% of the students from lower decile schools and 52% of the students from upper decile schools did so. Overall, 45% of students who could gain did so.

Multiplicative Strategies

Students were assessed on the strategies they were able to use to solve a range of multiplication and division problems. Students at stages 4 and 5 used counting or adding strategies to solve these problems. Initially, 46% of students were assessed as being at either stage 4 or stage 5. This fell to 29% in the final assessment.

To be assessed as being at stage 6, students needed to have a good grasp of their multiplication tables and be able to use this knowledge to solve given problems. Stage 7 required students to use multiplication facts with a range of part-whole strategies. The percentage of students attaining stage 6 or stage 7 rose from 50% in the initial assessment to 69% in the final assessment.

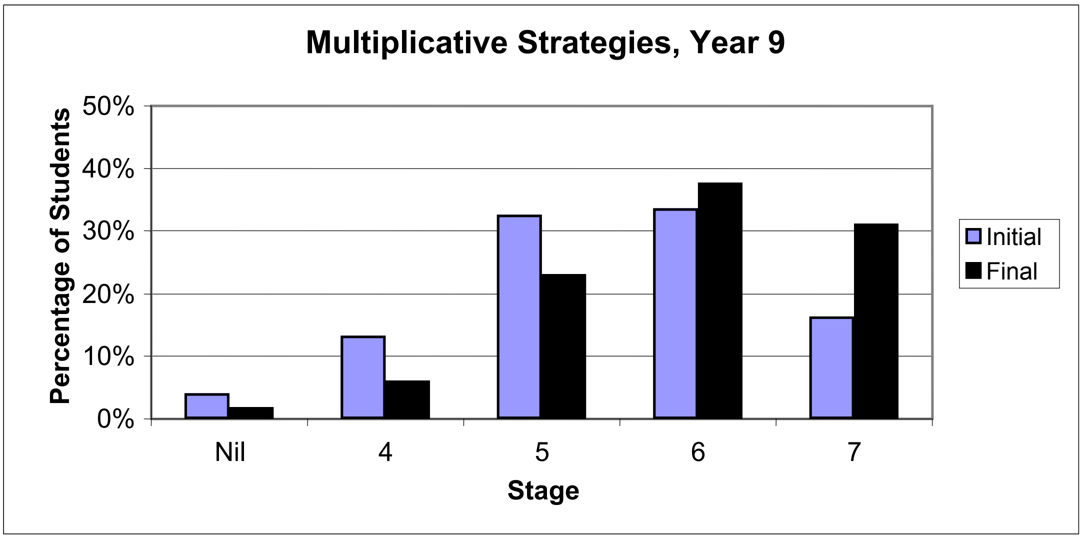
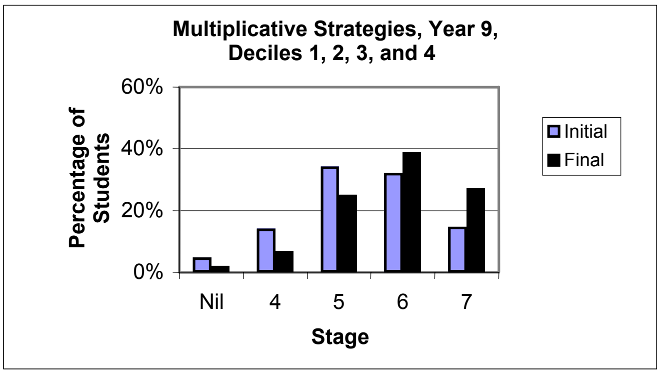


Figure 6.9 **Percentage of year 9 students at each stage on initial and final assessment of Multiplicative Strategies**

Multiplicative Strategies of Students from Schools at Different Decile Levels

The initial assessment results from the lower decile schools placed 53% of students at, or below, stage 5 (using additive part-whole strategies to solve multiplication or division problems). After a term's teaching, 34% of students were still unable to use methods other than counting or Additive Strategies to solve multiplication or division problems. Once again, final results for students from lower decile schools tended to match initial results for students from higher decile schools.



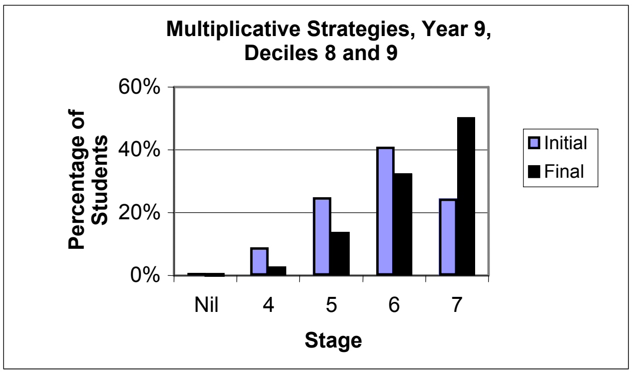


Figure 6.10 **Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Multiplicative Strategies**

Both groups of students showed a marked improvement overall between the initial and final assessments. The percentage of students from the lower and upper decile schools attaining stage 7 approximately doubled for both groups, between the initial and final assessments.

Percentage of Students Gaining One or More Stages on Multiplicative Strategies

Initially, 16% of all students were at the top stage of this scale. This was made up of 15% of students from lower decile schools and 24% of students from upper decile schools. Of the students who could make gains, 44% did so. Comparing results based on a decile level, we find that 42% of the students from the lower decile schools and 56% of the students from the upper decile schools who could make gains, did so.

Of the students who could make gains, only 42% of the students from the lower decile schools did so. In contrast, 56% of the students from the upper decile schools who could make gains, did so. Perhaps the higher percentage of students (53%) from the lower decile schools who were initially assessed at stage 5 or lower lacked a sound grasp of their multiplication tables and were unable to attain this knowledge within the one term time frame.

Table 6.5 Percentage of year 9 students gaining on Multiplicative Strategies

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	58%	44%	56%
Gain 1	33%	45%	35%
Gain 2	8%	11%	8%
Gain 3	1%	0%	1%

* Does not include those at ceiling

Ratio and Proportional Strategies

This is the most advanced scale in the Numeracy Project and the results reflect this.

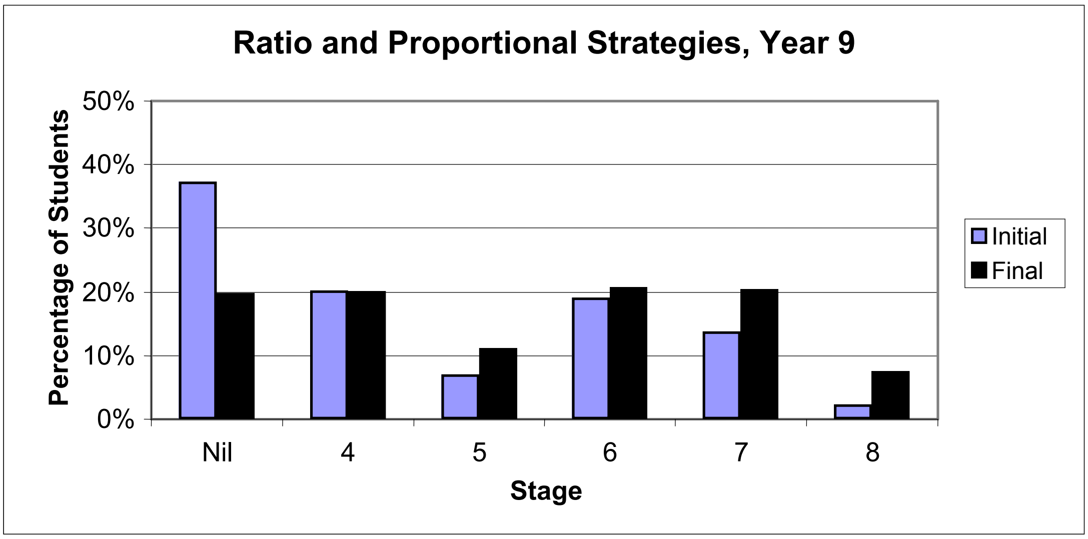


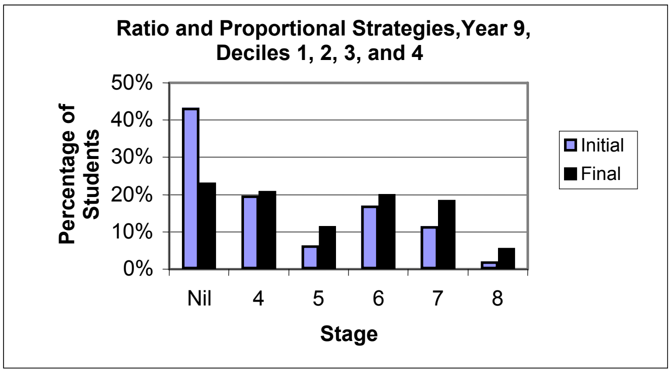
Figure 6.11 Percentage of year 9 students at each stage on initial and final assessment of Ratio and Proportional Strategies

On the first question in this assessment, students were asked to find a fraction of a whole number (stage 4), while at stage 8 students had to demonstrate the ability to use a range of part-whole strategies for solving proportional reasoning problems.

Teachers were initially shocked at the high percentage of students who could not find 1/3 of 24, the first question in this part of the assessment. On the initial assessment, 37% of students were unable to answer this question and were assessed as being below stage 4. This figure had reduced to 20% in the final assessment. A further 20% of year 9 students were assessed as being at stage 4 on the final assessment. A slightly smaller percentage of students gained on this scale than had on the other scales of the assessment, and a smaller percentage gained the top stage after one term. Whether this was due to less teaching time being spent on this chapter of the teaching programme or to this aspect of the Numeracy Project being conceptually more difficult for students is not known.

Ratio and Proportional Strategies of Students from Schools at Different Decile Levels

Separating results for students from lower and upper decile schools highlights a stark contrast in students' assessment results. Initially, 43% of students from the lower decile schools failed to reach stage 4 on this section of the assessment, compared with 8% of students from the higher decile schools. The initial modal stage for students from lower decile schools was nil and the modal stage for this group on final assessment moved to stage 4. For the students from upper decile schools the modal stage was initially stage 6 and this moved to stage 7 on the final assessment. This was the only scale where final assessment results for students from lower decile schools did not reach the initial assessment results for students from the higher decile schools.



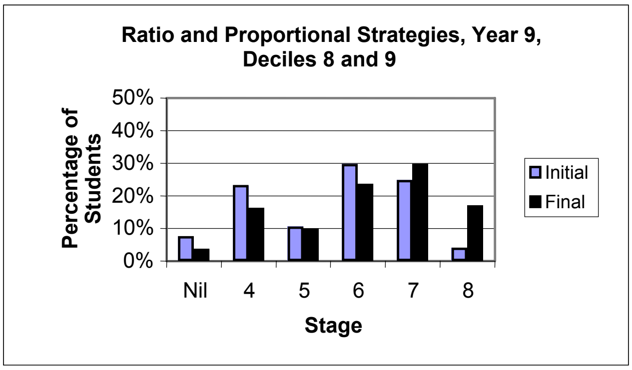


Figure 6.12 Percentage of year 9 students from lower and upper decile schools at each stage on initial and final assessment of Ratio and Proportional Strategies

At the final assessment, 55% of students from the lower decile schools and 30% of students from the upper decile schools were at stage 5 or lower. Stage 5 required students to find a fraction of a number using addition facts, while stage 6 required students to find a fraction of a number using a range of addition and multiplication facts. It is possible that students' lack of knowledge of multiplication facts, as mentioned in the previous section on Multiplicative Strategies, once again influenced students' results, with more than one term's teaching needed to effect a notable change.

Percentage of Students Gaining One or More Stages in Ratio and Proportional Strategies

Of the students who could progress, 42% from the lower decile schools and 48% from the upper decile school did so. Of note are the 23 students from the lower decile schools who gained four or five stages between the initial and final assessment.

Of concern are the students who made no gain, yet were not at the top stage. 58% of students from lower decile schools and 52% of students from higher decile schools fell into this category. As discussed previously, there could be several reasons for this state of events, including the emphasis teachers placed on this portion of the Number Framework in their teaching for the term and the ability of students to move appreciably on this scale in the space of one term.

Table 6.6 Percentage of year 9 students gaining on Ratio and Proportional Strategies

	Decile 1, 2, 3, and 4	Decile 8 and 9	Total
Gain 0*	58%	52%	57%
Gain 1	24%	36%	26%
Gain 2	10%	11%	10%
Gain 3	5%	1%	5%
Gain 4	2%	0%	1%

* Does not include those at ceiling

Summary

These results show that a large percentage of students from both upper and lower decile schools made gains between the initial and final assessment. Table 6.6 compares the percentage of students who made gains on each of the six scales.

As noted previously and discussed in Chapter 10, several of the schools fitted this numeracy initiative into their existing program rather than making numeracy the major topic during this period and adopting the grouping recommended on a regular basis. Many of the teachers interviewed commented that they felt that the individual assessment was one of the most useful features of the numeracy programme. Perhaps teachers' increased understanding of the stages their students were at enabled them to target their teaching of specific students on a day-to-day basis more effectively.

Table 6.7 Percentage of year 9 students gaining at least one stage on each of the six scales in NEST

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
Percentage of students gaining at least one stage	57%	52%	51%	45%	44%	43%

These figures are quite similar to the proportion of year 7 and 8 students making gains, as reported in Chapter 5. Over 50% of the students advanced in all three knowledge scales and approximately 45% gained on the strategy scales, within the space of one term. A higher proportion of the students from upper decile schools than from lower decile schools made gains on five of the six scales. The exception was Identification of Whole Numbers, where 83% of the students from upper schools were already at the top stage.

7. Analysis across Year Groups

One of the surprises from the analysis of the initial assessment scores, made and reported in September 2001, was the lack of difference in stages between the students in years 7 and 8 and those in year 9 (Irwin and Niederer, 2001). Those initial data were based on what had been reported by September 2001 for 1,894 year 7 and 8 students and 962 year 9 students (at that time it was not possible to differentiate year 7 from year 8 students from the data provided). This difference was interpreted as suggesting that the similarity in students' achievement might be related to the fact that the topics covered in this project had not been emphasised in school during those years.

In the sample that this report is based on, a few students had left their schools and did not have final assessments, so were not included. Results for several secondary schools that were not in the initial sample were now available, including more students from upper decile secondary schools. When the final data was available, it was possible to separate year 7 and year 8 cohorts, and show the initial comparison of three year cohorts. Although there were seven year 10 students their data has not been included in the analysis in this chapter.

The graphs below show the initial and final percentage of students at each stage on the six scales, for years 7, 8, and 9. There were 923 students in year 7, 950 students in year 8, and 1,451 students in year 9. The graphs show some differences between different year cohorts, which are discussed in the following chapters. However, the different year groups are judged to be remarkably similar overall on aspects assessed in the Numeracy Project, both initially and after intervention. Possible reasons for this are discussed.

Note that the vertical axes on the graphs for the different scales are not the same. The axis for Identification of Whole Numbers goes up to 90%, reflecting the high proportion of students at the top stages of this scale. The vertical axes for other scales vary from 30% to 60%. To some extent this reflects the number of stages within a scale that students could be distributed across.

Identification of Whole Numbers

This was the scale on which over 60% of students from all year groups were at the top stage initially, and 80% or more were at the top stage on the final assessment.

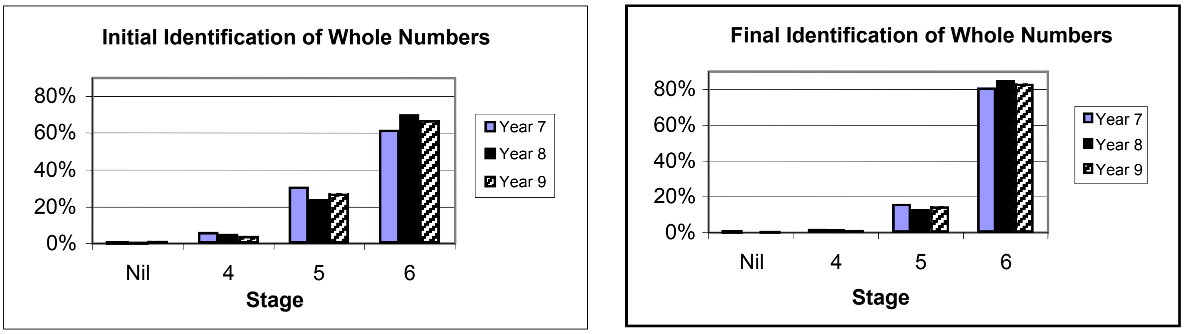


Figure 7.1 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final assessment of Identification of Whole Numbers

Just as the initial scores for years 7, 8 and 9 were similar; the final scores are also similar. There are no strong age related differences. The vast majority of students of all three year groups can reach the goals of this scale.

Identification of Fractions

In the initial assessment of Identification of Fractions the modal stage for all ages was stage 5 (identifying unit fractions). However, the modal stage at final assessment for all year groups was stage 6 (ordering unit fractions and identifying decimals).

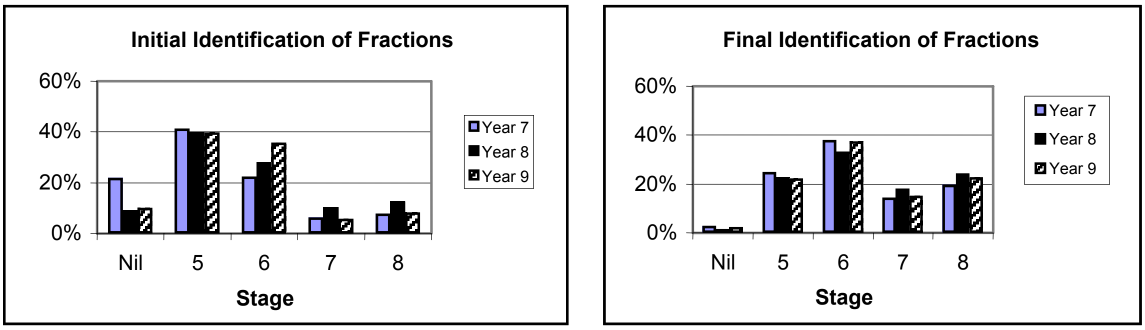


Figure 7.2 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final assessment on Identification of Fractions

A higher percentage of students in year 7 than in the older year groups failed to order simple unit fractions (nil) on the initial assessment. There was also a higher percentage of year 9 students scoring at stage 6 (ordering unit fractions and identifying decimals) initially than in the younger year groups. Overall, a higher percentage of students in year 8 scored at the top stages than did in other years.

Knowledge of Base 10 Grouping

All three year groups showed improvement between initial and final assessment on this scale. The year 7 students did less well initially than the year 8 or year 9 students. They also did less well than the older students in the final assessment, although they showed improvement.

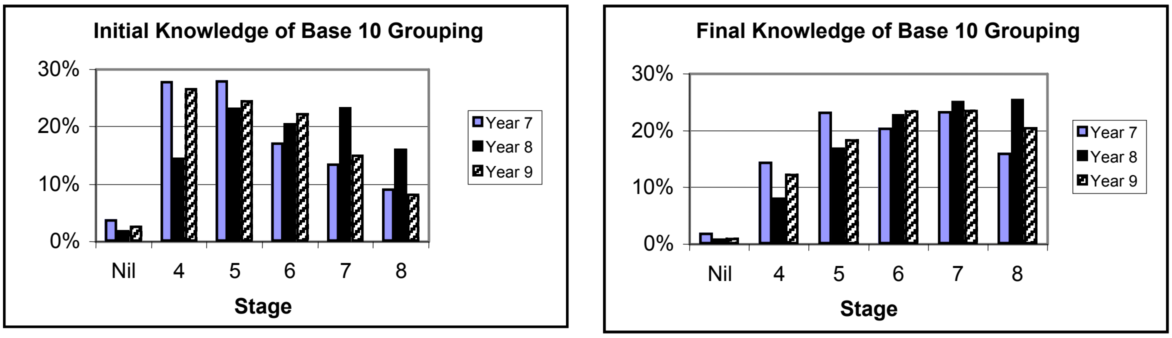


Figure 7.3 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final assessment on Knowledge of Base 10 Grouping

The year 8 students did somewhat better on this scale than the year 9 students for unknown reasons. This could be related to decimals having been taught recently to the year 8 students. It could also be that this is the final year in which numeracy has been given prominence.

Additive Strategies

The similarities between year groups on the graphs for Additive Strategies are more marked than are the differences.

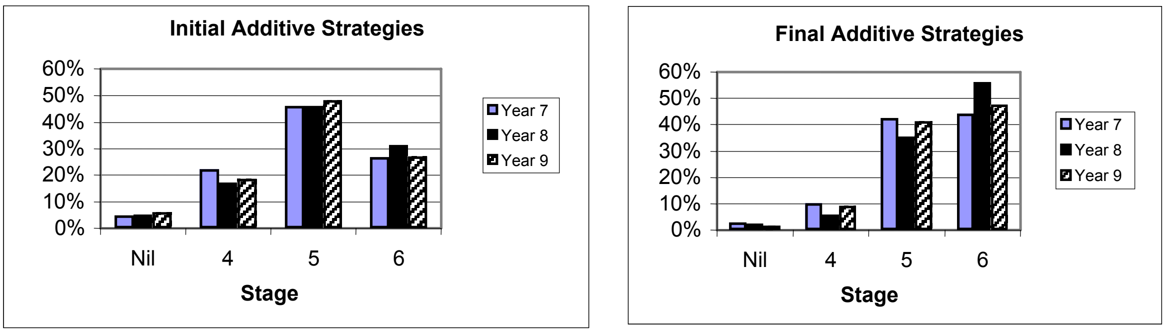


Figure 7.4 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final Additive Strategies

Mental strategies for addition have not been taught in New Zealand schools as a rule, so it is not surprising that there was little difference between various year groups in the proportion of students who added mentally by counting on, using some part-whole strategies, or using a variety of part-whole strategies before the start of the project. There was some difference between the year groups after one term of teaching, with the year 8 students showing the highest proportion of students moving to the top stage, using a variety of part-whole strategies.

Multiplicative Strategies

Strategies for mentally solving multiplication and division problems have also not been taught in schools, as a rule.

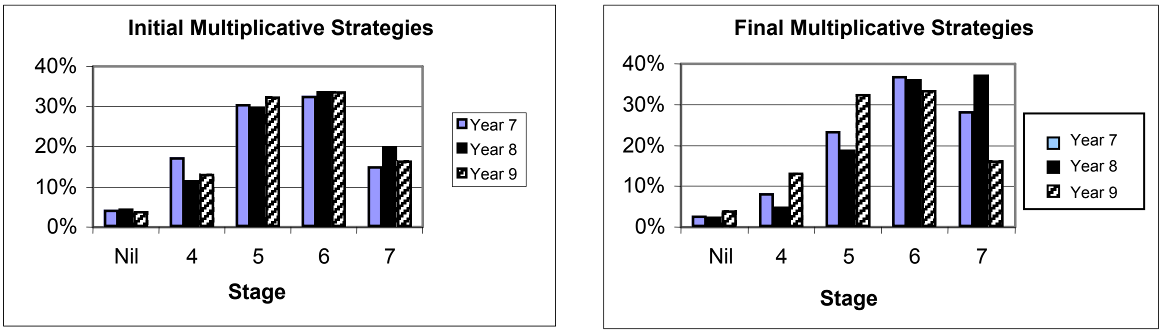


Figure 7.5 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final assessment of Multiplicative Strategies

Older students were judged to be at somewhat higher stages on this scale initially, but the differences were not marked. At the time of the final assessment, the bulk of the students had moved from stages 5 and 6 to stages 6 and 7. While 71% and 69% of year 8 and 9 students, respectively, were at the top two stages of this scale, only 62% of the year 7 students were at these two stages at the end of a term of teaching. As on other scales, the percentage of year 8 students at the top levels exceeded the percentages of year 7 and year 9 students.

Ratio and Proportional Strategies

Mental strategies for solving ratio and proportional problems have also not been taught, as a general rule.

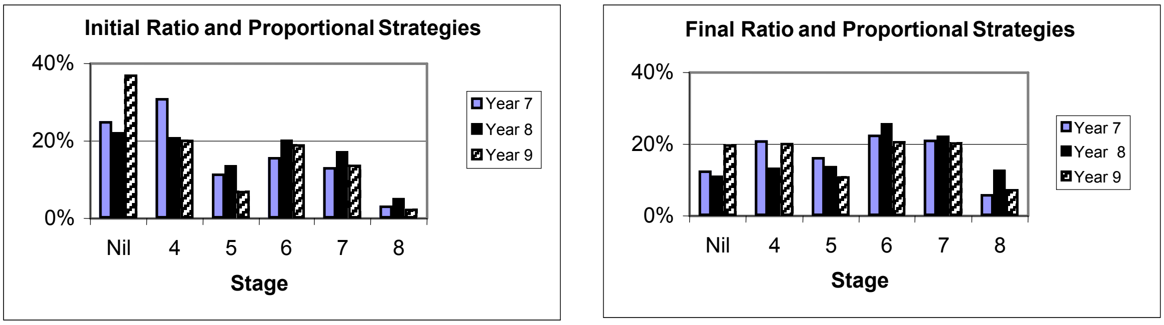


Figure 7.6 Percentage of students in years 7, 8 and 9 scoring at different stages in the initial and final assessment of Ratio and Proportional Strategies

The surprising factor shown on the first of these graphs is the very high proportion of year 9 students who were unable to find 1/3 of 24 beans. Teachers reported that they appeared not to know what the problem required them to do, even if given 24 beans to work with. This proportion decreased in the final assessment, but was not noticeably different from the year 7 students. A higher proportion of year 8 students were judged to be at the top levels of this scale on both the initial and final assessments.

There are several possible contributing factors to the relatively poor results on this scale. One is the known difficulty of this topic, often seen as the most difficult numerical concept for students to understand, certainly at the primary school level. A review of the curriculum in this sphere does not give the emphasis to this topic that these data suggest is needed. We do not know how much teaching time was spent in the project on proportional thinking, especially under the circumstances of teaching from The Number Framework only toward the end of the year. There were only a limited number of resources available in the materials for teachers covering this area. More time may be needed for professional development in this area. In 2002, more attention is being paid to this area, and educators are being asked to rethink how fractions are taught at different levels of the mathematics curriculum, and to make suggestions to the Curriculum Stocktake.

Summary

The overall picture from all of these graphs is the similarity of both initial and final stages for all three year groups. Where one year group had a higher proportion of students at higher stages than did the other year groups, it was year 8.

One can only speculate on reasons for this. Firstly, it could have been the result of random factors and might not be repeated in another sample of students. It could be that year 8 is the final year in which numeracy is given considerable attention. In particular, the teaching of decimals and percentages is emphasised in years 7 and 8 and usually given less emphasis in year 9. In year 9 there is no longer a strong emphasis on numerical concepts, and mental calculation is not usually fostered. Experience with older students and adults has shown that this knowledge is often forgotten. The fact that such a high proportion of year 9 students were unable to find 1/3 of 24 suggests that this skill is forgotten remarkably quickly.

Importantly, many of the secondary schools did not spend the full term teaching material from NEST. They had already covered numeracy and felt the need to spend time on other topics in the mathematics curriculum. The fact that they did not progress more than younger students did might be the result of less teaching.

8. Analysis of Gains Made from Different Initial Stages

An analysis by Thomas and Ward (2002) of the improvement made by younger students working on ENP who started at different stages, demonstrated that students starting from lower stages made more progress than did those starting from higher stages. Reports from teachers working on this project suggested that this relationship might also hold for these older students. They thought that students who started from lower levels made more marked progress than did those who started from higher stages. This statement was qualified by noting that some changes in students’ approaches as seen in class were not necessarily reflected by a change in a full stage on NESTA. It may have been that students at higher stages were beginning to understand and use new strategies but were not yet confident in their use.

The data from these older students provides some evidence that students who started at lower stages (and had more room for growth on these scales) made more progress than did students who started at higher stages, but the pattern is less clear than that found by Thomas and Ward (2002).

For these students, this aspect was analysed on strategy scales only. Each strategy scale is reported separately, showing which proportions made which amounts of progress from each stage.

An initial analysis was done of the difference in the intervals between stages, using Rasch analysis techniques. This is also reported in this section.

Additive Strategies

A higher percentage of year 9 students who initially failed to score at the lowest level (indicated as nil) made gains in the use of Additive Strategies than did of those who started at higher stages. This was not true of students in year 7 and year 8. Of these younger students, those who started at stage 4 made the most gain.

This irregularity may relate to the breadth of techniques covered by the nil category. This includes all students who did not show the ability to use counting to solve an addition problem. On earlier scales of the numeracy programme these skills are separated into stages 0, 1, 2, and 3. It might have been that a higher proportion of secondary school students who scored nil already had skills at stage 3 and were therefore ready to move ahead. Apart from this, a smaller proportion of those who started at stage 5 gained than did those who started at stage 4.

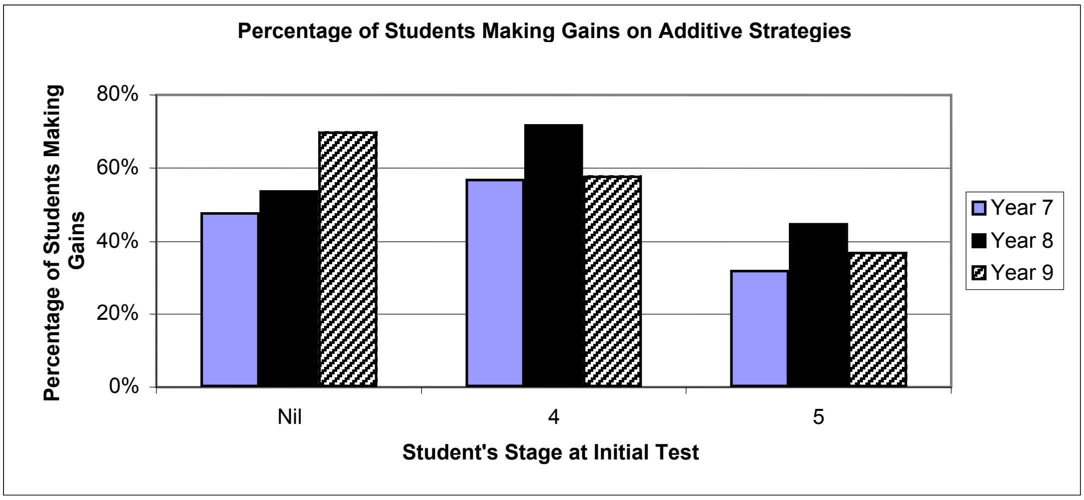


Figure 8.1 Percentage of year 7, 8 and 9 students making gains in Additive Strategies from initial stage to final assessment

The percentage of students starting at each level who gained one or more stages appears in Appendix S.

Multiplicative Strategies

On this scale it was also the case that the percentage gain was highest for students starting at stage 4. Diminishing proportions of students starting at stage 5 and stage 6 made progress.

The general pattern of student gains for stages 4, 5, and 6 is similar to that for young children as presented by Thomas and Ward (2002). This graph again points out that a higher proportion of year 8 students made gains than of students in years 7 and 9.

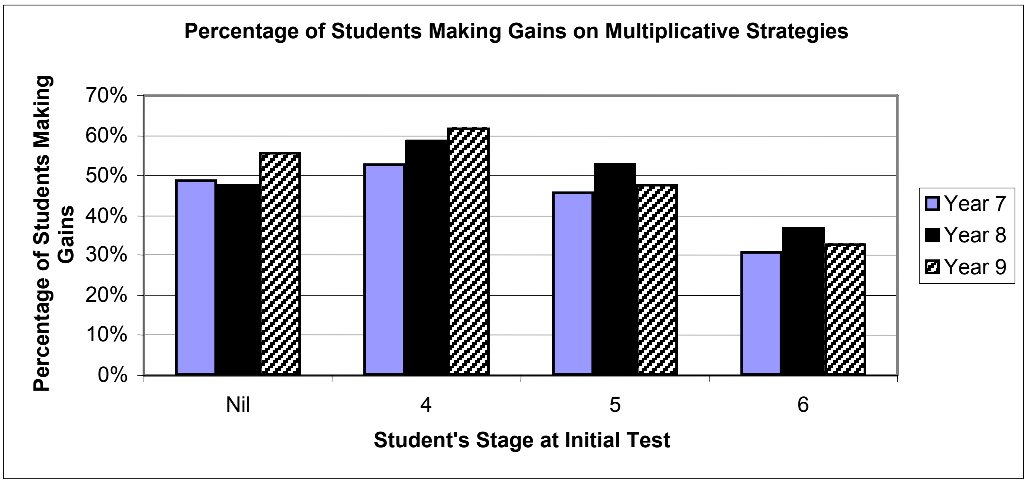


Figure 8.2 Percentage of year 7, 8 and 9 students making gains in Multiplicative Strategies from initial stage to final assessment

The percentage of students starting at each level who gained one or more stages appears in Appendix S.

Ratio and Proportional Strategies

For the most part, students who started at stage 4 made more progress than did those who started at more advanced stages. There were irregularities here. As on other strategy scales, students who started at nil made less progress than did those who started at higher stages. In addition a smaller proportion of year 9 students who started at stage 4 made progress than did those who started at stage 5. Also, a higher percentage of year 7 students starting at stage 6 made progress than of students who started at stage 5.

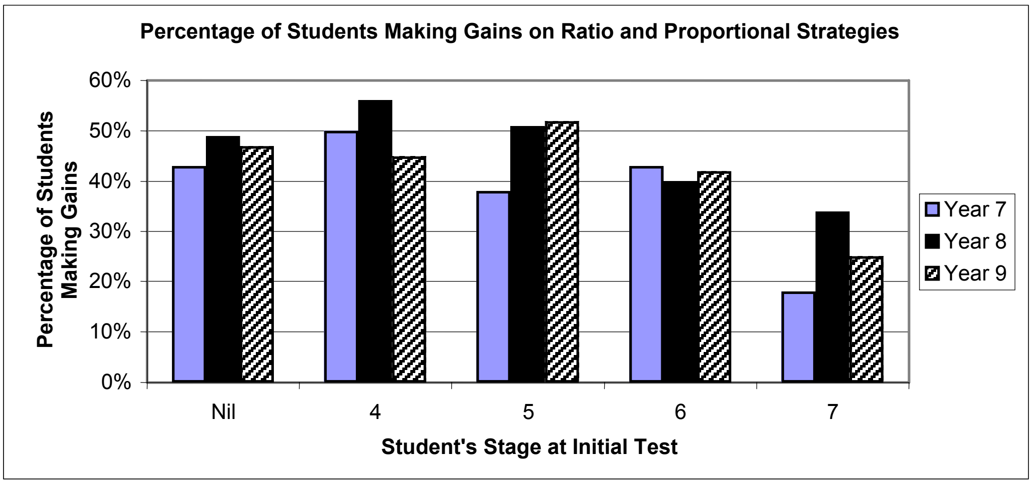


Figure 8.3 Percentage of year 7, 8 and 9 students making gains in Ratio and Proportional Strategies from initial stage to final assessment

The percentage of students starting at each level who gained one or more stages appears in Appendix S.

Results of a Rasch Analysis of the Data from One School

The data from one secondary school, for 176 students, was analysed as a trial, to see if this analysis would show difference in the intervals between the stages. This analysis was done on Identification of Fractions, Knowledge of Base 10 Grouping, and Ratio and Proportional Strategies, as these were the only scales that included stages 3 through 8. The question behind this analysis is the same as that addressed in the graphs above. It asks whether the difficulty in moving from stage 3 (nil) to stage 4, for example, is the same as the difficulty in moving from stage 7 to stage 8.

Rasch analysis (see Bond and Fox, 2001) offers one way of answering this question. The analysis provides a common equal-interval measure of both student ability and difficulty of the assessment. For theoretical reasons, the units of the scale are "logits" (the natural logarithm of the odds ratio). Logits are equally spaced on the logistic distribution, just as z-scores are equally spaced on the normal distribution.

The results are shown diagrammatically in Figure 8.4 for the three assessments on which students could score between 3 and 8. Each point locates the stage shown beside it on the logit scale.

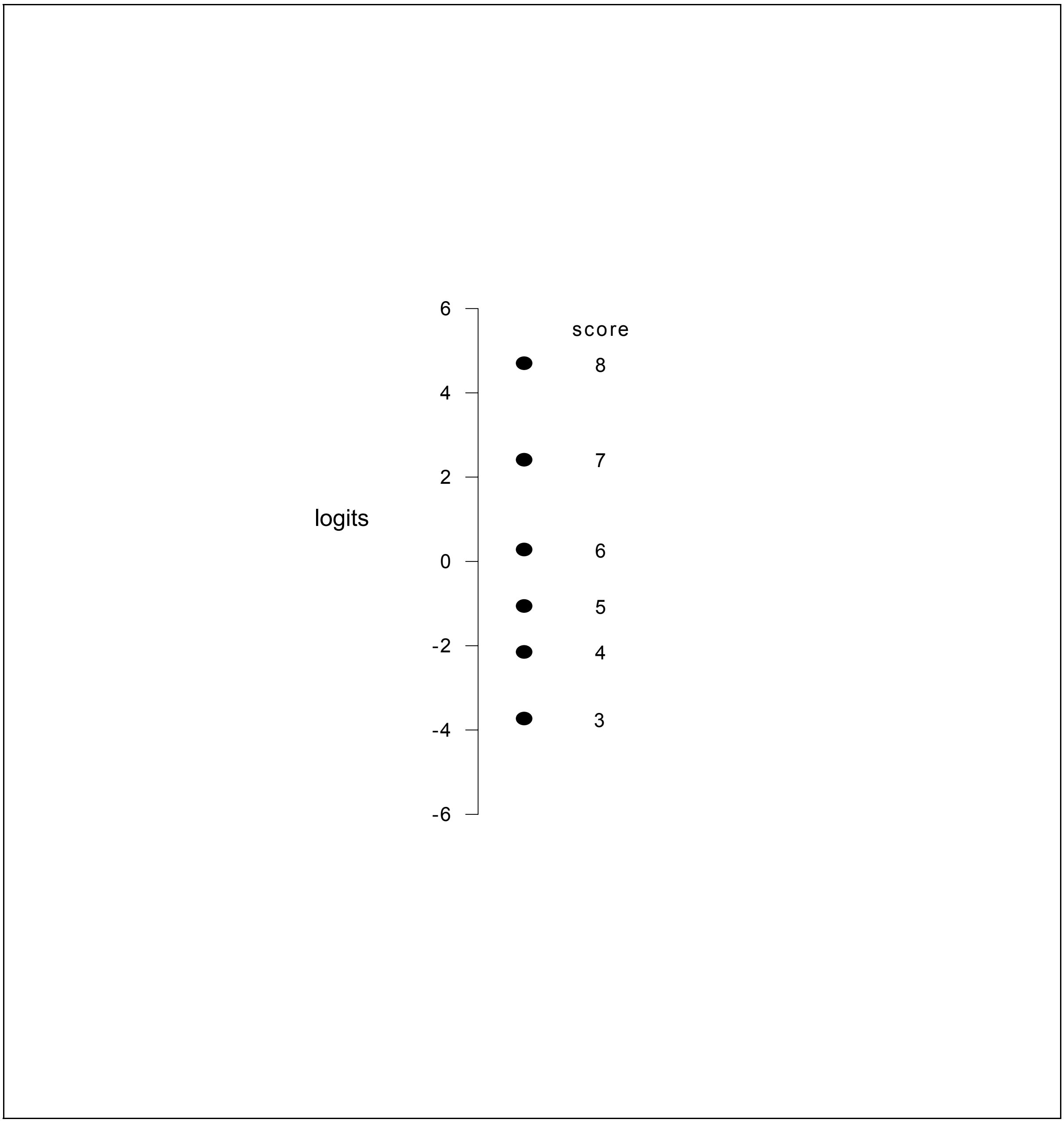


Figure 8.4 Placement of numeracy stages for the three scales on NESTA that give scores for stages 3 through 8.

From this analysis it can be seen that the scores are not equidistant. To move from stage 6 to stage 7, for example, entails a larger step than to move from stage 3 to stage 4. This analysis also appears in Appendix C.

Summary

Steps between stages on NESTA appear to be of unequal size. It is more challenging for students to move between stages 5 and 6, between stages 6 and 7 and between stages 7 and 8 than it is to move between stages 4 and 5. Evidence for this was presented by demonstrating the percentage of students starting at each point who gained one or more stages between the initial and final assessment. This evidence was based on the three strategy scales. Additional evidence came from a Rasch analysis of a sample from one school. This analysis was done on the three scales on which it was possible for students to be scored at all stages between 3 and 8. This included two knowledge scales and one strategy scale. In another year, all data can be analysed in this manner to see if this pattern of the step sizes holds across all scales and all years.

There was an anomaly in this pattern involving the students who moved from stage 3 (nil) to stage 4. A smaller percentage of these students made progress than of the students who moved from stage 4 to 5. We speculate that this is because this category was given to all students who failed to score at stage 4. On the assessment for the younger ages this includes the separate stages 0 through 3. At this age all of these stages are put together. In 2002 it will be possible to assess students at all relevant stages. This should provide information on this anomaly.

This analysis needs to be treated with some caution because of the limitations of the sample, and the fact that not all scales cover the same stages. In 2002 the students in these year groups will be able to score at all levels. Analysis of those data should confirm this finding, but we cannot be certain of that.

However, if this pattern of unequal size of the steps between stages is found to be generally true, it has important implications for teachers and school administrators in interpreting results. While it is encouraging to find that students have moved up one or more stages, this movement will take more time and more teaching for the higher stages. Not all advances are of equal value. It is highly likely that the concepts in proportional thinking, for example, are more difficult to teach and learn than Additive Strategies.

9. Consistency of Stage for Strategies in Different Operations

An assumption made in the construction of The Number Framework was that students would be able to demonstrate the same level of strategy on different mathematical operations. For example, if they were operating at stage 5 (early part-whole thinking) in addition, they should be able to separate numbers in a similar way for multiplicative or proportional reasoning. They should be able to transfer the understanding of the additive composition of numbers that they developed with one operation and apply this understanding to another operation. If they were at stage 4 (advanced counting), it would be unlikely that they would use more advanced strategies for other operations.

There are reasons why this might not be true in practice despite being reasonable in theory. These reasons relate to variables among the students, among the teachers who are interviewing students, and in the questions themselves. Students might respond differently to the same question despite being able to use similar strategies. Teachers with different levels of experience with The Number Framework might ask different additional questions to see if students could use a different strategy for the same problem. Individual questions in the assessment might trigger the use of different strategies.

There are two ways to analyse the extent to which students scored at different stages across different operations. One is to look at students who used more advanced strategies for Multiplicative and/or Ratio and Proportional Strategies than for Additive Strategies. Students who did this might not have been fully challenged by the additive tasks, for example, because they were very good at doing the traditional addition or subtraction algorithms mentally. This was true for some of the older students who had had years of experience with the algorithms. This was reported to be especially true of Asian students, although the data cannot be analysed for this characteristic, as it does not indicate which strategy any particular student used.

If students were using more advanced strategies on multiplicative and proportional problems than on additive ones then it would appear to be sensible to teach to their more advanced level of strategies. It might also be possible to create harder additive problems that encouraged strategy use, although this does not seem essential.

The other way of exploring this question is to look for students who demonstrated a lower level of strategy on multiplicative and proportional thinking than on additive thinking. This could happen because they had not seen the applicability of Additive Strategies to other domains. It could also be that there was more to think about in these more advanced domains and the students had reverted to less advanced strategies when faced with more difficult problems.

For these analyses, the scores of all students in years 7 through 10 were included.

Students Who Scored at a Higher Stage in Strategies for Multiplicative or Ratio and Proportional Problems than for Additive Problems

Only scores on the final assessment were compared for this analysis, as teachers were seen as being more certain about assessing different stages at that time. Table 9.1 shows the proportion of students who scored at higher levels on the multiplicative and ratio scales than on the additive scale. This was done only for stages 4 and 5 of the additive scale (advanced counting, early part-whole thinking). Stage 6 is the highest level possible on the additive scale, while the multiplicative and ratio scales go to stage 7 and 8. A competent student who scored at stage 6 on the additive scale could be expected to score at higher levels on the other scales.

Table 9.1 Percentage of all students, years 7 through 10, scoring at higher levels on multiplicative and ratio and proportional scales than on additive scales.

	Additive stage is lower than multiplicative stage*	Additive stage is lower than ratio and proportional stage*
Percentage of students	31%	16%

* Excluding those students on stage 6 of the additive scale

Therefore there were many students who scored at higher strategy stages on multiplicative and ratio problems than on additive problems. An examination of the data for these students shows that they were largely the more competent students with scores that included several sixes and sevens. They had higher means for all scores. They included some students who “caught on” to strategies well, sometimes moving from stage 3 on the first assessment of strategies for multiplying and proportional thinking to stage 6 or 7 on the second assessment. This may have been because they had good number sense. The fact that their additive stage is lower than their multiplicative or proportional stage does not appear to be a concern as they can think in a way that demonstrates that they have a part-whole understanding of number.

Students Who Scored at a Lower Stage in Strategies for Multiplicative or Ratio and Proportional Problems than for Additive Problems

The second analysis shows the number of students achieving at lower stages on multiplicative and ratio and proportional scales than on the additive scale.

Table 9.2 Percentage of all students, years 7 through 10, scoring at lower levels on multiplicative and ratio scales than on additive scales

	Additive stage is higher than multiplicative stage	Additive stage is higher than ratio and proportional stage
Percentage of students	7%	34%

The general achievement pattern of those who do less well on Multiplicative or Ratio and Proportional Strategies than on Additive Strategies is very different from that of the group of students represented in Table 9.1. Their scores overall were mostly 3 (nil), 4, or 5 and the mean of their scores was lower. Many of them were at stage 3 on several scales. These students appeared to be less competent in mathematics. Teaching had enabled them to gain some understanding of advanced counting or part-whole concepts in addition but they had not transferred this skill to other domains. As with all other indices, this score might be affected by the specific tasks that students were asked to do to demonstrate the strategies.

The much larger percentage of students (41%) who could use an advanced counting skill for addition but not for ratio tasks is probably indicative of how much more difficult it is for students to think about ratio tasks.

Summary

If this analysis is generally valid, teachers could expect to find that the students who have more difficulty than average with numerical skills would need to work on part-whole strategies in addition more intensively than other students would. Once these have been mastered, these students should be helped to transfer the part-whole skills used in addition to the solving of multiplicative and proportional problems.

On the other hand, if a competent student scores at a relatively low stage on Additive Strategies but higher stages on multiplicative and proportional reasoning, they might be quickly shown a few strategies for addition but be taught primarily to their level on the other, more difficult scales.

10. Overview of Interview and Questionnaire Data

In addition to the scores on the initial and final assessment, information was gathered through open-ended interviews, both in person and by telephone. Interviews were held with all facilitators, and with a selection of teachers from Auckland, Gisborne, Wellington, and Christchurch. Questionnaires were sent to all principals and heads of mathematics departments in secondary schools (who were asked if they would be willing to be interviewed by telephone). Questionnaires were not sent to all teachers because most facilitators had asked the teachers they worked with to fill out questionnaires, and it seemed inappropriate to question them further. The questions that guided the interviews and open-ended questionnaires appear in Appendix T.

Interviews with teachers, administrators, and facilitators of three schools are included in Chapter 11 with the case studies of these schools. These provide the best picture of what happened in a sample of schools for teachers, administrators and students, with some reference to how much the teachers appreciated the work done by facilitators. All three schools had different facilitators. The comments from people associated with these three schools were not included in this chapter.

Most of the quotations in this chapter came from secondary school participants. The level of school is indicated for each of the comments. Introducing this project, which is based on teaching in early primary school years, was more of a change and a challenge for this group. Few intermediate school teachers commented on grouping for example, and few commented on the use of equipment.

Main Themes

There were common themes in the feedback from facilitators, teachers, and principals. These included:

- The benefit of being able to interview each student individually and learning about their knowledge and strategies.

- Surprise at the gaps in students' knowledge. This was particularly true of secondary school teachers who had not realised that their students had an inadequate understanding of place value (as demonstrated in the Knowledge of Base 10 Grouping scale), of naming and ordering fractions, and of finding a fraction of a whole number.

- The complexity of teaching in groups, if teachers were not already accustomed to teaching in this way. This was particularly true for secondary schools.

- The challenge for teachers in learning new concepts at the same time as their students were learning them. Teachers valued strategies but found nomenclature such as "advanced part-whole" novel.

- Ongoing discussions on the relative merit of standard algorithms versus mental strategies.

- Difficulties with having student teachers in the room, particularly in full control, when these student teachers were not familiar with the project and often not competent in mathematics.

- Plans to continue with the Numeracy Project in 2002 but introduce it earlier in the year, and revisit concepts at regular intervals.

Facilitators' Involvement

Facilitators were unanimous in their view of the general benefit of the project. They worked many hours over their allotted time in providing workshops, helping teachers locate or make teaching equipment, providing in-class help, providing extra help for teachers who moved into the school during the course of the project, and helping ancillary teachers who took students for extra tuition. Some reported giving particular support to teachers who had difficulty with some of the concepts in the project. Several facilitators responded to questions or requests for help whenever they were asked. In a support role such as that provided by these facilitators it is very hard not to work more than the hours allotted to this part of their job, and the facilitators did appear overworked. However, it would be fair to summarise that these facilitators were very supportive of this new initiative and happy to give more of their time to it than was allotted.

Several facilitators, but not all, reported emphasising the theoretical model for teaching based on that of Pirie and Kieren (1989) described in Chapter 3.

Implementation in Different Schools

Facilitators provided most of the information on how the schools implemented the programme.

Schools spent different amounts of time on teaching the Numeracy Project. Most intermediate schools reported spending 10 to 14 weeks on it, doing numeracy between one and five days per week during that time.

Secondary schools reported spending between five days and 11 weeks on this project. For some this was continuing with their normal mathematics programme "with a NEST flavour" or working on numeracy among the other topics they were covering, while others utilised the project guidelines as fully as possible. Some secondary schools reported using "starters" as their main intervention. The results from some of the schools indicate that the professional judgement behind this variation may have been based on what the students needed. See the case study of School F in Chapter 11 for an example.

The nature of the intervention was somewhat determined by the traditional pedagogy of each school. For example:

"[One school] grouped students and had done so in the past. [The other school] was a very traditional school, [students] in rows, working silently from the textbook. They said that group work didn't work well. Advances there were in moving from dependence on texts and to getting students to offer answers, come up to the board. Impact may have been on how mathematics can be taught."

The facilitators felt that a major benefit for the teachers involved was professional development, while for the students it was the ability to work at an appropriate level and receive recognition for using mental strategies. One facilitator thought that the more capable students, who had used algorithms exclusively, moved easily to using strategies, once they were given these as starters and understood what was involved.

Difficulties in Implementation

Facilitators also commented on the proportions of teachers who were fully engaged in the project and of those who were still holding back for some reason. The proportions ranged from about one third being fully engaged to nearly all teachers being fully engaged. This differed from school to school, but not specifically by level or school location. Reasons for lack of full engagement included being new to teaching and still working on class management, being long-serving teachers and not eager to try something new, and lacking confidence in mathematics. All facilitators of intermediate schools commented that a few of their teachers needed some help with the mathematical concepts involved. Some reported discreetly providing this support. However, this appeared to be a subsidiary focus rather than a major one.

There were also difficulties mentioned by several facilitators. These related to factors outside the control of the project. They included changes in school leadership, teachers moving to new schools, variable organisation on the parts of those who had been given responsibility for this project in their schools, illnesses, a school fire, and in the case of two schools, the discovery that they were to be merged with other schools in the area. Some of the facilitators commented on difficulty in controlling students in some of the secondary schools that they worked with.

Comments from Secondary School Teachers, Heads of Mathematics Departments, and Administrators

As stated at the start of this chapter, most comments here relate to secondary schools, partly because of the way the data was collected. The teachers in three intermediate schools were interviewed on two occasions, but two of these three schools are discussed in Chapter 11, so their comments are excluded from this chapter.

The comments of secondary school teachers, heads of mathematics departments, and administrators are intermingled. All of the heads of departments who responded also taught on the programme and some of the administrators were deputy principals who also taught. All of the administrators and heads of departments were well informed about the project in their schools.

The Assessment

Most people commented on the benefit of the individual assessment. This was coupled with a concern that the time taken for this could not be sustained.

Intermediate school: *[The main advantage was] "getting to know what students know and how their thought processes operate. And having the time to do this. For the students, it was demonstrating their mathematical knowledge without peer pressure or the need to perform for the teacher."*

Secondary school: *"Knowing where the kids are at is a real advantage, and not just thinking, okay we have a year 9 class here. …You not only pick up their maths ability but bits of where they are at is valuable in terms of placement and attending to individual needs."*

Professional Development

Several teachers spoke of the main benefit of this project as professional development. In this regard, also see the comments from a teacher in School E (Chapter 11) who spoke of how the professional development in the project enabled her to make her teaching more focused.

One experienced intermediate school teacher commented: *"It certainly honed up my teaching skills. …It revitalised me and made me revisit, in more depth, what I have been doing in the last few years."*

Secondary school: *"It has been excellent professional development for staff in looking at what, why and how they teach. They are asking questions about students' processing rather than just whether they get the right answer."*

Secondary school: *"All the teachers are asking the kids why – not just accepting an answer – probing more. Other kids will say, "I didn't do it like that" – and will say how they did it. So all the kids are hearing and using different strategies."*

Grouping

Teaching style, including group work, was the focus of discussion for several teachers. Grouping was not an issue for teachers of years 7 and 8. For them, it was already a common practice and did not arouse comment. However, it was novel for most secondary school teachers. All comments on grouping came from secondary schools. One secondary school teacher reported having four groups in her class and having a roster in which she taught two groups each day. She did this for the first five weeks of the term but found it very intense and "*gave up"*. She did not return to it after this period, saying that she would go back to whole class teaching. Other comments on grouping are given below.

Secondary school: *"Grouping was a problem for some because of the number and size of the students in their classes. Space is a problem with 32 kids trying to group meant rearranging the room. I was lucky because there was an empty class next door, which I used. We do team teaching."*

Secondary school: *(Asked about grouping) "Yes, definitely. We recognise it is a good idea – but we ran into behavioural problems at times – not conducive to learning maths". (Would you try it again?) "Yes, maybe, but I'm doubtful of its workability."*

Secondary school: *"We made games for the kids to play – they really enjoyed them. … It was more work for the teachers, who were not used to doing group work. … Most teachers started with the whole class – then gathered a group around them that needed extra help. One teacher uses little groups the whole time. I did a mixture – a bit of both. I knew which kids were at the bottom end – I taught them individually."*

Secondary school: *"We grouped for different things – initially children's skill level, based on the pre assessment. Had a mixture of group sizes – from small to big. Grouping was based on results* *from the knowledge section of the test. We found we had kids with gaps in their knowledge of decimals and fractions in particular."*

Secondary school: "*Everyone was very positive – once we were immersed in it. There were several pleasant surprises – the kids reacted really positively to the change in teaching style and technique. Even the top kids reacted positively – no one said this is boring or sissy. I'd got out of the habit of group work, but I've got a really tough class of third formers who have just reacted very positively. There was a certain loosening up that occurred – an inevitable result of having to adlib at times.…One year 1 teacher who arrived in Term 2, thought it was just brilliant. … Another teacher – who is young – had difficulties with the group lessons. She is loosening up now. Teachers before were teaching the whole class – chalk and talk – with a few activities thrown in. This is still the main approach. But the Numeracy Project changed things for some of us. We felt the group work was very positive."*

Strategies

The use of strategies aroused comment from some teachers.

Intermediate school: *"My top group could get all but the last few problems right so we focused on those and they just lifted each other. In every maths period there was just – like an excitement. [The facilitator] suggested some resources and I would get them and the kids would go…they were feeding off each other."*

Secondary school: *"At the beginning of the project I was very sceptical about the strategies. But I'm quite sold on strategy work now and I'm using it with my fourth form as well. We will extend the numeracy approach for the fourth form next year."*

Using Equipment

Using equipment was a relatively new aspect for some intermediate teachers and several of the secondary school teachers.

Intermediate school: *"We are using equipment more. We bought a lot of equipment for the project." (Any especially useful?) "Yes, dice. Also cubes and beans. One of the most effective activities was when [the facilitator] took the class. She folded a piece of paper in half, then in quarters, etc., with students copying and folding their own piece of paper. They wrote on the paper as they went, writing the amount as a decimal as well as a fraction. They also did thirds, sixths, etc. By the end they had a lot of fractions and decimals on their paper and they could see the order they came in, and their relative sizes. In the final assessment, they could do the ordering part because of this."*

Secondary school: *"One real shock when I got to this school was the lack of apparatus – teachers were not used to using it. I am now building up resources. Each third form teacher now gets a tote tray of resources at the beginning of the year. Push for resources has been a combination of my expectations from the last school I was at plus the numeracy impetus."*

Secondary school: "*There are too few resources that require kids to work strategically. At first I was very angry and annoyed but by the end I was very positive."*

Changes needed in NEST

Several secondary school teachers commented on the need for changes in NEST. Among these were the inclusion of negative integers and algebra.

Future Planning

Planning for the following year was a good index of a school's support for the project. All schools said that they intended to continue with the project in 2002. At the time of the interviews they were not certain that they would get funding from the Ministry of Education for this function, but plans were being made regardless. Comments from teachers of year 7 and 8 students are given in the next chapter.

Secondary school: *"Time will be an issue. We plan to interview in weeks two and three. Re-interview the second week of term 2 – we really want to isolate the kids who need extra work and give it to them. The whole first term will be on number – we will revisit fractions later in the year. The principal is very supportive of numeracy. The children do maths three times a week. The assessment was done in non-contact time. No one wanted to interrupt class times. AP's helped with interviewing. We also had a part time teacher – we paid him to do some of the interviewing. I like to do it myself – you learn a lot about the kids. ... Next year, we will base our numeracy work for year 10 on this year's final assessment. The bottom two or three classes were very low. At least we know where to start now."*

Secondary school: *"We plan to test early in the year, and then teach numeracy in the first term for about four to five weeks – plus maintain it as the year goes on. The traditional way of thinking has changed – there will be more time spent on the basics – adding, subtracting, dividing, and multiplying. It has put algebra into the back seat – if you can't do number, you can't do algebra."*

Secondary school: "*Next year – we will test in week one, in the first few days. Teachers will be relieved to test children. We plan to spend the whole of the first term, in years 9 and 10, on number. Year 9 we will test then teach. Year 10 we will build on where they are at. At the end of next year we will spend 4-5 weeks on numeracy again – at the end of the term"*.

Secondary school: *"We have rejigged a new entrance test – no time for the verbal one – to pick out different areas. We will broadband the children into five groups – Level 2, Level 2/3, Level 3, Level 3/4, Level 4 – moving into Level 5 by the end of the year."*

Summary Comments

Some summary comments give the feel of teachers' reactions to the project.

Intermediate school: *"I'm looking forward to children coming through from primary school who have had the programme."*

Secondary school: *"This year was exciting and worthwhile, but frustrating. We want to do it better next year."*

Secondary school: "*Everyone was dubious at the start – but all positive comments now."*

Secondary school: "*I think it is a wonderful project – really good!"*

Summary

The teachers and administrators who returned questionnaires or were interviewed expressed enthusiasm about the project. They wrote or spoke of the benefits of the individual assessment, the emphasis on appropriate teaching, the use of equipment, and the professional development for themselves.

Secondary school teachers and administrators were more guarded in their response to the teaching proposed in the Numeracy Project. They were not universally enthusiastic about teaching in groups, about teaching strategies rather than algorithms, or about the fact that NEST did not cover some topics that they thought should be taught in secondary schools. However, all schools intended to use the programme in the following year, and some were revamping their year 9, year 10, and non-examination year 11 programmes to cover numeracy.

11. Case Studies of the Introduction of NEST in Three Schools

The implementation of NEST in three different schools is presented here as a set of case studies. These were all schools that appeared to be fully committed to the project, thus giving the project a good chance of succeeding. They were among the schools that the researchers visited on two occasions. The first two are schools for years 7 and 8, quite different in character, and the third is a secondary school.

These three case studies are presented in slightly different ways. In the first school there were two teachers involved, each of whom taught two mixed year 7 and 8 classes. We have presented an overview of the school and the views of the principal, salient comments from each of the teachers, and separate reports of the achievement of their year 7 and 8 students, as these showed interesting differences. Then the gains made by students in this school are presented in relation to other schools for years 7 and 8 in the same decile range. For the second school we have emphasised the deputy principal's summary of why this was a successful intervention in that school and of the factors which the administrators of the project could well attend to. In addition we have quoted one teacher from that school who felt less confident in mathematics initially, as her needs and growth may be similar to other teachers for whom mathematics has not been a favourite subject. This was the one decile 10 school, and gains made by students in this school are reported separately in Chapter 5. The third school, a secondary school, put an enormous amount of time and energy into the project, implementing it in a way that was compatible both with the goals of the project and the teaching organisation of a secondary school. They were rewarded for their efforts with above average gains for their students when compared with other schools in the same decile range. The case study of this school present the views of the principal, of the head of the mathematics department, and of other teachers on different topics, both before and after the intervention. The results of students in this school are then compared with others in the same decile range.

School E, Years 7 and 8

This was a relatively new decile 4 school. The classes involved in NEST were the two top classes in this full primary school. Two mixed year 7 and 8 classes returned results for a total of 62 students. This school had only recently added year 7 and 8 classes.

This school was unusual in the project in that the full school, from year 1 to year 8, was involved in the Numeracy Project. Years 1 to 3 had taken part in Count Me in Too in 2000, and years 4 to 6 were involved in ANP in 2001. The two teachers in the year 7 and 8 classes participated in initial NEST workshops with their associated secondary school and then planned their mathematics programme jointly with the teachers of year 4 to 6 classes in their own school. This involvement of the whole school meant that all teachers talked about the same sequence of numerical development.

The principal was most enthusiastic when discussing the implementation of all three Numeracy Projects in her school. Like many intermediate school administrators spoken with, she reported that numeracy was the focus for the school this year. On our first visit to the school, an evening for parents that described the Numeracy Projects was being planned. The school had a generally positive feel about it, giving the impression that the teachers there were capable and enjoyed their jobs.

Several factors contributed to the potential success of the Numeracy Project in this school. The full school and all of the teachers were involved. The teachers were either currently receiving in-school professional development or had received this the previous year. There was enthusiastic support from administrators, outreach to parents, and as reported below, teachers who appeared to understand the project and to be happy with the direction of the project.

The two classes were 77% European, 8% Māori (5 students), 5% Pacific Island students (3 students), 8% "other" (5 students) and 2% Asian (1 student).

Both of the teachers involved in NEST were experienced teachers and confident about their own mathematics at this level. The facilitator commented on the fact that, as a full primary school already involved in ENP and ANP, equipment was in common use. School decisions had been made about new equipment that was needed. Interviews with the teachers and principal are presented first, followed by an analysis of the results for this school.

The two teachers and the principal of this school were interviewed in August and in November 2001. The facilitator for this school was also interviewed twice. In August, interviews with all students had been completed and the teachers had begun to implement the programme. The views of the principal and facilitator have been given above.

Teachers' Views

Assessment

Both teachers found the assessment interesting and useful.

Teacher 1 (20.08.01*) "[The assessment is] really fantastic. It allowed me to sit, listen to, and analyse the children's thinking. I could really tell how quickly they picked up how easily they picked up a concept …They are like a running record, a really detailed picture of how children are doing."*

Teacher 1 (14.11.01) *"…the assessment especially. I had a wide range in my class and it confirmed and verified the programme."*

Teacher 2 (20.08.01) *"[The assessment] was time consuming initially. It took a little time to get used to it so you could do it quickly. Otherwise it was very good. Quite targeted … let you see that the children had gaps in their learning. … I was surprised that some of the kids who were quite far ahead in maths had little gaps. … Others at the lower end of the scale did as I expected."*

Teacher 2 *(14.11.01) "…It pinpointed the gaps. … Strategies were better than knowledge – problems with mastery of their basic facts …[The second assessment showed that] some children had moved up, particularly in fractions… . A couple of children had really moved on, some stayed in the same place. The really bottom children didn't move a lot but they felt more successful in targeted groups. The top and middle had about the same amount of movement."*

Programme Implementation

NEST formed the basis of the entire mathematics programme for the third term in this school, and work from NEST was done five times per week. The teachers reported also using the programme for the last two weeks of Term 2, and two weeks in Term 4. Most work was done in groups in both classes.

Teacher 1 (20.08.01) *"We use the ANP resources five days a week. Because of my junior school experience I run three groups anyway, one on teaching, one on practice, and one of problem solving. They rotate. ... I feel comfortable giving the groups games. I start with a maintenance activity, then sometimes a whole class activity, and then I split them into groups. I [already] use a lot of problem solving and in order to solve problems I teach about strategies you can use."*

Teacher 1 (14.11.01) *"The programme supported movement to the next step – I found that very empowering. It encouraged me to talk more about strategies and to use the equipment more."*

Teacher 2 (20.08.01)*"...I have a spread from [stages] 3 to 8 in my room, and within group differences as well. ...I always had three maths groups, and right now I have five – one extension group who are working independently and two Year 6 students who come for mathematics and are also working independently....At the moment I am grouping them according to where they were on the assessment. We are working through the sheets that came with the programme and supplementing those. It is a quite logical progression. We know that once they have mastered one stage we are ready to move them on with a transition.."*

Teacher 2 (14.11.01) *"I particularly like using physical materials with the more able, not just the less able."*

Use of the NEST/ANP Resource

Teacher 1 did not mention difficulty in understanding how to use the resource, but Teacher 2 did mention having some difficulty.

Teacher 1 (20.08.01) *"This week the focus is multiplication, so I take the strategy and knowledge parts of the resource, and I highlight and date the resources I am using with the children. ...I take a couple of activities out and teach [them] to the children. I use the resources in ANP, <u>Figure it Out</u>, <u>Math Matters 1 and 2</u> ...I don't just use one thing."*

Teacher 2 (14.11.01) *"I had a tough time coming to grips with the folder [of resources]. [For next year] I really need to come to grips with the folder. If it were simplified that would help. Perhaps putting the blackline masters in with the lessons would help. The print was tiny – bigger print would help."*

The school had made the decision to hire someone to make extra materials for them, rather than take teachers away from their classrooms. However, Teacher 1 saw a disadvantage to this:

(20.08.01) *"We have also made a lot of resources. We decided as a school to hire someone to make the resources so that there wouldn't be too many people in and out of classes. One of the drawbacks was that we weren't talking about the resources as we made them. ... Someone came in with [a new resource] and said "Have a look at this! I think a lot of teacher talk is needed to get you to use new resources."*

Professional Development

The teachers spoke more about professional development within the school than that provided in the initial workshops. This included both the support from other teachers and modelling by the facilitator.

Teacher 1 (20.8.01) *"Because of the professional development in this programme my teaching has become a wee bit more focused... We have Quality Learning Circles in our syndicate meetings (of seven teachers) and we are focusing on maths. We come along with a resource that we use and share it with the rest of the syndicate. We are getting ideas from other people.* (14.11.01) *...[The facilitator] was amazing. She modelled expert lessons and gave us something to aim for. She was very good at moving children on. ...One of the most useful aspects was learning about stages."*

Teacher 2 (20.8.01) *"I'm sure it will get more user-friendly with time.... It is interesting and really good professional development. It makes me think."*

Problems

Like the teachers in all other schools, the teachers in this school would like to start next year with the emphasis on numeracy. These teachers were rather less disturbed by starting late in the year than were some teachers in other schools, possibly because they could see the programme in action already in other classes in their school. Both teachers said that the time that the testing took was a problem, as it took longer than they had expected. They were also concerned that the second assessment did not reflect all of the progress that their students had made. Money for resources was an issue, as was understanding the folder. The teachers in this school raised one difficulty that was mentioned by several other teachers. This was the difficulty of initiating the programme with student teachers in the room.

In summary, one teacher said:

"We really feel that we are on the way to having a great numeracy programme in this school."

Student Progress

The results support the teachers' statements that progress was made especially in knowledge of fractions. The following tables (11.1 and 11.2) give the numbers of students as well as the percentages, because with a small number of students the percentages alone can be misleading.

On initial assessment, 16 (57%) of year 7 students were at the top level in Identification of Whole Numbers. None were at the ceiling in Identification of Fractions or Knowledge of Base 10 Grouping. Seven (25%) were at the top level in Additive Strategies. Six (21%) were at the top level in Multiplicative Strategies, and none were at the top level in Ratio and Proportional Strategies.

Table 11.1 Number and percentage of year 7 students who gained stages

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
Gain 0*	6 (50%)	4 (14%)	18 (64%)	9 (43%)	14 (70%)	13 (46%)
Gain 1	4 (33%)	17 (61%)	5 (18%)	11 (52%)	5 (25%)	12 (43%)
Gain 2	2 (17%)	5 (18%)	1 (4%)	1 (5%)	1 (5%)	1 (4%)
Gain 3	0	1 (4%)	3 (11%)	0	0	2 (7%)
Gain 4	0	1 (4%)	1 (4%)	0	0	0

* Not including students already at the top stage of a scale

Of the year 7 students for whom progress was possible, 50% progressed one or two stages on Identification of Whole Numbers, where the majority of students already scored at the top stage. For Identification of Fractions, 86% gained at least one stage. On Knowledge of Base 10 Grouping, 36% gained while the majority made no progress. In strategies, 57% gained in Additive Strategies, 30% gained in Multiplicative Strategies, and 54% gained in Ratio and Proportional Strategies.

Of the year 8 students, 16 students (47%) were at the top level in Identification of Whole Numbers initially, one (3%) was at the top stage in Identification of Fractions, and two (6%) were at the top stage initially in Knowledge of Base 10 Grouping. In strategies, 10 (29%) were at the top stage in Additive Strategies initially, six (18%) were at the top stage in Multiplicative Strategies and none were in the top stage of Ratio and Proportional Strategies. Table 11.2 shows the progress made by those not already at ceiling.

Table 11.2 Number and percentage of year 8 students who gained stages

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
Gain 0*	12 (67%)	14 (42%)	14 (44%)	13 (54%)	14 (63%)	20 (59%)
Gain 1	4 (22%)	15 (45%)	11 (34%)	8 (33%)	6 (27%)	10 (29%)
Gain 2	2 (11%)	2 (6%)	3 (9%)	3 (13%)	0	9 (13%)
Gain 3	0	2 (6%)	3 (9%)	0	2 (9%)	1 (3%)
Gain 4	0	0	1 (3%)	0	0	0

* This does not include students already at the top stage of a scale

Of the year 8 students, 33% of those who could gain did so in Identification of Whole Numbers, 58% gained on Identification of Fractions, and 56% gained on Knowledge of Base 10 Grouping. On the strategy scales 46% improved on Additive Strategies, 36% improved on Multiplicative Strategies, and 41% improved in the use of Ratio and Proportional Strategies.

For both year 7 and year 8 students, the greatest gain was made on the subtest for knowledge of fractions. These results were not known when teachers were interviewed, so they were not asked if this reflected more teaching of this topic, although this is a possible cause for the gains in this field.

Four students in each year group gained three or four stages on the Knowledge of Base 10 Grouping. These may be the gains that teachers were referring to when talking about the great gains made by some students. However, 64% of year 7 students and 44% of year 8 students did not make gains in this field. Similarly, a high proportion of both year 7 and year 8 students failed to make progress in Multiplicative Strategies. Again, it would be interesting to know if less time was spent on teaching these topics. Strategies for solving multiplicative and ratio problems are the most difficult parts of the programme. It could be that it takes longer than one term for students to learn these strategies and for teachers to become confident in teaching them.

Put together, the proportion of students in each year progressing at least one stage was as follows in Table 11.3.

Table 11.3 The proportion of year 7 and year 8 students progressing at least one stage on each of the scales.

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
Year 7	50%	86%	36%	57%	30%	54%
Year 8	33%	58%	56%	46%	29%	41%

The differences in some of these percentages, for example 86% of year 7 students gaining and 58% of year 8 students gaining, may reflect the fact that year 7 and year 8 students were in the same room, and no doubt mixed in teaching groups. This arrangement would give year 7 students more chance to hear and practice advanced concepts.

The next table, Table 11.4, compares the proportion of students from School E (n=64) gaining one or more stages with the proportions of all decile 3 and 4 schools (n=1,314) making this gain.

Table 11.4 Comparison of all students from School E with all students from decile 3 and 4 schools in the study including School E

	School E		All decile 3 and 4 schools	
	At ceiling	Percent gaining	At ceiling	Percent gaining
Identification of Whole Numbers	50%	19%	60%	21%
Identification of Fractions	2%	67%	4%	21%
Knowledge of Base 10 Grouping	3%	45%	4%	50%
Additive Strategies	27%	36%	21%	37%
Multiplicative Strategies	19%	22%	11%	39%
Ratio and Proportional Strategies	0%	45%	2%	42%

This table shows some difference between the percentage of students at School E that were at the top stages of different scales initially from the average for decile 3 and 4 schools. School E had fewer students at the top stage of Identification of Whole Numbers and a larger proportion of students were at ceiling on Additive and Multiplicative Strategies. The higher proportion of students at the top stage for Additive and Multiplicative Strategies may relate to the teachers' statements that they were already talking about strategies when doing problem solving, although the small number of students involved in these percentages means that the figures have to be treated with caution.

Markedly more of the students at this school made progress on Identification of Fractions than was average for schools at this level. On other scales a slightly lower percentage progressed one or more stages. This suggests that, apart from Identification of Fractions, this school was similar to others of the same decile range in the amount of growth. It may have been that the factors that led to their success were the same as those in other schools in this decile range.

Summary

The two teachers in this school were very enthusiastic about the project. Their students made good progress in one area and average progress for their decile range in other areas. This somewhat limited progress could reflect the difficulty of these concepts for both learning and teaching,

The conditions in this school would appear to be optimal for success in NEST. It is not possible to identify with certainty what made a difference for these students, but possible factors were:

- The support of all of the teachers in the school for this professional development programme. Britt, Irwin, and Ritchie (2001) demonstrated in another professional development project in mathematics that conversations between teachers engaged in trying to improve their teaching could be a major factor in the success of a project.
- Thoughtful decisions about making and using equipment for the students.
- Teachers who were already teaching in groups and knowledgeable about mathematics in general, while learning about The Number Framework and how to move students ahead on it.
- The support of the principal and parents for the programme.

It is possible that the excellent support provided within this school will show students to have benefited more in the second year of the programme.

School F

Intermediate F is the decile 10 school whose results are presented in Chapter 5. It was a large school, with 21 teachers involved in the project. This included 20 class teachers and one teacher employed to withdraw students for special help. Fifty-two percent of the students at this school were European, 34% were Asian, 10% were classified as "other", and 1% and 3% were Māori and Pacific Islanders respectively – a quite atypical distribution for a school in New Zealand.

The students at this school started at higher stages than did those in lower decile schools and ended at higher stages after one term on the project.

An interesting factor in this school was the delivery of the programme, which was markedly different from that in other intermediate schools. Teachers in this school said that they had covered numeracy early in the year. Those teachers interviewed indicated that they spent about two days per week on NEST during the third term of the year and reported spending more time on the programme with less able students and less time with more able students. Teachers did not plan as a whole school but did so in groups of two or four. In some cases this was effectively individual planning. Most of the teachers interviewed in this school considered themselves competent in teaching mathematics, although one discussed being less confident and competent in mathematics than in other areas of the curriculum. The success of the students in this school may reflect the value of having teachers who know the programme relatively well adapt it to the needs of their students.

The success of students in this school is shown in the graphs for the upper decile year 7 and 8 school, presented in the comparisons for each scale in Chapter 5.

The deputy principal (who oversaw the project) and the teachers spoken with were universally pleased with the project, and full of praise for their facilitator whom they felt went well beyond her allotted time in helping them, being willing to come over at almost any time to help out. Her summary of factors leading to the success of NEST in their school, factors that limited its success, and her overall evaluation of the project and what might be improved were particularly insightful and are given here. They are not direct quotes but summarise her views. She has read this summary of her views and agreed with its accuracy.

Factors Leading to Success

1. The whole school had participated in Infolink in 2000 (a technology project), and saw the benefit of this whole-school, whole-year type of professional development, which was a first-time experience for the school. Senior staff saw this as a good way of getting a common knowledge base and common focus.

2. They had reviewed the school's mathematics programme last year which had showed them that although the curriculum was being delivered pretty well, some teachers needed on-going professional development in this field. They realised that their resources were not being used as fully as they could be, by people who didn't know what they were appropriate for.

3. As a school they were already talking about children's thinking strategies. Five teachers had been to the Breakthrough conference on thinking skills and ten have now visited the Navigator schools in Victoria. They had been reflecting on why what they did worked. The theoretical, constructivist model was there.

4. Although teachers had a range of experience levels, several had taught from year 1 up and were experienced with running records, as an example of taking a close look at the strategies that one child uses while much else is going on, and knew the spiral of students' learning.

5. They had a part-time teacher (not a teacher aide) to work with those students who had some knowledge in mathematics but many gaps and confusions in their understanding.

She withdrew these students for short periods in a small group for teaching. She was included in the training, as the project was seen as particularly appropriate for these students.

6. They had an excellent facilitator who spent much more than the 0.1 of her time that was allotted on this project, and was reinforced by the enthusiasm of the teachers.

Different positive comments came from other teachers interviewed. The views of one teacher have been selected as she may have been typical of teachers in other schools who were less confident in mathematics. This teacher was particularly positive about the project. Her reflections were a good indication of the benefit of the project as professional development.

This teacher discussed how useful the information from the assessment, the workshops, the teaching notes, the explanations, and the activities had been to her. Reflecting on her past and present teaching, she commented that she had not used the same approach in mathematics as in other subjects. For example, if she were teaching a unit on rocks, she would start by asking the children what they knew, and they would learn together. She did not expect that she would know everything that the students did. She did not do this in mathematics. She always started with ten straight-forward problems for practice. In other subjects, when they finished a unit they celebrated their knowledge, yet she did not do this in mathematics. She realised that she had taught mathematics as a series of things that needed to be done. In the past she had seen teachers looking through resources and wondered why they hadn't already planned what they were going to do. Now she realised that they were hunting for resources to meet a particular need and found herself looking in the same way.

At the end of every year she asked the students which subject they had liked most, and this year several said mathematics, which she couldn't remember happening before.

"In this project I felt I was teaching properly… [much more as she had in her other subjects]. I had used equipment in the junior school and in a lower decile intermediate, When I arrived at [this school] I didn't see equipment being used as a matter of course and thus thought that I was doing maths wrong! There has been an emphasis on equipment throughout this project and I could understand the reasons for using it and see benefits for my students. It was wonderful to see students' eyes light up after making fraction strips. I want to mention how much I enjoy using equipment at this level."

In the second administration of NESTA, she slowed herself down and asked more questions about how students did the problems, questions that were the result of her greater knowledge of the programme. She was more interested in their answers now that she knew what she was looking for. Her attitude toward mathematics had shifted. She wants to spend more time in 2002 on The Number Framework so she can note students' strategies as they work. She will try starting classes with NumberSENSE starters rather than a quick quiz (McIntosh, Reys, and Reys, 1997). She has been looking at the nzmaths website (http://www.nzmaths.co.nz/) and will continue to use that more.

Other teachers at Intermediate F made positive comments that were similar to those of this teacher.

Concerns about NEST and How They Were, or Might Be, Dealt with

The deputy principal had also thought about what caused difficulties in the project, and how these were overcome, or how they might be overcome. These were as follows:

1. Initially they hadn't realised how time-consuming NEST would be. The teachers' enthusiasm for the project led them to persevere with it. When they saw this they dropped a different proposed review area for 2001.

2. There was a range of teachers at the school: two relatively inexperienced (Year 1–3) teachers and others with nine or more years of experience. Of these, nine teachers had only taught in intermediate schools, so did not have a picture of the normal development of understanding of mathematical skills and concepts. It would be beneficial if more teachers at this level had taught at all levels.

3. The validity of the initial scores might be questioned, as teachers had a limited understanding of the programme at that point. Therefore they needed to be cautious in interpreting 2001 data. Most teachers were on a big learning curve and those without experience in taking running records in reading found the demands of testing difficult.

4. Some teachers still needed help in understanding the mathematics behind the Numeracy Projects. There was a need for teachers who were not comfortable with mathematics to top up their own mathematical knowledge. The facilitator was seen as having done a good job of providing some extra help where needed. The deputy principal would like there to be a core of the project that all teachers do, and discretionary additional parts of the project that schools could direct teachers to as needed.

5. There were staff changes during the year, but the facilitator and others on the staff were willing and ready to help the new staff understand NEST.

Other impediments mentioned by other teachers were:

6. Complexities of the school timetable, and the number of children who were in and out of the classroom for various reasons. "*You had to make sure that maths was done in the children's busy schedule.*"

7. Difficulty in providing teaching time for all groups when the project was worked on only twice per week. This meant that individual groups might receive instruction only once in two weeks.

8. Having student teachers in the room whose own mathematics was not adequate.

9. Introduction in Term 3 when they had done numeracy in Terms 1 and 2.

10. Fitting NEST into a programme while still covering all the other strands.

Major Effects of NEST in This Intermediate School

The main effect of the project was reported to have been the teachers' enthusiasm. The facilitator was particularly praised because she glowed with enthusiasm for the subject and transmitted that enthusiasm to the staff. She moved some staff members from feeling that they really did not want another full-staff involvement similar to the one the school was involved in during the previous year to displaying major enthusiasm. Staff believed that students were certain to benefit from this enthusiasm. Students were also thought to benefit from teachers looking at this field in a new way, "unpacking" understanding, and having the resources to take them forward.

Most teachers were convinced that NEST was the way to go in numeracy. They were excited – and horrified – by the initial assessment results. Teachers were talking about students' numeracy at the photocopier, sharing their ideas and resources. Teachers increased in confidence to start delving into what students didn't know and teaching from what they knew. They now accepted different strategies for solving problems, as one teacher who had taught extension classes already did.

Existing resources were now being used more effectively. Many resources had been in the school and had been introduced to the staff at general meetings, but were not widely used. Teachers now saw the need to use things like the 100s board, fraction boards/kits, counters, and dice.

The school had given considerable thought to planning for the use of NEST in 2002. Two of their three contributing schools will have had ANP, so those children should be familiar with strategies, and the school should have access to their final assessment data. They will use final year 7 data for the beginning of year 8, and will only check pre-tested children if they have a concern. Year 7 and other new teachers will be released to do the assessment. As the first term is full of things like school camps, they will start the NEST programme in Term 2, although some aspects of the programme will also be taught in Terms 1, 3 and 4 in association with other strands of the curriculum.

Planning will continue to be done in groups of two or four teachers as before. Money has been set aside for photocopying and laminating. Bins of resources have been made up so that teachers can pick them up from the resource room ready for use. The deputy principal did not know whether all teachers would start next year with teaching strategies or go back to what they have done before. They will have to wait and see. The deputy principal was unsure whether many of the teachers had made the transition to encouraging imaging, for example, for problems involving fractions. Although imaging had been encouraged by the facilitator, with reference to the Pirie-Kieren model, no teachers mentioned this in interviews.

Other teachers interviewed were all positive about the project and eager to carry it on next year.

In summary, this was a high decile school attended by students who were generally competent, and taught by teachers who were also competent. The teachers responded enthusiastically and reflectively to the numeracy initiative. Although the teachers were initially surprised at the gaps in students' knowledge, an unexpectedly high proportion of these students reached the top stages of the numeracy scale.

Secondary School O

This was a suburban school classified as decile 3. There were 10 year 9 classes in this school, all of which were involved in the project. Of the 177 year 9 students who were interviewed on both the initial and final assessments, 39% were European, 33% were Māori, 20% were from Pacific Island nations, 6% were classified as Asian and 2% were classified as "other". Females made up 47% of this group and males made up 53%.

Some months before the Numeracy Project was expanded to include years 7 through 10, the mathematics department in School O had identified numeracy as an area of the curriculum that needed attention, and had approached the Ministry of Education and the School Support Service in their region on this issue. As soon as the project was approved they were asked if they would like to be involved, and they agreed. They delayed the teaching of numeracy from Term 1 because they knew that the Numeracy Project would be introduced in their school. They based their year 9 teaching entirely on the NEST for all of Term 3 and one week of Term 4. From information gained from personal and telephone interviews and from questionnaires, they appear to have been the secondary school who put the most thought and time into the project.

Information in this case study was put together from interviews with the head of department, four of the teachers, the school's principal, and the facilitator. All were interviewed twice: in August 2001 after they had been involved in teaching on the project for four weeks, and in November 2001 after they had collected their final data. In addition, initial and final data for this school were analysed separately.

All the teachers interviewed and the head of department reported that they worked together closely as a department, sharing problems and suggesting ways of solving them.

Views of Teachers, the Head of the Mathematics Department, and the Principal

The Assessment

The initial and most positive reaction from teachers interviewed was the benefit to them from being able to spend 20 minutes with each of their year 9 students for the initial assessment. One commented that this was the first time, and probably the last, in an entire teaching career that there had been time to talk with each student and find out what they did and did not know. The head of department said, "*If nothing else happens in the whole project the assessment in itself would be worthwhile.*" In relation to the results of the assessment, one teacher said that it confirmed what was suspected about the weaker students but gave them some surprises with their more advanced students. Another reported being surprised at the strategies that some of the weaker students were using. Yet another commented on students who were not at similar stages across the different scales, so that she was regrouping students for different activities. All of these comments indicated that they were thinking carefully about individual students and how to meet their needs.

Both the principal and the members of the mathematics department were shocked to find what their students did not know. They had known that numeracy needed attention, but did not realise that many of their students could not read unit fractions (about 1/3 of the initial assessments), could not read six digit numbers (about 1/3 of the initial assessments) and had a poor understanding of place value. Nearly half of the students could not say how many $10 were in $4,520 and 2/3 did not know how many $10 were in $82,600. Over 40% could not give 1/3 of 24. These and other depressing statistics galvanised the schools' determination to work on numeracy. The principal commented,

"Some of the findings blew me out of the water. Place value, keeping track of five places, we had taken for granted. Students had a veneer of knowledge.…Schools have to respond to kids wherever they are."

Teaching

All teachers interviewed reported that they were putting a great deal of extra effort into finding appropriate teaching materials for this project. They worked with the facilitator to make sets of worksheets that would cover what needed to be taught to three groups for each class. One teacher found material on the Internet and another built upon her experience with the British Numeracy Project. At the end of the project they continued to report that finding appropriate teaching material was a considerable strain. Most teachers said that they had not been trained to teach place value or unit fractions, as this was part of the primary school syllabus.

All teachers moved to some group teaching. In most cases, this meant that they taught one group while the other groups worked from appropriate worksheets. Management was an issue, as the students in their classes were fairly demanding.

One teacher said: *"They are finding the work within their means, so I can actually sit down with one or two or three students. It is that that is reaping the benefits. I am able to listen to them and hear what is going on in their heads and help them with the best strategy for them, rather than doing one thing with the whole class."*

It was seen as particularly difficult to leave the two lower groups unsupervised when the top group was being taught. Two methods of overcoming this difficulty that were mentioned were a plan already in operation in which there were two teachers in each classroom on one occasion a week, and one occasion when a Resource Teacher for Learning and Behaviour (RTLB) took one group and the teacher took another. Despite the difficulties, teachers reported enjoying teaching groups rather than the whole class.

The head of the mathematics department summarised the benefits of the project, saying:

"Most people would say that their classes are happier. That doesn't mean they are more saintly, but certainly they are happier because they have things they can do. The kids in the bottom group are much happier. It has been most successful for them."

To balance out this statement on the benefit to the bottom group, one teacher found the project most beneficial for the middle group, and another felt that the top group might have been short changed, because of the management issues mentioned above.

Need for Professional Support

Teachers did not feel that all their needs as teachers had been met. One teacher reported finding it *"time-consuming, and to some extent draining ... We could have spent more time preparing appropriate worksheets."*

Another teacher commented that *"[to be] successful you need more ongoing support for teachers on almost a daily basis. You are in the middle of it before you know what you need. It would be good to have someone available."*

Despite this, the head of the department summarised that:

"Even in our worst moments of chaos and hard work, there would be no one saying 'Why did you get us to do this?'"

Views after the Project

After the final assessment the same people were interviewed again, and made many of the same comments, for example about the workload, the need for more resources, the need for more appropriate training, and about the students being happier in mathematics classes. New summarising comments are given below.

The principal reported that he liked the grouping, the cooperative learning and the reaction to students' needs. He had observed that the starter activities gave teachers time to move around among students. He also saw teachers flicking back and forth between high and low level thinking as appropriate. He wrote: *"The classes that I observed were highly focused and engaged with the work. Motivational levels were high – no off-task behaviours were observed."*

In interview he said: *"I'm a constructivist. Much of the teaching I have seen concurs with my views of what teaching is. It was highly interactive."*

He had received positive reports on the project from students and the parents once they got over their initial shock at seeing their children's needs. He reported that the Board of Trustees was very supportive. He saw the success of project at this school as being due to the work and attitude of the mathematics staff, the grouping of students by their learning needs, and explicit targeting in teaching. He saw his supportive role as giving teachers the wherewithal and the reins to do their job, for example by asking the assistant principal to sort out relief for the teachers.

The head of the mathematics department reported that the project had gone extremely well. She said that there were real advantages for teachers in knowing that students could understand what they were teaching and that this project had given them the freedom to do this, a comment also made by one of the teachers. Teachers learned about teaching; using different patterns in class. She reported that they had not broken the mould of using algorithms. The members of the mathematics department continued to discuss this. She felt that it had been a disadvantage for the top students in mixed classes to do the NEST work in that they were not extended.

Most teachers reported on the improvement of their students, although one was disappointed that her students showed less improvement than she expected, possibly because of the break between teaching and assessing. They felt that the programme was particularly beneficial for lower achieving students. One teacher thought that they had done well on teaching knowledge aspects but rather lost sight of strategies. In part this was because of limited teaching materials available for strategies. In addition she claimed that the students did not like strategies, and preferred to use known algorithms.

Another teacher who was a strong advocate of strategies commented that students who had used mental strategies in class returned to the use of standard algorithms (often incorrectly) in a written test. Other teachers commented on the support that they had been given by other members of the department, and were particularly grateful to those teachers who found materials that they could teach from. The price for all this hard work was reported by one teacher as: *"some loss of enthusiasm toward the end of 10 weeks, when you are working everyday until 7 pm."*

Teachers mentioned their plans for teaching numeracy in 2002 in the initial interviews but these plans were now advanced. Plans for 2002 indicate that the project has changed the way in which this school will teach its year 9 and year 10 students in future, as well as their non-examination year 11 students. They were certain that they would spend a considerable amount of time on numeracy in year 9 and touch it again in year 10 as necessary. They will delay topics often introduced in year 9 until year 10. Numeracy will be the basis of the programme for non-examination year 11 students. This will not be an "add on", as some schools seem to have suggested, but a core item in their mathematics programme.

Summary

In summary, the factors that were likely to contribute to the effectiveness of NEST in this school were as follows:

- The mathematics staff of this secondary school had already identified numeracy as an area that needed attention for their year 9 students.
- The mathematics staff worked cooperatively in identifying issues and finding ways of meeting challenges. This included having an established system in which each teacher worked with another in his or her classroom once a week to provide help and feedback.
- The mathematics staff were ready to innovate to make a project initially designed for primary schools work for their students.
- The head of department was very hard-working. She led staff, located teaching materials and thoughtfully implemented the project in her own class. It is hard to over-stress the workload she took on herself in relation to this project.
- The teaching staff included one member who had been a primary school teacher and who had been trying to get group work introduced for some time and a teacher who had taught students in Britain who had been involved with the Numeracy Standards and valued strategies.
- The principal valued the goals of this project and supported the mathematics staff.
- The teachers enjoyed seeing the progress that their students were making even when this progress did not show on the final assessment.
- The students enjoyed having work that they could do, making them easier to manage in class.

On-going factors that will need attention, in the eyes of the staff of this school, include:

- Much more teaching material appropriate for this age group, especially for teaching strategies.
- More support for teachers attempting a very different type of teaching. This includes methods for teaching in groups while maintaining a general working atmosphere.
- A decision, at some point, on the place of strategies and of algorithms in teaching numeracy.
- Extension of the use of the strategies developed in numeracy to other strands of the curriculum. For example, one teacher said that her students were now much better at spotting patterns in algebra.

Progress Made during the Project as Shown by Gain between Initial and Final Assessments

Progress on the assessment items confirms the views expressed by these teachers and administrators. The results for this school showed greater improvement than was the case for other schools in their decile range. This school kept its own data on the number of students failing particular items, on the initial and final assessment. This information shows how well the students overcame these particular difficulties.

Table 11.5 The number of students at Secondary School O who failed particular items on the initial and final assessments.

Note that there were 213 students in the initial assessment, 25 of whom were in an extension class. There were 177 students in the second assessment. Results of data on these 177 students is presented in Table 11.6

Item	Number failing on the initial assessment N=213	Number failing on the final assessment N=177
Name 7,049	25	3
Name 164,014	72	16
Name 1/2 1/3 1/4	75	3
How many $10 in $230	41	6
How many $10 in $4,520	109	33
How many $10 in $82,600	140	56
47 + 25 (mentally)	36	9
53 – 26 (mentally)	77	30
18 x 6 (given 17 x 6)	92	31
72 ÷ 4 (mentally)	137	75
1/3 of 24	89	34

Results from the data on initial and final assessments is given below, first showing the percentage of students gaining one or more stage, and then giving this data in summary form in relation to other schools of similar decile rating (N=1,205).

Table 11.6 The percentage of students in School O who were at ceiling or gained from zero to five stages on each of the six scales of the NESTA

	Identification of Whole Numbers	Identification of Fractions	Knowledge of Base 10 Grouping	Additive Strategies	Multiplicative Strategies	Ratio and Proportional Strategies
At ceiling	58%	5%	2%	15%	6%	1%
Gain 0*	8%	23%	32%	36%	45%	49%
Gain 1	32%	24%	31%	44%	41%	29%
Gain 2	2%	24%	18%	5%	7%	10%
Gain 3		19%	12%		1%	9%
Gain 4		3%	4%		1%	3%
Gain 5		2%	1%			

* This does not include students already at the top stage of a scale

Table 11.7 Comparison of all students from Secondary School O with all students from decile 1 to 4 schools in the study

	Secondary School O		Other decile 1- 4 Secondary Schools	
	At ceiling	Percent Gaining at least 1 stage	At ceiling	Percent Gaining at least 1 stage
Identification of Whole Numbers	58%	34%	64%	21%
Identification of Fractions	5%	72%	6%	48%
Knowledge of Base 10 Grouping	2%	66%	6%	46%
Additive Strategies	15%	49%	26%	33%
Multiplicative Strategies	6%	50%	15%	36%
Ratio and Proportional Strategies	1%	51%	2%	40%

This table shows that a smaller percentage of students from Secondary School O started the project at the top stage than was the average for schools in these lower deciles, on all six scales. This was particularly true for Additive and Multiplicative Strategies. However, a higher percentage of students gained at least one stage than was the average for these schools. This could be due to several factors, such as more conservative initial assessments. However, it seems likely that the concerted effort that this school put into implementing the project had a marked effect on students' progress.

All in all, this secondary school provides a model for an effective innovation in numeracy. They put a great deal of time and energy into the project, and adapted it to their needs, where necessary.

Summary

All three of these schools demonstrated a high level of commitment to and enthusiasm for this numeracy initiative. The students in two of them made remarkable progress, and those at the third school were well above average in one of the six scales. This may have been the focus of their teaching.

The fact that the implementation model in all three was different demonstrates the value of encouraging teachers to take initiative in seeing how best to integrate the project into their teaching programmes.

12. Summary and Implications for Further Research

This project demonstrated the need of students in years 7 – 9 for a numeracy programme. It showed up unexpected weaknesses, especially in the understanding of fractions, the ability to find a fraction of a whole number, and the meaning of large numbers. Results showed that these skills and understandings could be taught once teachers were aware of students' needs.

Students' difficulties with fractions have drawn attention to the fact that these may not be well covered in primary school nor in the mathematics curriculum (Learning Media, 1992). This is a matter that the Curriculum Stocktake needs to attend to.

The higher stages of the framework, which involve students' ability to think about multiplication and division flexibly and apply these skills to problems of ratio and proportion, are not as easy to teach and learn.

The fact that larger proportions of year 8 students than year 9 students scored at high stages on most scales in this project is an issue that requires further investigation. There could be many factors contributing to this, as there are many changes between the last year of primary school and the first year of secondary school. One of these factors is the training of the teachers. Several secondary school teachers commented that they were not trained to teach place value and fractions and realized that they needed to be able to teach these concepts. A major factor in this study may have been the amount of teaching in numeracy that students received at year 8 and year 9. Issues in this transition can be explored further.

In this project teachers were encouraged to use their professional judgement in choosing what, when, and how to help students learn the skills and understandings in this framework. As this evaluation did not capture what and how topics were taught, we cannot comment on exactly what made the difference to students who did make gains. More regularity in teaching is expected in 2002, when the programme is introduced earlier in the year. This may make it possible to draw more conclusions on the relationship between teaching and learning.

One difference in implementation between years 7 and 8 and year 9 was the extent to which students were taught in groups. School O, a secondary school, taught in groups throughout, but this was not the case for most secondary schools. On the other hand, it was a common teaching organisation for years 7 and 8. While theoretically it is an important part of the programme, it may be difficult for secondary teachers, who teach up to 150 different students a day at several levels, to change to teaching their year 9 students this way. An exploration of the costs and benefits of group teaching in secondary schools would be of interest.

The Number Framework puts heavy emphasis on mental calculation. This was particularly a challenge for older students who were competent in use of the usual written vertical method of calculating. Some students held to their belief that this vertical algorithmic method was the most efficient method. Several teachers, especially at the secondary school level, also reported being dubious about the value of calculating mentally, although several realised that they used mental part-whole methods themselves. The overall goal of The Number Framework seems to be flexibility of thought in solving problems, based on use of part-whole strategies and number sense. Written calculation can also play a part in a students' flexibility in calculating. It would be a shame if whether to use algorithms became the debate, rather than a discussion of the place of algorithms in the full set of strategies that students could use.

The role of the development of the numerical knowledge and strategies promoted in this project in relation to wider mathematics is a subject worthy of further exploration. Some teachers spoke of the connections between concepts encouraged in numeracy and relevant aspects of algebra or geometry. There is a need for comparative studies of students who have been taught through this project and those who have not, on tests of mathematical application that are wider than the goals of this project.

This evaluation did not look in detail at individual teachers' knowledge of the concepts that were to be taught or their initial knowledge of the best ways to teach these concepts. As students showed less improvement in multiplicative concepts in ratio and proportion, it might be that teachers also found these concepts difficult, or difficult to teach. A possible research project in this field could be to develop a programme to help teachers both understand and teach these concepts, and look at the resulting effect on their students' understanding. That is not to assume that slower progress in this domain is necessarily related to lack of teaching. There is both theoretical and research evidence that this is a relatively difficult area (e.g., Carpenter, Fennema, and Romberg, eds., 1993).

Differences in the achievement of students by decile, found also in the National Educational Monitoring Project (Flockton and Crooks, 1997) and the evaluation of the Early Number Project (Thomas and Ward, 2002) need to be an ongoing focus of attention. Although students from low decile schools make marked progress, a smaller proportion of them reach the top stages of The Number Framework than do students from higher decile schools. It may be that an intervention similar to that by Phillips, McNaughton and MacDonald (2001) in literacy, starting before school and including parents as an integral part, would help address this problem.

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Appendix A Stages of The Number Framework

(Included for younger students)

Stage 0	Pre-counting. Students at this level cannot count a small group of objects.
Stage 1	Counting from one on materials. Students at this stage can count and can form a set of up to 10 objects by counting each one. They cannot solve simple addition problems by joining these sets.
Stage 2	Adding by counting from one with materials. These students can add four counters and two counters by counting all of them.
Stage 3	Counting from one by imaging the objects to be counted. These students use counting but do not need to see objects in order to add.

(Included in the assessment of students in years 7 through 10)

Stage 4	Advanced counting. Students at this stage solve addition problems by "counting on". For example, for 8 + 3 they say "Eight, nine, ten, eleven." to get the answer 11. They can also count by 10s.
Stage 5	Early additive part-whole thinking. At this stage students recognise that addition problems can be solved efficiently by breaking up numbers into their component parts. They may do this by breaking up a number into parts. For example, 9 + 7 is equal to 10 + 6.
Stage 6	Advanced additive / early multiplicative part-whole thinking. Students at this stage use a variety of methods to break up numbers for addition problems and may solve multiplication problems by using these part-whole addition strategies. For example, they may mentally work out that 63 – 29 can be solved mentally by thinking that 63 – 30 = 33, and adding one (perhaps by using a visualised number line) to give 34.
Stage 7	Advanced multiplicative / early proportional part-whole thinking. At this stage students can use their understanding of multiplication to break up numbers. For example, they may realise that 50 x 124 is the same as 100 x 62, so it equals 6,200.
Stage 8	Advanced proportional thinking. Students at this stage can use a range of multiplication and division strategies to solve proportional problems. This includes finding a percentage of a whole number. Students who can do this might find 15% of 240 by first finding 10% of 240 (24) and then adding half of this (12). When these two percentages are added together they give the correct answer (36).

Appendix B The Numeracy Project Assessment Sheets Used by Teachers

NEST Numeracy Project Assessment: Individual Student Record

Student Name	(Last Name)	(First names)	Teacher	

Date of Birth		Year level	7 8 9 10	Ethnicity	E M P A O	Gender	M F

Knowledge Questions

Whole Number ID and Sequence	4 The student cannot identify numbers up to 1000.		5 The student identifies numbers in the range 1 to 1000, and can give the number one after and one before. Ref: Q 1		6 The student identifies numbers in the range 1 to 1 000 000 and can give the number one after and one before any whole number. Ref: Q 1					
	I	F	I	F	I	F	I	F	I	F

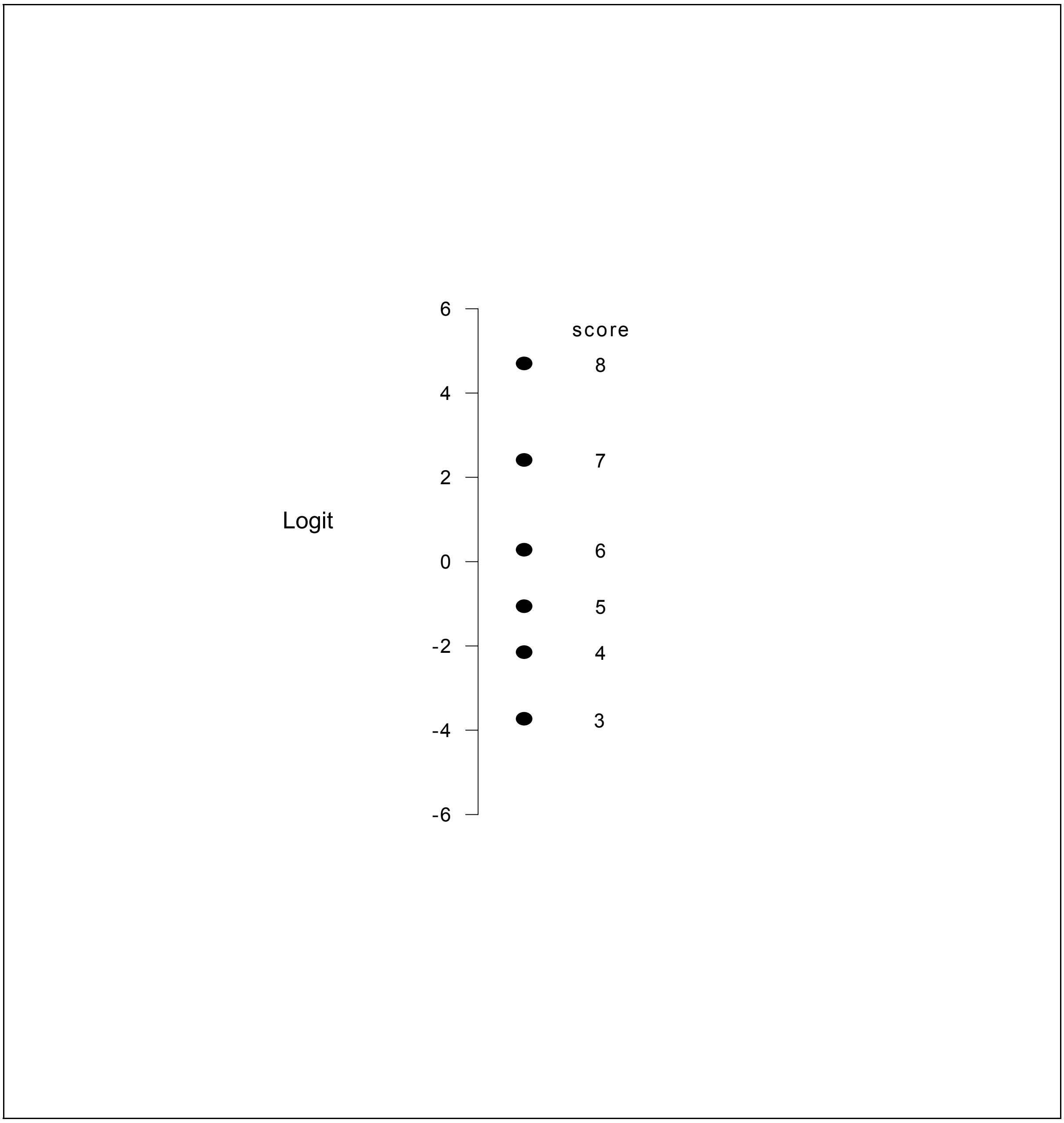
Fraction, Decimal, %age ID and Sequence	4 The student cannot identify unit fractions.		5 The student identifies unit fractions (eg. 1/4, 1/3). Ref: Q 2		6 The student identifies decimals to two places and orders unit fractions. Ref: Q 2 and 3		7 The child can order decimals to 3 places, and orders fractions with different numerators and denominators. Ref: Q 2 and 3		8 The student orders fractions, decimals, and percentages. Ref: Q 3	
	I	F	I	F	I	F	I	F	I	F

Grouping	4 The student finds the tens in numbers to 100 by repeated counting, ie. 10, 20, 30, 40,… Ref: Q 4		5 The student finds how many tens are in numbers to 1000 using knowledge that ten tens are one hundred, eg. for 230: 10, 20, 23. Ref: Q 4		6 The student knows how many tens are in any whole number. Ref: Q 4		7 The student knows how many tens and hundreds are in any whole number. They find the number of tenths in a decimal number using knowledge that ten tenths make one, eg. for 2.40: 10, 20, 24. Ref: Q 4, 5 and 6		8 The student knows how many tenths, thousandths are in any decimal number. They know the effect of multiplying, dividing, by powers of ten. Ref: Q 6, 7, 8 and 9	
	I	F	I	F	I	F	I	F	I	F

Strategy Questions

Stage	4 Advanced Counting	5 Early Additive	6 Advanced Additive	7 Advanced Multiplicative	8 Advanced Proportional
Add^n & Sub^n/PV	The student solves addition and subtraction problems by counting on or counting back, eg. 29 + 7 as 29, 30, 31, 32, 33, 34, 35, 36. They also use ten counts to solve addition problems, eg. 47 + 25 as 47, 57, 67, 68, 69, 70, 71, 72. Ref: Q 10.	The student solves addition and subtraction problems using a limited range of part-whole strategies like doubles or standard place value partitioning, eg. 47 + 25 as (40 + 20) + (7 + 5) Ref: Q 10, 11 and 12.	The student solves multi-digit addition and subtraction problems using a full *range* of part-whole strategies. Ref: Q 11, 12, 13 and 14.		
	I | F	I | F	I | F	I | F	I | F
Mult^n & Div^n	The student solves multiplication problems by skip counting or a combination of skip counting and counting in ones, eg. 5, 10, 15, 20… Ref: Q 15.	The student solves multiplication problems using repeated addition, eg. for 6 x 5:5 + 5 = 10, 10 + 10 + 10 = 30 Ref: Q 15.	The student solves multiplication problems by deriving from known multiplication facts, Eg., (17 x 6) + 6 = 18 x 6. Ref: Q 16.	The student uses a range of part-whole strategies to solve multiplication and division problems involving multi-digit numbers. These strategies include the distributive property, eg. 24 x 6 as (20 x 6) + (4 x 6), and compensation, eg. 72 ÷ 4 as (80 ÷ 4) – (8 ÷ 4). Ref: Q 17 and 18.	
	I | F	I | F	I | F	I | F	I | F
Fractions	The student finds a fraction of a number by equal sharing of objects. Ref: Q 19 and 20.	The student finds a fraction of a number using addition facts, eg. 1/3 of 24 as 8 + 8 + 8. Ref: Q 19 and 20.	The student finds a fraction of a number using a combination of addition facts and multiplication, eg. (3/4 of 28 as 14 + 14 = 28, 7 x 2 = 14 so 1/4 of 28 is 7, 14 + 7 = 21. Ref: Q 19 and 20.	The student finds a fraction of a number using multiplication and division, eg. 3/4 of 28 as 28 ÷ 4 = 7, 23x 7 = 21. They also solve proportions problems using multiplication, eg. 3:8 as ?:40, 8 x 5 = 40 so 5 x 3 = 15. The child also renames fractions, decimals, and percentages. Ref: Q 20, 21 and 22.	The student finds a fraction of a number using a range of part-whole strategies based on multiplication and division and solves proportions problems using ratio. The child also finds percentages of a given amount. Ref Q 22, 23 and 24.
	I | F	I | F	I | F	I | F	I | F

Appendix C Rasch Analysis of the Achievement of 176 Students on the Three Scales for Which Scores Range from 3 to 8



Appendix D Ethnicity of Students

	Year 7 and 8		Year 9	
	Number	Percentage	Number	Percentage
Asian	206	11%	37	3%
European	1,124	60%	749	52%
Māori	379	20%	394	27%
Other	88	5%	48	3%
Pacific Island	74	4%	223	15%
Total	1,871	100%	1,451	100%

Appendix E Sex of Students

	Year 7 and 8		Year 9	
	Number	Percentage	Number	Percentage
Male	982	52%	654	45%
Female	889	48%	797	55%
Total	1,871	100%	1,451	100%

Appendix F Number of Students from Schools in Each Decile

Decile	Year 7 and 8		Year 9	
	Number	Percentage	Number	Percentage
1			142	10%
2			423	29%
3	698	37%	308	21%
4	615	33%	332	23%
8			117	8%
9			129	9%
10	558	30%	129	
Total	1,871	100%	1,451	100%

Appendix G Knowledge Assessment of Year 7 and 8 Students

i) Number of Year 7 and 8 Students at Each Stage at Initial and Final Assessment on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3 (nil) *	12	8	-	-	63	30
4	106	39	327	86	508	226
5	528	294	785	484	481	404
6	1,225	1,530	446	653	350	414
7	-	-	141	279	281	432
8	-	-	172	369	188	365
Total	1,871	1,871	1,871	1,871	1,871	1,871

* For Identification of Fractions the first scored stage was stage 5

ii) Percentage of Year 7 and 8 Students at Each Stage at Initial and Final Assessment on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3 (nil)*	1%	0%	-	-	3%	2%
4	6%	2%	18%	5%	27%	12%
5	28%	16%	42%	26%	26%	22%
6	65%	82%	24%	35%	19%	22%
7	-	-	8%	15%	15%	23%
8	-	-	9%	20%	10%	20%
Total	100%	100%	100%	100%	100%	100%

* For Identification of Fractions the first scored stage was stage 5

Appendix H Strategy Assessment of Year 7 and 8 Students

i) Number of Year 7 and 8 Students at Each Stage at Initial and Final Assessment on Strategy Scales

	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	106	52	105	54	491	266
4	403	169	306	162	463	325
5	848	760	562	411	237	279
6	514	890	595	673	329	441
7	-	-	303	571	275	386
8	-	-	-	-	76	174
Total	1,871	1,871	1,871	1,871	1,871	1,871

ii) Percentage of Year 7 and 8 Students at Each Stage at Initial and Final Assessment on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	6%	3%	6%	3%	26%	14%
4	22%	9%	16%	9%	25%	17%
5	45%	41%	30%	22%	13%	15%
6	27%	48%	32%	36%	18%	24%
7	-	-	16%	30%	15%	21%
8	-	-	-	-	4%	9%
Total	100%	100%	100%	100%	100%	100%

Appendix I Knowledge Assessment of Year 9 Students

i) Number of Year 9 Students at Each Stage at Initial and Final Assessment on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3 (nil) *	20	4	-	-	40	17
4	63	22	148	35	387	180
5	396	215	577	323	359	269
6	972	1,210	521	543	324	342
7	-	-	85	219	220	343
8	-	-	120	331	121	300
Total	1,451	1,451	1,451	1,451	1,451	1,451

* For Identification of Fractions the first scored stage was stage 5

ii) Percentage of Year 9 Students at Each Stage at Initial and Final Assessment on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3 (nil)*	1%	0%	-	-	3%	1%
4	4%	2%	10%	2%	27%	12%
5	27%	15%	40%	22%	25%	19%
6	67%	83%	36%	37%	22%	24%
7	-	-	6%	15%	15%	24%
8	-	-	8%	23%	8%	21%
Total	100%	100%	100%	100%	100%	100%

* For Identification of Fractions the first scored stage was stage 5

Appendix J Strategy Assessment of Year 9 Students

i) Number of Year 9 Students at Each Stage at Initial and Final Assessment on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	88	26	59	26	541	288
4	269	133	193	89	294	293
5	701	599	473	336	103	162
6	393	693	488	547	278	301
7	0	0	238	453	200	296
8	0	0	0	0	35	111
Total	1,451	1,451	1,451	1,451	1,451	1,451

ii) Percentage of Year 9 Students at Each Stage at Initial and Final Assessment on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	3%	2%	4%	2%	37%	20%
4	27%	9%	13%	6%	20%	20%
5	25%	41%	33%	23%	7%	11%
6	22%	48%	34%	38%	19%	21%
7	15%	0%	16%	31%	14%	20%
8	8%	0%	0%	0%	2%	8%
Total	100%	100%	100%	100%	100%	100%

Appendix K Number and percentage of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage at Initial and Final Assessment on Knowledge Scales

i) Number of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	9	6	-	-	54	27
4	96	36	308	77	438	197
5	427	252	616	387	386	337
6	781	1,019	295	510	244	321
7	-	-	61	192	140	284
8	-	-	33	147	51	147
Total	1,313	1,313	1,313	1,313	1,313	1,313

* For Identification of Fractions the first scored stage was stage 5

ii) Percentage of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	1%	0%	-	-	4%	2%
4	7%	3%	23%	6%	33%	15%
5	32%	19%	47%	29%	29%	26%
6	59%	77%	22%	39%	19%	24%
7	-	-	5%	15%	11%	22%
8	-	-	3%	11%	4%	11%
Total	100%	100%	100%	100%	100%	100%

* For Identification of Fractions the first scored stage was stage 5

Appendix L Number and percentage of Students from Decile 3 and 4 Schools in Year 7 and 8 at Each Stage at Initial and Final Assessment on Strategy Scales

i) Number of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	84	43	94	49	409	237
4	346	143	277	148	384	280
5	612	597	414	328	188	222
6	271	530	383	481	192	305
7	0	0	145	307	121	207
8	0	0	0	0	19	62
Total	1,313	1,313	1,313	1,313	1,313	1,313

ii) Percentage of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	6%	3%	7%	4%	31%	18%
4	26%	11%	21%	11%	29%	21%
5	47%	45%	31%	25%	14%	17%
6	21%	40%	29%	37%	15%	23%
7	0%	0%	11%	23%	9%	16%
8	0%	0%	0%	0%	1%	5%
Total	100%	100%	100%	100%	100%	100%

Appendix M Number and Percentage of Year 7 and 8 Students from a Decile 10 School at Each Stage on Knowledge Scales

i) Number of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)	3	2	-	-	9	3
4	10	3	19	9	70	29
5	101	42	169	97	95	67
6	444	511	151	143	106	93
7	-	-	80	87	141	148
8	-	-	139	222	137	218
Total	558	558	558	558	558	558

ii) Percentage of Year 7 and 8 Students from Decile 3 and 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)	1%	0%	-	-	2%	1%
4	2%	1%	3%	2%	13%	5%
5	18%	8%	30%	17%	17%	12%
6	80%	92%	27%	26%	19%	17%
7	-	-	14%	16%	25%	27%
8	-	-	25%	40%	25%	39%
Total	100%	100%	100%	100%	100%	100%

Appendix N Number and Percentage of Year 7 and 8 Students from a Decile 10 School at Each Stage on Strategy Scales

i) Number of Year 7 and 8 Students from a Decile 10 School at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
3(nil)	22	9	11	5	82	29
4	57	26	29	14	79	45
5	236	163	148	83	49	57
6	243	360	212	192	137	136
7	-	-	158	264	154	179
8	-	-	-	-	57	112
Total	558	558	558	558	558	558

ii) Percentage of Year 7 and 8 Students from a Decile 10 School at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
3(nil)	4%	2%	2%	1%	15%	5%
4	10%	5%	5%	3%	14%	8%
5	42%	29%	27%	15%	9%	10%
6	44%	65%	38%	34%	25%	24%
7	0%	0%	28%	47%	28%	32%
8	0%	0%	0%	0%	10%	20%
Total	100%	100%	100%	100%	100%	100%

Appendix O Number and Percentage of Year 9 Students from Decile 1 through 4 Schools at Each Stage on Knowledge Scales

i) Number of Year 9 Students from Decile 1 through 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	20	4	-	-	38	16
4	61	20	138	31	358	168
5	356	195	497	276	313	243
6	768	986	426	472	264	299
7	-	-	68	185	161	284
8	-	-	76	241	71	195
Total	1,205	1,205	1,205	1,205	1,205	1,205

* For Identification of Fractions failure to complete the first scored stage of the scale was level 4

ii) Percentage of Year 9 Students from Decile 1 through 4 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	2%	0%	-	-	3%	1%
4	5%	2%	11%	3%	30%	14%
5	30%	16%	41%	23%	26%	20%
6	64%	82%	35%	39%	22%	25%
7	-	-	6%	15%	13%	24%
8	-	-	6%	20%	6%	16%
Total	100%	100%	100%	100%	100%	100%

* For Identification of Fractions failure to complete the first scored stage of the scale was level 4

Appendix P Number and Percentage of Year 9 Students from Decile 1 through 4 Schools at Each Stage on Strategy Scales

i) Number of Year 9 Students from Decile 1 through 4 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
3(nil)	87	25	57	25	522	279
4	240	123	171	82	237	253
5	566	516	412	302	77	138
6	312	541	387	467	205	243
7	-	-	178	329	139	223
8	-	-	-	-	25	69
Total	1,205	1,205	1,205	1,205	1,205	1,205

(ii) Percentage of Students from Decile 1 through 4 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
3(nil)	7%	2%	5%	2%	43%	23%
4	20%	10%	14%	7%	20%	21%
5	47%	43%	34%	25%	6%	11%
6	26%	45%	32%	39%	17%	20%
7	-	-	15%	27%	12%	19%
8	-	-	-	-	2%	6%
Total	100%	100%	100%	100%	100%	100%

Appendix Q Number and Percentage of Year 9 Students from Decile 8 and 9 Schools at Each Stage on Knowledge scales

i) Number of Year 9 Students from Decile 8 and 9 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	0	0	-	-	2	1
4	2	2	10	4	29	12
5	40	20	80	47	46	26
6	204	224	95	71	60	43
7	-	-	17	34	59	59
8	-	-	44	90	50	105
Total	246	246	246	246	246	246

* For Identification of Fractions the first scored stage was stage 5

ii) Percentage of Students from Decile 8 and 9 Schools at Each Stage on Knowledge Scales

Stage	Initial Identification of Whole Numbers	Final Identification of Whole Numbers	Initial Identification of Fractions	Final Identification of Fractions	Initial Knowledge of Base 10 Grouping	Final Knowledge of Base 10 Grouping
3(nil)*	0%	0%	-	-	1%	0%
4	1%	1%	4%	2%	12%	5%
5	16%	8%	33%	19%	19%	11%
6	83%	91%	39%	29%	24%	17%
7	0%	0%	7%	14%	24%	24%
8	0%	0%	18%	37%	20%	43%
Total	100%	100%	100%	100%	100%	100%

* For Identification of Fractions the first scored stage was stage 5

Appendix R Number and Percentage of Year 9 Students from Decile 8 and 9 Schools at Each Stage on Strategy Scales

i) Number of Year 9 Students from Decile 8 and 9 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	1	1	2	1	19	9
4	29	10	22	7	57	40
5	135	83	61	34	26	24
6	81	152	101	80	73	58
7	-	-	60	124	61	73
8	-	-	-	-	10	42
Total	246	246	246	246	246	246

ii) Percentage of Students from Decile 8 and 9 Schools at Each Stage on Strategy Scales

Stage	Initial Additive Strategy	Final Additive Strategy	Initial Multiplicative Strategy	Final Multiplicative Strategy	Initial Ratio and Propor-tional Strategy	Final Ratio and Propor-tional Strategy
Nil	0%	0%	1%	0%	8%	4%
4	12%	4%	9%	3%	23%	16%
5	55%	34%	25%	14%	11%	10%
6	33%	62%	41%	33%	30%	24%
7	-	-	24%	50%	25%	30%
8	-	-	-	-	4%	17%
Total	100%	100%	100%	100%	100%	100%

Appendix S Percentage of Students Starting at Each Stage Shown with the Number of Stages They Gained

i) Percentage of Year 7 Students Making Gains in Additive Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5
Gain 0	52%	43%	68%
Gain 1	23%	49%	32%
Gain 2	25%	8%	
Gain 3	0%		

ii) Percentage of Year 8 Students Making Gains in Additive Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5
Gain 0	46%	28%	55%
Gain 1	22%	58%	45%
Gain 2	26%	14%	
Gain 3	6%		

iii) Percentage of Year 9 Students Making Gains in Additive Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5
Gain 0	30%	42%	63%
Gain 1	24%	47%	37%
Gain 2	38%	12%	
Gain 3	9%		

iv) Percentage of Year 7 Students Making Gains in Multiplicative Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6
Gain 0	51%	47%	54%	69%
Gain 1	32%	36%	39%	31%
Gain 2	12%	15%	6%	
Gain 3	3%	2%		
Gain 4	2%			

v) Percentage of Year 8 Students Making Gains in Multiplicative Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6
Gain 0	52%	41%	47%	63%
Gain 1	15%	39%	44%	37%
Gain 2	11%	13%	9%	
Gain 3	13%	7%		
Gain 4	9%			

vi) Percentage of Year 9 students Making Gains in Multiplicative Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6
Gain 0	44%	38%	52%	67%
Gain 1	25%	40%	37%	33%
Gain 2	19%	19%	11%	
Gain 3	10%	2%		
Gain 4	2%			

vii) Percentage of Year 7 Students Making Gains in Ratio and Proportional Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6	Start at Stage 7
Gain 0	57%	50%	62%	57%	82%
Gain 1	23%	24%	31%	40%	18%
Gain 2	6%	22%	7%	3%	
Gain 3	10%	5%	0%		
Gain 4	4%	0%			
Gain 5	0%				

viii) Percentage of Year 8 Students Making Gains in Ratio and Proportional Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6	Start at Stage 7
Gain 0	51%	44%	49%	60%	66%
Gain 1	19%	23%	40%	34%	34%
Gain 2	11%	23%	9%	6%	
Gain 3	16%	10%	2%		
Gain 4	3%	2%			
Gain 5	0%				

ix) Percentage of Year 9 Students Making Gains in Ratio and Proportional Strategies from Initial Stage to Final Assessment

	Start at Nil	Start at Stage 4	Start at Stage 5	Start at Stage 6	Start at Stage 7
Gain 0	53%	55%	48%	58%	75%
Gain 1	24%	19%	37%	37%	25%
Gain 2	11%	20%	12%	5%	
Gain 3	8%	5%	4%		
Gain 4	3%	1%			
Gain 5	1%				

Appendix T Interview Schedules and Questionnaires

Note that both interview schedules and questionnaires were open ended and used only as guides to conversation.

Initial interview schedule used with teachers and heads of mathematics departments:

- What has been your impression of the project so far?
- Comment on the suitability of the assessment for your students.
- How will you use this assessment information?
- How will you integrate this into your teaching programme?
- What concerns have you?

Second interview schedule used with teachers and heads of mathematics departments:

- How have you used the programme? How often?
- What have been the best, most useful aspects? Less useful?
- What activities did you find effective for students? Less effective?
- What progress did your students make?
- Are there difficulties with the project that you think should be addressed?
- What plans do you have for using the project in 2002?

Interview schedule used with facilitators:

- Tell me in general how the project is going.
- What aspects of NEST have teachers seen as particularly useful?
- Has any aspect been seen as less appropriate for this age group?
- What differences have you observed between schools or between teachers in their responses to the project?

Questionnaire sent to principals:

1. School characteristics:
   Number of students in the school
   Urban / suburban / rural
   Decile level
   Number of teachers involved in NEST
   Year levels involved

2. Why did your school take part in this project?

3. What have been the main effects from your point of view?

4. What could have been improved about the project?

5. What does your school intend to do in 2002 in relation to the project?

6. What issues underlie your decision for 2002?

7. What feedback have you had from: Students
Teachers
Parents
Board of Trustees

8. Any other comments?

9. If you are willing to take part in a 10-minute telephone interview, please give:
Telephone number
Preferred week and day
Preferred time of day