Exploring Issues in
Mathematics Education

An Evaluation of the Early Numeracy Project 2002

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Acknowledgments
This evaluation was funded by the New Zealand Ministry of Education.

Sincerest thanks are extended to the students, teachers, and principals who participated so willingly in the evaluation of the project.

Thanks also to Dr Jenny Young-Loveridge, The University of Waikato, for peer reviewing the final draft.

First published 2003 by the:

Ministry of Education
PO Box 1666
Wellington
New Zealand

ISBN 0-478-18705-X
Dewey number 372.707

Further copies may be ordered from Learning Media Customer Services, Box 3293, Wellington.
Freephone 0800 800 565, freefax 0800 800 570. Please quote item number 18705.

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Cover design: Learning Media Limited and Base2 Limited
Production: Learning Media Limited, PO Box 3293, Wellington, New Zealand.

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Executive Summary

The Numeracy Development Project sits within the context of the Ministry of Education's Literacy and Numeracy strategy. Three themes underpin that strategy: clarifying expectations, improving professional capability and involving the community.

The quantitative data collected on over 200,000 students' number strategies and knowledge as part of the Numeracy Project since 2000 provides a rich source of evidence with which to clarify expectations of student achievement.

This report describes the impact of the Early Numeracy Project on the 18,392 year 0-3 students who participated in 2002. It also reports on the longer-term sustainability of the project as investigated in 18 schools that first participated in the project in 2000 or 2001.

Key findings

Student achievement

- The project had a positive impact on students' number strategies at all year levels with the percentage of students at the higher stages of the Number Framework increasing over the duration of the project. This improvement was greater than that which would have been expected prior to the implementation of the project.

- The quantitative data collected on over 18,000 students provides a valuable source of information on which to establish realistic expectations of achievement.
    - Eighty percent of year 0-1 students are able to count from one to solve addition and subtraction problems with a further 15% able to count-on or count-back to solve such problems.
    - By the end of year 2, 40% of the students are able to use advanced counting strategies with 15% using early additive part-whole strategies to solve addition and subtraction problems.
    - Thirty-five percent of year 3 students are able to use at least one part-whole strategy to solve problems involving addition or subtraction.

- The progress that students' make is strongly linked to their starting stage on the Number Framework with those beginning at the lower stages progressing through more stages than those who begin at the upper stages. These findings provide further support for the fact that the stages on the Number Framework are not of equal size. The steps between the lower stages are smaller and consequently easier for students.

- The progress that students make on the Number Framework is linked to year level, gender, ethnicity and the decile level of the school.
    - Older students make greater progress than younger students from all stages on the Number Framework.

- Boys tend to make greater progress than girls from the higher stages of the Number Framework.
- Students in higher decile schools make greater progress than students in lower decile schools.
- The progress made by Pasifika students from the higher stages of the Number Framework is consistently lower than that made by Māori and NZ European students.

- Students who make the transition from counting from one to advanced counting strategies are stronger in their knowledge of the forward and backward number word sequence to 100.

- Students who make the transition from counting to part-whole strategies are stronger in their knowledge of the sequence of numbers to 1000 and are more likely to know their groupings with 10s.

<u>Lesson study group</u>

- Teachers involved in the trial of the lesson study group professional development approach found it to be significant in developing their own professional knowledge in mathematics, enabling them to clarify their own thinking and consolidate existing ideas.

- The participants of the lesson study group felt it would not be possible to sustain the approach in their own school without outside facilitation as the facilitator offered invaluable clarification, direction and knowledge.

<u>Longitudinal study</u>

- Teachers who have been involved in the project for one or two years believe the project has had a positive impact on students' ability, both in number and in mathematics in general. This increased ability was attributed to students' increased enjoyment and confidence, due to a feeling of success, and increased teacher expectation.

- The mean strategy stage of students in schools which participated in 2000 or 2001 is consistently lower in 2002 than 2001. This finding is inconsistent with the perceptions of the teachers and principals about the improved numeracy outcomes of their students.

- Teachers in the longitudinal study found the project to be significant in developing their professional knowledge in mathematics, increasing their awareness of the stages of development students are likely to follow and enlarging their focus on the strategies students are using to solve number problems.

- Schools in the longitudinal study had developed a number of different strategies to help ensure progress was maintained. These included the compilation of resource material to support new and existing staff, the use of staff meetings to review recent issues and the development of systems to monitor the teaching and learning process.

Chapter One: Introduction

Philosophy Behind the Numeracy Development Projects

The underlying philosophy behind the Ministry of Education's Numeracy Development Projects is that:

> *Teachers are key figures in changing the way in which mathematics is taught and learned in schools. Their subject matter and pedagogical knowledge are critical factors in the teaching of mathematics for understanding. The effective teacher of mathematics has a thorough and deep understanding of the subject matter to be taught, how students are likely to learn it, and the difficulties and misunderstandings they are likely to encounter.* (Ministry of Education, 2002a, page i)

The Numeracy Development Projects are positioned within the framework of the Ministry of Education's Literacy and Numeracy Strategy. The strategy functions to provide alignment and consistency across the range of policies and projects focused on improving student achievement in literacy and numeracy. Three key themes of the strategy have been used as an organising framework. They are:

- raising expectations for learners' progress and achievement;
- lifting professional capability throughout the system so that everyone plays their part in ensuring that the interaction between teacher and learner is as effective as possible; and
- developing community capability – encouraging and supporting family, whanau and others to help learners.

Improving student performance in mathematics through strengthening the professional capability of their teachers is the focus of the Numeracy Development Projects. A key feature of the projects is their "dynamic and evolutionary approach to implementation" (Ministry of Education 2002a, page i). The current Numeracy Development Project builds on the findings and experience associated with the projects that operated in 2000 and 2001. The evaluation reports of the projects to date highlight the success of the professional development approach used (Higgins, 2000, 2001; Irwin and Niederer, 2002; Thomas and Ward, 2001, 2002). The Numeracy Development Projects have contributed to what we know about:

- *children's learning and thinking strategies in early mathematics;*
- *effective identification of, and response to, children's learning needs;*
- *the characteristics of professional development programmes that change teaching practice; and*
- *effective facilitation.* (Ministry of Education, 2002a, page i)

Approximately 7680 teachers have participated in the Numeracy Development Projects from 2000 to 2002. It is intended that by 2005 most teachers of year 1 to 3 students, the majority of the teachers of year 4 to 6 students, and many of the teachers of year 7 and 8 students will have had the opportunity to participate in the numeracy projects.

Early Numeracy Project and its development since CMIT

The Early Numeracy Development Project (ENP) 2002 is a continuation of the ENP 2001 and builds on the success of the Count Me In Too Pilot Project (CMIT), which was implemented in 2000.

CMIT incorporated frameworks of number from the original CMIT programme: a major, school-based and systemic initiative of the New South Wales Department of Education and Training. Understanding for this programme was also drawn from the Mathematics Recovery Programme (Wright et al, 1996) which itself draws heavily on the work of Les Steffe and others at the University of Georgia, USA, (Steffe and Cobb, 1988). Important aspects of this pilot project were the Learning Framework for Number and the Schedule for Early Number Assessment.

ENP 2001 included several new developments. The Learning Framework for Number and the diagnostic assessment tool were reviewed and up-dated and draft teacher resource folders and videos were developed. The professional development programme, a key feature of the project's implementation, was also reviewed and refined. Further revisions of the programme have occurred in 2002. The most notable of these changes concern the diagnostic interview now referred to as the Numeracy Project Assessment (NumPA). This assessment has been developed to include the use of strategy window questions, allowing teachers to determine which of three test forms to use, streamlining the assessment process. The development of resource material has also been ongoing with eight booklets of teaching materials replacing the resource folder.

An important component of the Numeracy Development Projects has been the collection of quantitative data on students' number strategies and knowledge. This information has been collected by teachers through the diagnostic interview at the start and end of the project and forwarded to a secure website. Results for both the CMIT pilot project and the ENP 2001 were impressive (Thomas and Ward, 2001, 2002). In both years student progress was evaluated using the associated number framework and significant progress was observed irrespective of age, region, decile or ethnicity. Although all students made similar gains, there were marked differences between the subgroups when their profiles, expressed as stages on the number frameworks, were compared. In both 2000 and 2001 the number profiles of students were higher in higher decile schools or when students were of New Zealand European or Asian descent.

A key finding of Thomas and Ward's (2002) evaluation was that the numbers of stages gained by students was a function of their starting stage on the Number Framework. Students starting at lower stages of the framework made greater gains, which supported the notion that the framework stages are not of equal size and that the higher stages represent "bigger steps for students to make (Thomas and Ward, 2002, p. ii)." The results from 2000 and 2001 also suggest that the transition from early to advanced additive strategies is challenging and one that a significant proportion of students in the ENP are unable to make, irrespective of years of participation in the project.

The professional development programme was generally very well received by participating teachers and principals in 2000 and 2001. The teachers reported developments in their professional knowledge as a result of their involvement in the project and changed their classroom practices to accommodate their new knowledge and understandings. They were overwhelmingly positive about the project; recording increases in confidence and enthusiasm for mathematics teaching.

Description of the Number Framework

The Number Framework forms the core of the ENP by providing teachers with:

- *an effective means to assess students' current levels of thinking in number;*
- *guidance for instruction;*
- *knowledge of how children acquire number concepts and an increased understanding of how they can assist children to progress.* (Ministry of Education, 2001)

The Number Framework contains two main sections: Strategy and Knowledge. The strategy section looks at how students solve number problems, focusing on the mental processes they employ. Nine different strategy stages are described, with increasing levels of sophistication. The nine stages fit within two broader bands: Counting and Part-Whole. Each stage also contains three operational domains: addition and subtraction, multiplication and division and proportion and ratios.

The knowledge section outlines the key items of knowledge children need to acquire and has been categorised under five content domains: number identification, number sequence and order, grouping/place value, basic facts, and written recording.

The two sections of strategy and knowledge are seen as dependent on one another, with students needing to make progress in both areas simultaneously. For a more detailed description of the Number Framework see Appendix A.

Description of the Numeracy Project Assessment (NumPA)

The Numeracy Project Assessment (NumPA) is a diagnostic tool designed to give teachers quality information about the knowledge and mental strategies of their students, as aligned to the Number Framework.

Individual interviews are conducted with the role of the teacher to elicit a student's most sophisticated strategy and then determine where each response is categorised within the Number Framework. The NumPA tool has three interview forms of varying difficulty. The teacher determines the appropriate interview form to use for each student following their response to the "strategy window questions". The introduction of strategy windows was new to the 2002 NumPA. The NumPA tool enables teachers to develop a number profile for each student. This profile has two main components, operational strategies and number knowledge. The operational strategies are broken down into the three domains from the Number Framework:

- addition and subtraction;
- multiplication and division; and
- proportion and ratios.

The number knowledge component is assessed on six domains:

- facility with forward number word sequences (FNWS);
- facility with backward number word sequences (BNWS);
- ability to identify numerals (NID);
- ability to use fractions (Fractions);
- ability to use decimals and percentages (Decimals); and
- understanding of the place value nature of the number system (Grouping and Place Value).

Having teachers assess and monitor the development of children through the NumPA diagnostic interviews is an integral and mandatory component of the ENP. Teachers use the initial and subsequent assessments to make decisions regarding learning experiences necessary for individual children and groups of children, to help them advance through the stages of the Number Framework.

A copy of the NumPA strategy window is contained in Appendix B. Copies of the full interview are available on the nzmaths website (www.nzmaths.co.nz/Numeracy/project_material.htm).

Outline of this evaluation report

This report evaluates the short and longer term impact of the ENP on the participating principals, teachers, and students. Chapter Two outlines the methodology used to address the research questions. Chapters Three through Five summarise the quantitative data collected on the participating students. Chapter Six uses the evidence from the student results to suggest expectations for achievement. Chapters Seven and Eight report on a study of the sustainability of the project. This sustainability study included the trial of a professional development approach known as the Lesson Study Group, and an analysis of the ongoing progress of the project in schools that participated in either CMIT in 2000 or ENP in 2001.

Chapter Two: Methodology

The 2002 ENP evaluation comprised three main aims. The first focused on the progress of students whose teachers were participants in ENP during 2002. The second was an examination of the progress of achievement of students in a sample of schools who participated in either 2000 or 2001. The final aim investigates the use of a "lesson study group" approach as a means of ongoing professional development in early numeracy.

Aims and Research Questions of the ENP Research Evaluation

The first aim was to examine the progress of the students participating in ENP during 2002. The progress of students, arguably one of the most important aspects of the project, was designed to address the following questions:

- What progress do students make on the Number Framework?
- Is the progress of students linked to age, ethnicity, region, school decile level or gender?
- How does progress in 2002 compare to progress in the two previous years of the project?

The second aim involved a more comprehensive look at the impacts and sustainability of the project by investigating a sample of schools that first participated in 2000 or 2001. The research questions linked to this part of the evaluation are:

- What impact has participation in the Numeracy Projects had on student achievement in mathematics beyond the Number Framework?
- What progress do students make on the Number Framework in the years following participation in the Numeracy Projects?
- Is the project able to be sustained once the professional development programme has concluded?

The third aim was to investigate the effectiveness of a "lesson study group" as a means of ongoing professional development for a small number of teachers who participated in the project in 2000. This third part of the 2002 evaluation seeks to answer the following research question:

- Are lesson study groups an effective, enjoyable and sustainable professional development approach?

Design and Methodology

The investigation had three approaches. The first approach involved the collection of data from all year 0-3 students who were participating in 2002. The second approach involved the students, teachers and principals from 20 schools who had participated in CMIT in 2000 or ENP in 2001. The third was a case-study approach involving seven teachers from two schools who had participated in CMIT in 2000.

Approach One: Participating Students in 2002

The research evaluation of ENP in 2002 involved all of the schools that were accepted into the ENP project through a School Support Services contract offered on behalf of the Ministry of Education. This included approximately 26,000 year 0-3 students in 634 schools. All students were assessed twice using the NumPA tool, first at the completion of the teachers' training workshops on the assessment tool and second after the 15-20 weeks of the teaching programme that follows the initial NumPA. Each participating school was required to submit the results of both the initial and final NumPA to a secure website. In addition to the results of the NumPA, the following personal information was collected about each student: gender, date of birth, school year level and ethnicity. The date of birth data was used to calculate the age of each student as at the 1st of May 2002. As the students were linked to schools, their performance could also be reported with respect to region and decile. For the purposes of this report the deciles have been grouped into three bands. Deciles one to three form the low decile band, deciles four to seven form the medium decile band and deciles eight to ten form the high decile band.

This report examines the results of 18,392 students in schools that had submitted the initial and final NumPA results by November 1, 2002. This represents 71% of the students participating in ENP during 2002. The remaining schools were expected to enter final results by the end of the school year and do not form part of this evaluation. Tables 2.1 and 2.2 illustrate the biographic and demographic profiles of the students, which form the sample of students for this report. Approximately 2% of schools did not return decile information. The students from these schools have been included in Table 2.2 but excluded from all other results which are reported. Table 2.3 reports on the spread of the students according to region. The regions are defined by the school support service that was responsible for the delivery of the ENP professional development under contract to the NZ Ministry of Education.

Table 2.1: Profile of ENP students by age and year level

	Year level			
Age	1	2	3	Total
5	33%	0%		10%
6	67%	34%	0%	31%
7	1%	66%	33%	34%
8+	0%	0%	67%	25%
Total	5491	6014	6887	18392

Table 2.2: Profile of ENP students by ethnicity and school decile

	Decile group				
Ethnicity	None given	Low	Medium	High	Total
NZ European	71%	31%	68%	85%	61%
Māori	8%	39%	16%	5%	21%
Pasifika	7%	24%	5%	1%	10%
Asian	10%	3%	6%	5%	5%
Other	5%	3%	5%	4%	4%
Total	376	6207	6757	5052	18392

Table 2.3: Profile of ENP students by region

Region	Frequency
Auckland	37%
Christchurch	25%
Massey	4%
Dunedin	5%
Waikato	18%
Wellington	10%
Total	18392

Table 2.4: Profile of ENP students by gender

Gender	Frequency
Female	49%
Male	51%
Total	18392

Approach Two: Wider impacts and sustainability

While the 2000 and 2001 evaluations examined student progress in numerical development it was measured solely using the assessment tools inherent in the project. One of the aims of the 2002 investigation was to examine the wider impact of the project by looking at student progress in mathematics as measured by other assessment tools and measures used by schools. Schools who had participated in either the CMIT pilot project in 2000 or the ENP in 2001 were selected and invited to take part in this component of the evaluation.

The first sample comprised nine schools from the CMIT pilot project. The case-study schools were selected from three regions (Auckland, Wellington and Dunedin). Three schools from Auckland, three from Wellington and three from Dunedin were randomly selected from a list of CMIT pilot project schools from these regions. All of the schools accepted the invitation to participate; however, one of the schools opted to withdraw from the evaluation after the first visit citing non-project related issues within the school. The remaining eight schools will be subsequently referred to as the CMIT 2000 schools

The principal and/or teacher with responsibility for maths in each school were interviewed on two occasions for their views about any wider impacts of the project and its sustainability. The interviews were conducted in June and September and followed the framework contained in Appendix G.

Questionnaires were sent to all teachers in the remaining eight CMIT 2000 schools (see Appendix H). The questionnaires were designed to collect relevant demographic and biographical details about participants and to elicit perceptions about the wider impacts of the project and its sustainability. The schools were also asked to submit the strategy stages, as assessed on the NumPA, for each of their year 0-3 students to the project website by November 1. Tables 2.4 and 2.5 illustrate the biographic and demographic profiles of the students from these CMIT 2000 schools.

Table 2.5: Profile of students from CMIT 2000 schools by age and year level

	Year group			
Age	1	2	3	Total
5	53%	0%	0%	20%
6	46%	40%	1%	30%
7	0%	60%	33%	29%
8	0%	1%	66%	21%
Total	429	324	364	1135

Table 2.6: Profile of students from CMIT 2000 schools by decile and ethnicity

	Decile group			
Ethnicity	Low	Medium	High	Total
NZ European	38%	85%	74%	65%
Māori	29%	12%	6%	15%
Pasifika	20%	1%	4%	9%
Asian	6%	1%	14%	8%
Other	8%	1%	2%	4%
Total	391	318	426	1135

The second sample was selected from schools that participated in ENP in 2001. These schools will be subsequently referred to as the ENP 2001 schools. Initially ten schools were randomly selected and invited to participate with six of these schools accepting. An additional four schools were randomly selected with these accepting the invitation to participate. One further school was invited to participate following the recommendation of a facilitator on the basis that they were known to be already investigating the wider impact of the project within their school. Questionnaires (see Appendix H) were sent to the year 0-3 teachers in the eleven ENP 2001 schools. The year 4 to 6 teachers and their students were not included in the sample for this research component as they were participating in Advanced Numeracy Project (ANP) during 2002. The schools were also asked to submit the strategy stage from the Number Framework for each of their year 0-3 students to the project website by November 1. The letter of invitation indicated that a full NumPA assessment was not required as it was assumed that most classes would be grouped according to strategy stages on the Number Framework using a previous version of the assessment tool. Tables 2.6 and 2.7 illustrate the biographic and demographic profiles of the students from the eight ENP 2001 schools which had submitted data by November 1.

Table 2.7: Profile of students from ENP 2001 schools by age and year level

	Year group			
Age	1	2	3	Total
5	48%	0%		18%
6	52%	38%		30%
7	0%	61%	32%	28%
8		0%	67%	23%
Total	470	344	413	1227

Table 2.8: Profile of students from ENP 2001 schools by decile and ethnicity

	Decile group			
Ethnicity	Low	Medium	High	Total
NZ European	7%	67%	66%	48%
Māori	64%	20%	6%	31%
Pasifika	25%	4%	1%	10%
Asian	1%	5%	19%	7%
Other	2%	4%	8%	4%
Total	387	556	284	1227

<u>Approach Three: Lesson Study Group</u>

The year 0-3 teachers from two schools within the same city were invited to participate in the lesson study group component of the research. The schools had participated in CMIT in 2000, and in 2001 one of the teachers in both schools had participated in the effective teacher case study (Thomas and Ward, 2002). The seven lesson study group teachers met on twelve occasions spread throughout terms two and three. The meetings were held outside school hours and lasted about an hour. The aim of the meetings was to collaboratively plan, implement, evaluate and revise "lessons" linked to the use of tens frames, a key piece of equipment used in ENP. The meetings were attended by two of the researchers of this report. The planned role of the researchers was to take meeting notes, transcribe audio tapes, and to update the lesson plans collectively developed by the teachers.

Chapter Three: Progress Made by Year 0-1 Students

This is the first of three results chapters which report on the impact of the project on students by year level. As data are reported on approximately 18,500 students, there are large numbers in each sample, even when they are analysed by subgroups according to biographic and demographic variables. Sample size raises issues related to practical versus statistical significance. With such large samples, even the smallest differences can be statistically significant. Throughout the following chapters the results are reported in terms of the real or practical significance of any observed difference between groups.

The findings presented in this chapter are organised into four sections. The first section presents an overview of the findings in terms of the percentages of students reaching the various stages on the strategy section of the Number Framework. The second section focuses on the students' number strategies and looks at the progress of students with reference to their starting points on the Number Framework. The progress of students in relation to the variables of gender, ethnicity, school region and school decile are examined. The third section details the progress that students made on the knowledge aspects of the Number Framework. The fourth section explores the relationships that exist between the strategy and knowledge aspects of the Number Framework in relation to the transition from emergent, to count from one, to advanced counting strategies.

Impact of the Project on Students' Number Strategies

Consistent with the findings of previous years (Thomas and Ward, 2001, 2002) the progress of year 0-1 students was substantial. It is clear from Table 3.1 that the project had a positive impact on students' number strategies with the percentage of students at the lowest stages on the framework decreasing.

Table 3.1: Percentage of year 0-1 students by stage on the additive domain at the start and end of the project

	Initial additive	Final additive
n=	5491	5491
0: Emergent	26%	6%
1: One to one counting	31%	14%
2: Counting from one on materials	33%	45%
3: Counting from one by imaging	6%	17%
4: Advanced counting	4%	17%
5: Early additive part-whole	0%	1%
6: Advanced Additive part whole		0%

At the start of the project 26% of the year 0-1 students were assessed as emergent on the additive domain. By the end of the project only 6% of the students were still unable to form a set of up to 10 objects. The percentage of students who were able to count-on or count-back to solve addition or subtraction problems had increased from 4% to 18%. Appendix C details the percentages of year 0-1 students by stage at the end of the project on the various domains of the Number Framework as a function of gender, ethnicity, and school decile level. Tables 3.2 to 3.4 present the mean stage of the addition strategy subgroups at the beginning and end of the project as an indication of where differences exist. All subgroups were found to make a gain of around a stage, with the differences primarily around the start and end points for each subgroup. Māori and Pasifika students, and those from lower decile schools, started and finished at a lower mean stage than the other students. The largest gain was made by students from high decile schools (1.11), and the lowest was made by the 124 students from schools with no decile information (0.86). That the mean initial additive stage of Pasifika students is below 1, or 0.34 of a stage below the national mean is a cause for concern, especially as they also finish the project 0.36 of a stage below the national mean.

Table 3.2: Mean additive stage at the start and end of the project by gender for year 0-1 students

Gender	N	Initial additive	Final additive
Female	2690	1.3	2.3
Male	2801	1.3	2.3
Total	5491	1.3	2.3

Table 3.3: Mean additive stage at the start and end of the project by decile band for year 0-1 students

Decile group	N	Initial additive	Final additive
Low	1717	1.2	2.1
Medium	1966	1.3	2.2
High	1684	1.5	2.6
None given	124	1.4	2.3
Total	5491	1.3	2.3

Table 3.4: Mean additive stage at the start and end of the project by ethnicity for year 0-1 students

Ethnicity	N	Initial additive	Final additive
NZ European	3365	1.4	2.4
Māori	1103	1.1	2.0
Pasifika	565	1.0	1.9
Asian	269	1.6	2.6
Other	189	1.3	2.2
Total	5491	1.3	2.3

Students were not assessed in the multiplicative and proportional domains unless they were at the counting from one stage on the additive domain. Consequently, 99% of the year 0-1 students were not rated on the other two domains at the start of the project. By the end of the project 8% of the students were rated on these domains as shown in Table 3.5.

Table 3.5: Percentage of year 0-1 students by stage on the multiplicative domain at the start and end of the project.

	Initial multiplicative	Final multiplicative
n=	5491	5491
Not Rated	99%	92%
2-3: Counting from one	0%	2%
4: Advanced counting	0%	5%
5: Early additive part-whole		0%
6: Advanced Additive part whole		0%

Although there was no control group as such, the year 2 students at the start of the project provide a contrast for the year 1 students at the end the project. At the end of the project, the year 0-1 students were still about 6 months younger than the year 2 students at the start of the project; however, they were performing at a similar level to the year 2 students. Table 3.6 shows this comparison.

Table 3.6: Percentage of year 0-1 (final) and year 2 (initial) students by stage on the additive domain

	Year 0-1 Final additive	Year 2 Initial additive
n=	5491	6014
0: Emergent	6%	6%
1: One to one counting	14%	14%
2: Counting from one on materials	45%	42%
3: Counting from one by imaging	17%	13%
4: Advanced counting	17%	22%
5: Early additive part-whole	1%	2%
6: Advanced Additive part whole	0%	

Patterns of Progress on the Number Framework (Additive Strategies)

The 2001 evaluation of the ENP found that the number of stages gained by students varied as a function of their initial stage on the framework (Thomas and Ward, 2002). This means that it is not appropriate to compare the gains of various subgroups unless they have the same starting stage on the framework. This section examines progress on the strategy section of the Number Framework in relation to starting points.

Table 3.7 shows the percentage of students at each stage of the framework at the end of the project in relation to their starting point. The percentages in bold are the students who remained at the same stage throughout the project. The percentage of students who remain at the same stage increases the higher the initial stage on the framework. Eighty percent of the students initially at Stage 0 or Stage 1 moved up at least one stage compared to the 17% who moved up from Stage 4. In italics are the small percentages of students whose final stage is lower than their initial stage. It is interesting to note the 8% of students who dropped from Stage 3 to Stage 2. This may be partly explained by the relatively "fine" difference between counting with the support of materials and counting by imaging. Figure 3.1 provides a visual representation of the same data. As there were just three students who began the project at early additive part-whole their progress is not reported in the following analysis.

Table 3.7: Final additive stage by initial additive stage for year 0-1 students

	Initial additive stage						
Final additive stage	0	1	2	3	4	5	Total
0: Emergent	**21%**	*1%*	*1%*				6%
1: One to one counting	30%	**17%**	*2%*	*0%*			14%
2: Counting from one on materials	40%	57%	**48%**	*8%*	*3%*		45%
3: Counting from one by imaging	6%	16%	26%	**34%**	*3%*		17%
4: Advanced counting	3%	9%	23%	53%	**78%**		17%
5: Early additive part-whole		0%	1%	4%	16%	**100%**	1%
6: Advanced Additive part whole					1%		0%
Total	1431	1707	1822	331	197	3	5491

Figure 3.1: Final additive stage by initial additive stage for year 0-1 students

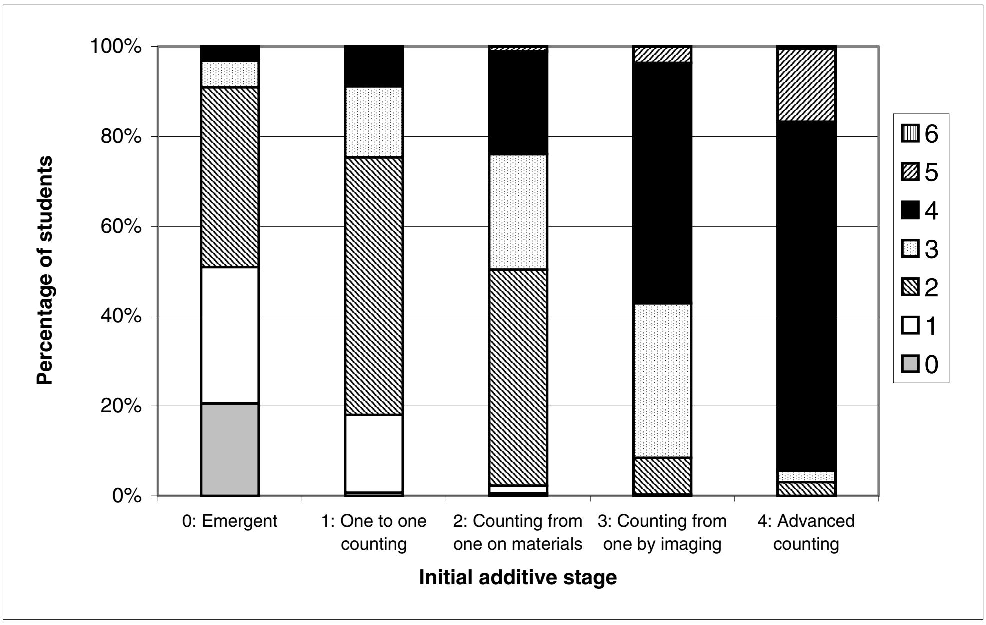


Figure 3.2 illustrates the patterns of progress when the year 0-1 students are analysed according to gender. The pattern for year 0-1 girls was very similar to year 0-1 boys for those who began the project in the lowest two stages. However, the boys make greater gains at the higher stages with 21% of the boys making the transition from advanced

counting to early additive part-whole, compared with 9% of the girls. Similarly, 8% more boys made the transition from count from one by imaging to advanced counting.

Figure 3.2: Final additive stage by initial additive stage and gender for year 0-1 students

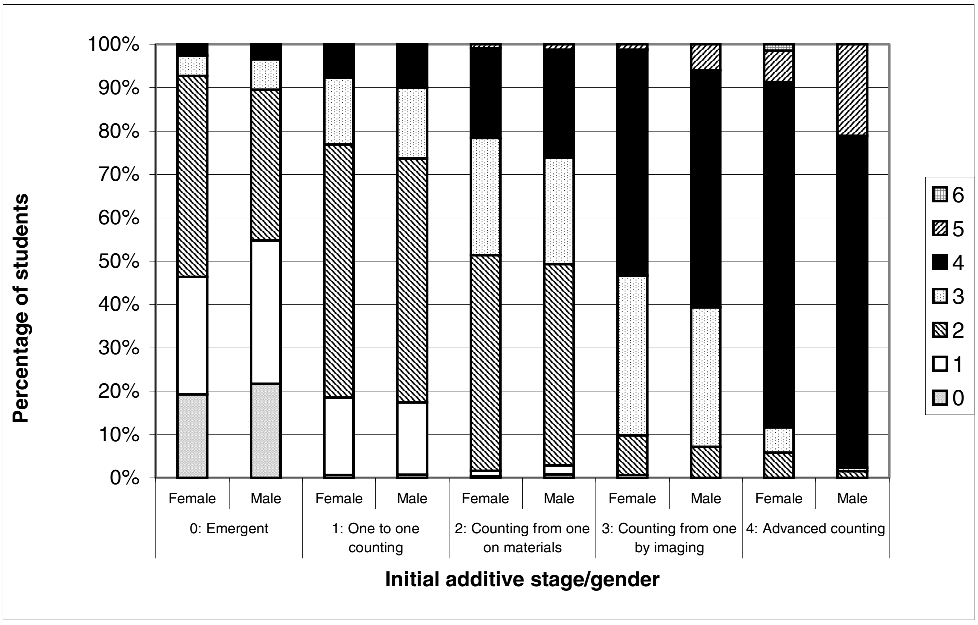


Figure 3.3 presents the number of stages gained by the year 0-1 students when the results are analysed according to ethnicity and starting stage. Consistent with the findings of the ENP 2001 evaluation the gains made by Māori and Pasifika students from the first three stages were slightly but consistently lower than the gains made by the other groups (Thomas and Ward, 2002). At Stage 3 and Stage 4 the Pasifika students progress at a similar level to the NZ European and Asian students and outperform the Māori students. A focus on the students who were emergent at the start of the project shows that 9% of the Asian students were still emergent at the end compared to the overall average of 21%.

Figure 3.3: Final additive stage by initial additive stage and ethnicity for year 0-1 students

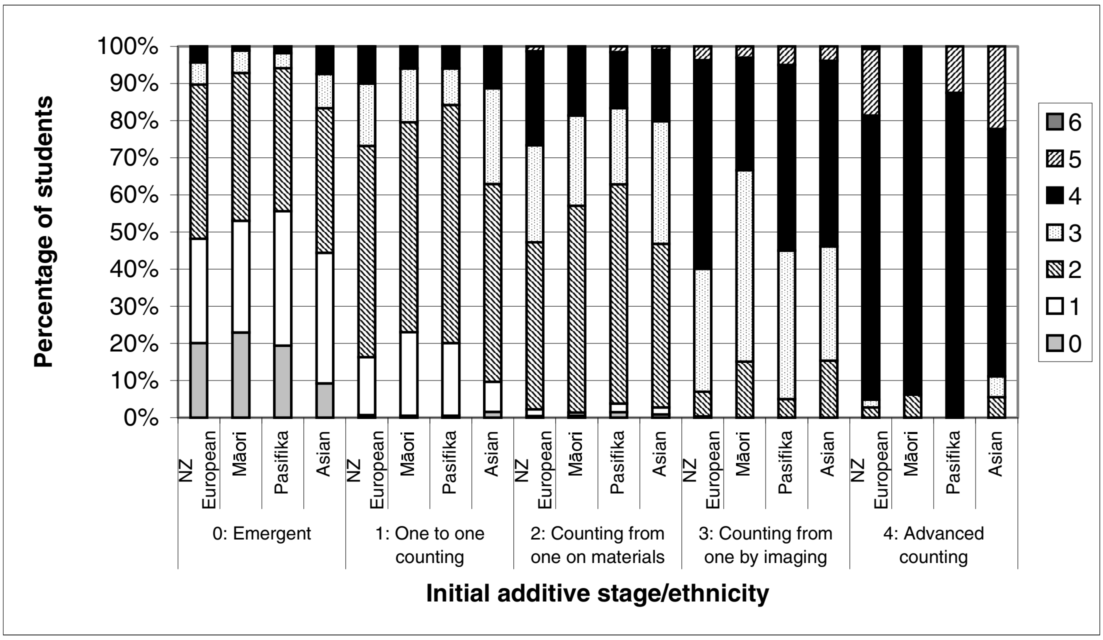
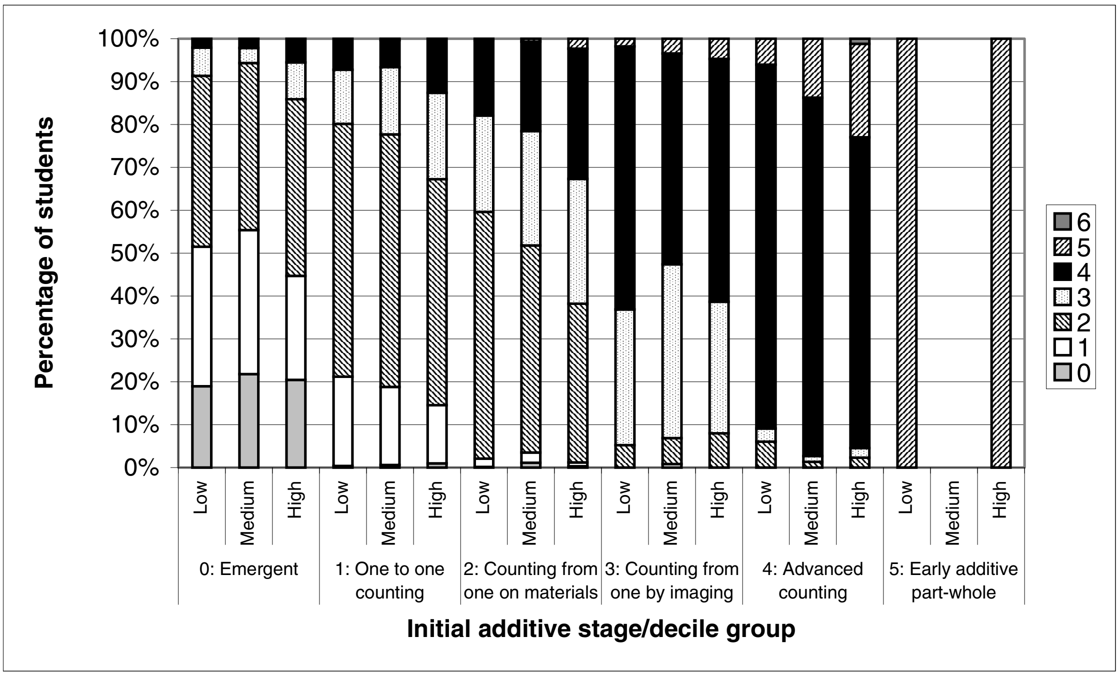


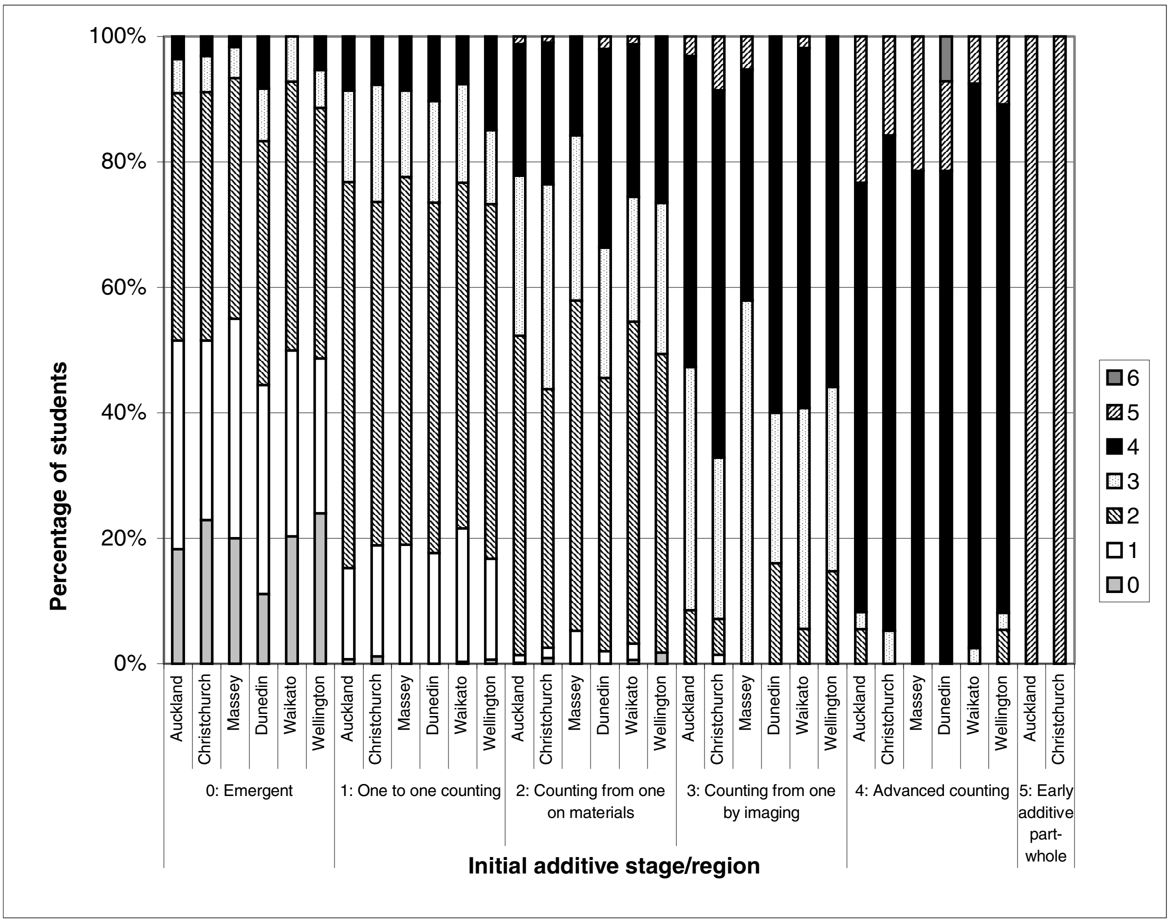
Figure 3.4 presents the mean number of stages gained by the year 0-1 students when their progress is analysed according to the decile level of their school and starting stage. The impact of decile is seen most noticeably in the students who began the project at Stage 2 or Stage 4. Thirty-three percent of students from high decile schools moved from Stage 2 to Stage 4, compared with 22% for medium and 18% for low decile schools. A similar difference is seen at Stage 4, where 23% of the students in high decile schools made the transition to early additive part-whole, compared to 14% in the medium and 6% in the low decile schools. The impact of decile on progress is less noticeable at Stages 0, 1, and 3.

Figure 3.4: Final additive stage by initial additive stage and decile group for year 0-1 students



As presented by Figure 3.5, the pattern of progress when analysed by region presents more similarities than differences. One noticeable difference occurs at Stage 3 where 67% of the students in the Christchurch region make the transition to at least Stage 4 compared to 44% for Massey and 53% to 60% for the other four regions.

Figure 3.5: Final additive stage by initial additive stage and region for year 0-1 students



Impact of the Project on Students' Number Knowledge

This section examines the progress that students make on four of the six knowledge domains as assessed by the NumPA. As fewer than 5% of the year 0-1 students were assessed on the fraction and decimal domains, these results are not discussed in this section. Tables 3.8 to 3.11 present the proportion of year 0-1 students at each stage on the framework at the initial and final assessments. The initial results for the year 2 students who form the contrast group are also shown.

Table 3.8 shows that 12% of the year 0-1 students began the project unfamiliar with the forward number word sequence (FNWS) in the range 0 to 10. By the end of the project this had fallen to 2%. Seventy-five percent of the students knew the forward number word sequence at least to 20 by the final assessment, compared to 28% percent at the initial assessment. The final results for the year 0-1 students are very similar to the initial results for the year 2 students despite the 6-month difference in ages.

Table 3.8: Percentage of year 0-1 students by stage on the FNWS domain at the start and end of the project and the contrast year 2 students

	Initial FNWS	Final FNWS	Year 2 Initial FNWS
n=	5491	5491	6014
0: Emergent	12%	2%	2%
1: Initial to 10	29%	7%	8%
2: To 10	31%	16%	21%
3: To 20	19%	32%	26%
4: To 100	9%	40%	39%
5: To 1000	0%	3%	4%
6: To 1000000		0%	0%

Table 3.9 shows similar patterns of improvement for the backward number word sequence (BNWS). At the start of the project, 37% of the year 0-1 students were classified as emergent. This had dropped to 6% by the end of the project. Correspondingly, the proportion of students who knew the BNWS to at least 100 had increased from 4% to 29%.

Table 3.9: Percentage of year 0-1 students by stage on the BNWS domain at the start and end of the project and the contrast year 2 students

	Initial BNWS	Final BNWS	Year 2 Initial BNWS
n=	5491	5491	6014
0: Emergent	37%	6%	8%
1: Initial to 10	23%	12%	14%
2: To 10	26%	27%	29%
3: To 20	9%	25%	20%
4: To 100	4%	27%	26%
5: To 1000	0%	2%	3%
6: To 1000000		0%	0%

It is clear from Table 3.10 that the project had a positive impact on students' ability to identify numbers with the percentage of students at the lower stages on the framework decreasing while those at the upper stages increased. Twenty-four percent of the students were unable to identify numbers in the range 0-10 at the initial assessment. This had dropped to 3% by the final assessment. Correspondingly, the proportion of students who were able to identify numbers to at least 100 had increased from 20% to 61% by the final assessment. Discounting the year 2 students who were not assessed on this domain, the final results for year 0-1 are again very similar to the initial year 2 results.

Table 3.10: Percentage of year 0-1 students by stage on the Numeral Identification domain at the start and end of the project and the contrast year 2 students

	Initial NID	Final NID	Year 2 Initial NID
n=	5491	5491	6014
Not Assessed	1%	4%	12%
0: Emergent	24%	3%	3%
1: To 10	37%	16%	14%
2: To 20	18%	15%	14%
3: To 100	17%	44%	42%
4: To 1000	3%	17%	15%

As shown by Table 3.11, the proportion of students assessed as using groupings of numbers increased from 8 to 51% over the project. Year 0-1 students with six months involvement in the ENP clearly outperformed year 2 students entering the project with 51% assessed as using grouping strategies, compared to 32% for year 2 students.

Table 3.11: Percentage of year 0-1 students by stage on the Grouping domain at the start and end of the project and the contrast year 2 students.

	Initial Group	Final Group	Year 2 Initial Group
n=	5491	5491	6014
Not Assessed	2%	1%	1%
0-1: Non-Grouping	90%	48%	66%
2-3: With 5s and within 10	7%	32%	23%
4: With 10s	1%	18%	9%
5: 10s in 100		1%	0%
6: 10s and 100s		0%	0%
7: 10s 100s and 1000s			0%

The Relationship Between Strategies and Knowledge

This section examines the relationship between the students' use of strategies and their number knowledge. More specifically, it compares the knowledge profile of students (initially Stage 2 or 3) who progressed to Stage 4 (advanced counting) with those who remained at or below Stage 3 (count from one by imaging). It is clear from Table 3.12 that these two groups differed in the four knowledge domains displayed.

Table 3.12: Comparing the number knowledge of initially count from one students (Stage 2 or 3) who became advanced counters (Stage 4+) with those who remained count from one

	Remained count from one	Became advanced counters
n=	1529	624
FNWS		
0: Emergent	0%	
1: Initial to 10	2%	0%
2: To 10	13%	4%
3: To 20	35%	12%
4: To 100	49%	72%
5: To 1000	1%	11%
6: To 1000000	0%	1%
BNWS		
0: Emergent	2%	0%
1: Initial to 10	7%	1%
2: To 10	27%	8%
3: To 20	32%	17%
4: To 100	31%	65%
5: To 1000	1%	8%
6: To 1000000		1%
NID		
Not Assessed	1%	21%
0: Emergent	1%	0%
1: To 10	10%	1%
2: To 20	13%	4%
3: To 100	56%	36%
4: To 1000	19%	38%
Groupings		
Not Assessed	1%	0%
0-1: Non-Grouping	41%	12%
2-3: With 5s and within 10	40%	32%
4: With 10s	18%	53%
5: 10s in 100	0%	3%
6: 10s and 100s		0%

Almost twice as many students who progressed knew their forward number word sequence to at least 100 compared to those who remained at Stage 3. A similar difference is seen in the backward number word sequence with 74% of those who became advanced counters assessed as knowing the sequence to at least 100, compared to 32% of those who remained counting from one. Students who made the transition were also stronger in their knowledge of groupings with 88% assessed as able to group at least with 5s compared to 58% of those who did not make the transition.

Chapter Four: Progress Made by Year 2 Students

The findings presented in this chapter are organised in a similar way to Chapter Three. The first section presents an overview of the findings in terms of the percentages of students reaching the different stages on the Strategy Section of the Number Framework. The second section again focuses on the students' number strategies but looks at the progress of students with reference to their initial stage on the Number Framework. The progress of students in relation to the variables of gender, ethnicity, school region and school decile are examined. The third section details the progress that students made on the knowledge domains of the Number Framework. The fourth section explores the relationships that exist between the strategy and knowledge aspects of the Number Framework in relation to the transition to advanced counting strategies.

Impact of the Project on Students' Number Strategies

A large proportion of year 2 students made progress over the duration of the project in their use of number strategies. As shown by Table 3.1 the percentages of students at the upper stages on the framework for additive strategies increased while those at the lower stages decreased. The additive domain of the framework is focused on strategies that students use to add and subtract numbers mentally.

Table 4.1: Percentage of year 2 students by stage on the additive domain at the start and end of the project.

	Initial additive	Final additive	Year 3 Initial additive
n=	6014	6014	6887
0: Emergent	6%	2%	2%
1: One to one counting	14%	3%	6%
2: Counting from one on materials	42%	23%	22%
3: Counting from one by imaging	13%	17%	12%
4: Advanced counting	22%	43%	46%
5: Early additive part-whole	2%	11%	12%
6: Advanced Additive part whole		1%	1%

At the start of the project 20% of the year 2 students were at stage 0 or 1. These students are assessed as unable to count objects to solve simple addition or subtraction problems. By the end of the project only 5% of the students were still unable to count objects to solve addition or subtraction problems. At the higher end of the framework the percentage of students who were able to count-on or count-back (Stage 4) to solve addition or subtraction problems had increased from 22% to 43%.

Although there was no control group as such, the year 3 students at the start of the project provide a contrast for the year 2 students at the end the project. Although the year 3 are still about 6 months older at the start of the project than the year 2 students, at the end they performed at a similar level. The third column of results in Table 4.1 presents the initial results of the year 3 students.

Appendix D details the percentages of year 2 students at the end of the project by stage on the various domains of the Number Framework in terms of the subgroups studied. Tables 4.2 to 4.4 present the mean stage of the addition strategy subgroups at the beginning and end of the project as an indication of where differences exist.

The only subgroups which did not make a mean gain of around a stage were low decile schools (0.86) and Māori students (0.83). This is particularly concerning as these groups both start lower than most on the framework, where the expected gain is higher due to the smaller size of stages. Pasifika students, who started lowest of all on average (1.93) made a gain of almost a stage and were comparable with Māori students by the end of the project.

Table 4.2: Mean additive stage at the start and end of the project by gender for year 2 students

Gender	N	Initial additive	Final additive
Female	2953	2.3	3.3
Male	3061	2.4	3.4
Total	6014	2.4	3.3

Table 4.3: Mean additive stage at the start and end of the project by decile band for year 2 students

Decile group	N	Initial additive	Final additive
None given	112	2.6	3.7
Low decile	2091	2.1	3.0
Medium decile	2160	2.4	3.4
High decile	1651	2.6	3.6
Total	6014	2.4	3.3

Table 4.4: Mean additive stage at the start and end of the project by ethnicity for year 2 students

Ethnicity	N	Initial additive	Final additive
NZ European	3605	2.5	3.5
Māori	1200	2.1	2.9
Pasifika	683	1.9	2.9
Asian	286	2.7	3.8
Other	240	2.4	3.4
Total	6014	2.4	3.3

The multiplicative domain of the framework looks at the strategies that students could use to multiply or divide numbers mentally. This domain includes stages where the students use counting strategies in order to solve a multiplication problem. For example, if a student skip counts they are assessed as being at Stage 4. True multiplicative strategies, which involve working with groups of numbers, begin at Stage 6. Table 4.5 shows the proportions of year 2 students at each stage of the multiplicative domain. The year 3 initial results are also included as a way of contrasting the progress that would have been made by students prior to the implementation of the Numeracy Project. In a similar finding to that reported in Table 4.1, the year 2 students at the end of the project are similar in performance to the older contrast group.

Table 4.5: Percentage of year 2 students by stage on the multiplicative domains at the start and end of the project.

	Initial multiplicative	Final multiplicative	Year 3 Initial Multiplicative
n=	6014	6014	6887
Not Rated	89%	63%	54%
2-3: Counting from one	6%	8%	17%
4: Advanced counting	4%	23%	23%
5: Early additive part-whole	1%	5%	5%
6: Advanced additive part whole	0%	1%	1%
7: Advanced multiplicative part-whole		0%	0%

The proportional domain of the strategy framework is the most challenging (Irwin and Niederer, 2001) and just 11% of the year 2 students at the start of the project were assessed against it. Table 4.6 presents the percentage of year 2 students at each stage of the proportional domain at the start and end of the project and the year 3 students' initial results. A comparison of Tables 4.1, 4.5, and 4.6 shows that between three and eleven percent of the students were assessed at the early additive part-whole stage or above in each of the three operational domains by the end of the project.

Table 4.6: Percentage of year 2 students by stage on the proportional domains at the start and end of the project.

	Initial proportional	Final proportional	Year 3 Initial Proportional
n=	6014	6014	6887
Not Rated	89%	63%	55%
1: Unequal sharing	6%	8%	20%
2-4: Equal Sharing	5%	26%	23%
5: Early additive part-whole	0%	3%	3%
6: Advanced additive part-whole	0%	1%	0%
7: Advanced multiplicative part-whole		0%	0%
8: Advanced proportional part-whole			

Patterns of Progress on the Number Framework (Strategies)

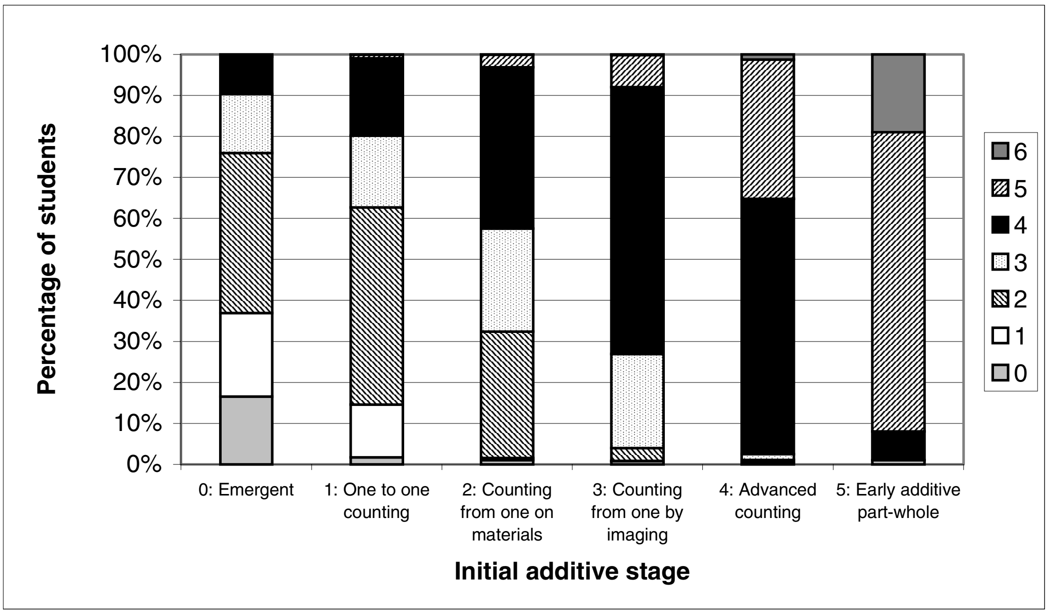
This section examines progress as a function of starting points on the framework in relation to gender, ethnicity, school decile, and region.

Table 4.7 shows the percentage of students at each stage of the additive domain at the end of the project in relation to their starting stage. The percentages in bold are the students who remained at the same stage throughout the project. In italics are the small percentages of students whose final stage is lower than their initial stage. Consistent with the year 0-1 findings, the students who began the project at the lower levels of the framework moved up the highest number of stages. It is of concern to note that 16% of the year 2 students who were emergent at the start of the project were still emergent at the end. These students represent 1% of the year 2 students. On the other hand, 10% of the initially emergent students progressed at least 4 stages to advanced counting or early additive. Of the students who began the project as advanced counters 35% made the transition to part-whole strategies (Stage 5 or 6). The bars in Figure 4.1 present the same data.

Table 4.7: Final additive stage by initial additive stage for year 2 students

	Initial additive stage						
Final additive stage	0	1	2	3	4	5	Total
0: Emergent	**16%**	*2%*	*1%*	*1%*	*0%*	*1%*	2%
1: One to one counting	20%	**13%**	*1%*	*0%*			3%
2: Counting from one on materials	39%	48%	**31%**	*3%*	*1%*		23%
3: Counting from one by imaging	14%	18%	25%	**23%**	*1%*		17%
4: Advanced counting	9%	19%	39%	65%	**62%**	*7%*	43%
5: Early additive part-whole	1%	1%	3%	8%	34%	**73%**	11%
6: Advanced Additive part whole		0%	0%	0%	1%	19%	1%
Total	382	838	2545	805	1344	100	6014

Figure 4.1: Final additive stage by initial additive stage for year 2 students



Figures 4.2 to 4.5 illustrate the final additive stage of the various subgroups of year 2 students as a function of their starting stage. Figure 4.2 presents the pattern of progress by gender and shows that year 2 boys and girls make similar progress in the first three

stages. The boys then make greater gains than the girls with 12% more boys than girls making the transition from advanced counting to early additive part-whole strategies. The difference appears even greater at Stage 5 with 22% of the boys moving to advanced additive part-whole compared to 6% of the girls. However, the sample size is small, with just 19 of the 6014 year 2 students.

Figure 4.2: Final additive stage by initial additive stage and gender for year 2 students

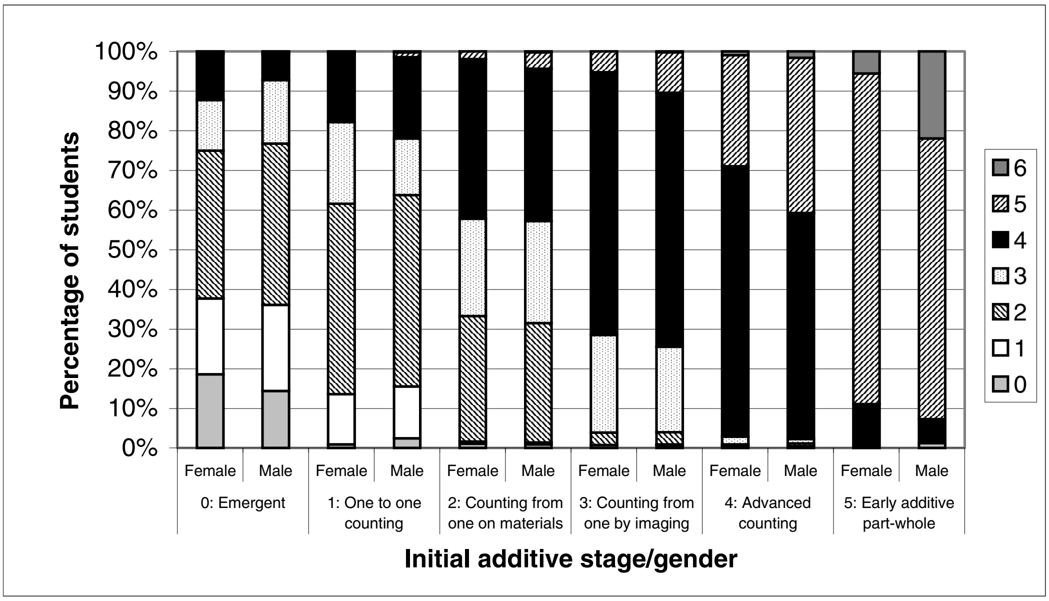


Figure 4.3 presents the final additive stage of the year 2 students when the results are analysed according to ethnicity and starting stage. The progress made by Māori and Pasifika students was consistently lower than that made by the other groups for Stages 2, 3, and 4. At Stage 0 the Māori students made similar progress to the NZ European and Asian students and outperformed the Pasifika students. At Stage 1 the Asian students made greater progress than the other three subgroups who all performed similarly. The difference in progress is more obvious at the higher stages where, for example, 40% of NZ European students made the transition from advanced counting to early additive part-whole compared to 23% of Māori and 22% of Pasifika students.

Figure 4.3: Final additive stage by initial additive stage and ethnicity for year 2 students

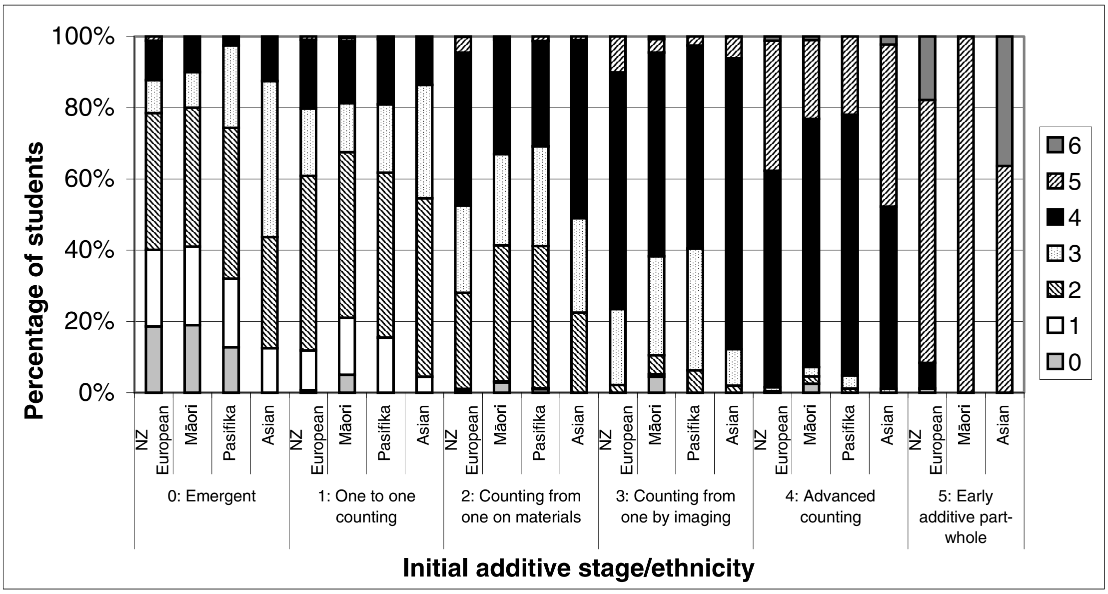
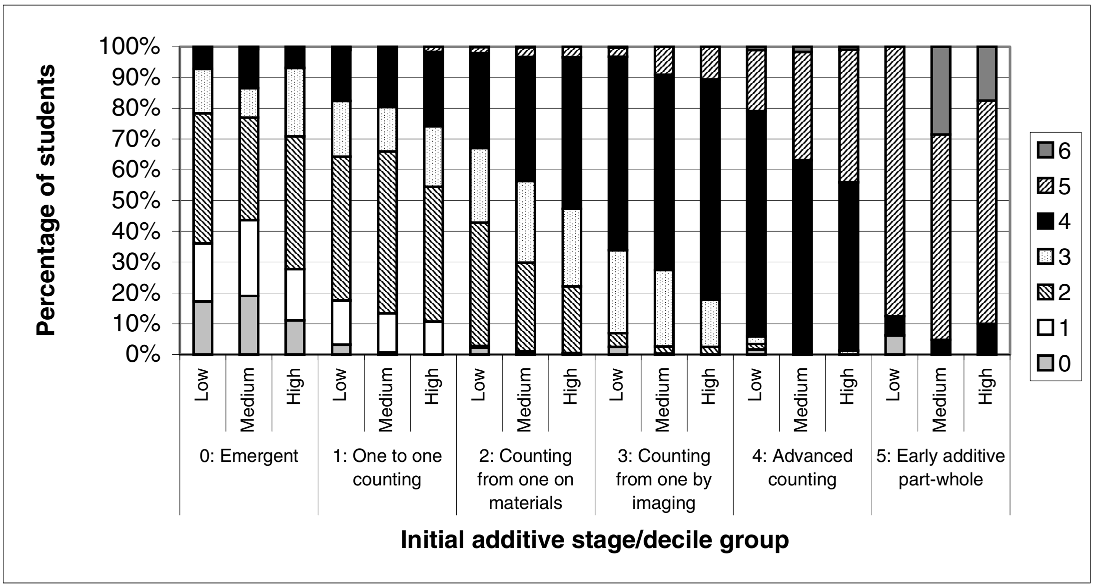


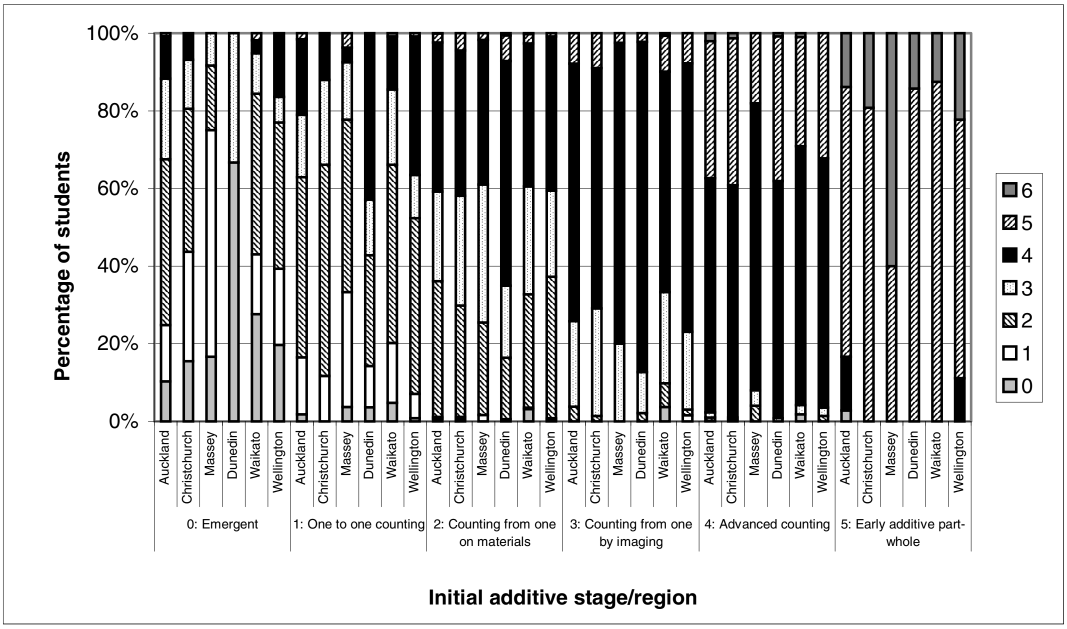
Figure 4.4 presents the final additive stage of the year 2 students when their progress is analysed according to the decile level of their school and starting stage. Students from high decile schools consistently rate higher than those from medium decile schools, with low decile schools worse still. The exception to this is at Stage 5 where 12 out of the 42 (29%) students from medium decile schools made the transition to advanced additive part-whole compared to 7 of the 40 (18%) students from high decile schools.

Figure 4.4: Final additive stage by initial additive stage and decile group for year 2 students



As presented by Figure 4.5 there are some interesting fluctuations in the final stage of students when progress is analysed by region and in relation to their starting stage. One such fluctuation is at Stage 2 where 65% of students in Dunedin made the transition to Stage 4 compared to approximately 40% across the other regions. Another large fluctuation occurs at Stage 5 where 60% of students in the Massey region move to Stage 6 compared to approximately 20% in the other regions. This fluctuation is largely explained by the sample size as just 5 students were initially classified at Stage 5 in Massey.

Figure 4.5: Final additive stage by initial additive stage and region for year 2 students



Impact of the Project on Students' Number Knowledge

This section examines the progress that students make on five of the six knowledge domains as assessed by the NumPA. As fewer than 2% of the year 2 students were assessed on the decimal domain, these results are not reported in this section. Tables 4.8 to 4.12 present the proportion of year 2 students at each stage on the framework at the initial and final assessments. The initial results for the year 3 students who form the contrast group are also shown.

Table 4.8 shows that at the start of the project 69% of the year 2 students were able to identify the number after given numbers in the range 0 to 20. By the end of the project this had increased to 93%.

Table 4.8: Percentage of year 2 students by stage on the FNWS domain at the start and end of the project and the contrast year 3 students

	Initial FNWS	Final FNWS	Year 3 Initial FNWS
n=	6014	6014	6887
0: Emergent	2%	1%	1%
1: Initial to 10	8%	1%	2%
2: To 10	21%	5%	6%
3: To 20	26%	15%	14%
4: To 100	39%	55%	54%
5: To 1000	4%	21%	22%
6: To 1000000	0%	2%	2%

Table 4.9 shows similar patterns of improvement for the backward number word sequence.

Table 4.9: Percentage of year 2 students by stage on the BNWS domain at the start and end of the project and the contrast year 3 students

	Initial BNWS	Final BNWS	Year 3 Initial BNWS
n=	6014	6014	6887
0: Emergent	8%	2%	3%
1: Initial to 10	14%	3%	4%
2: To 10	29%	11%	12%
3: To 20	20%	17%	15%
4: To 100	26%	47%	44%
5: To 1000	3%	18%	20%
6: To 1000000	0%	2%	2%

Table 4.10 illustrates the positive impact that the project has on the students' ability to identify numbers. As discussed in Chapter 2 the NumPA interview has three forms. Students who are assessed using Forms B and C are not asked numeral identification questions. By the end of the project the proportion of year 2 students not assessed in this domain had increased to 27%. These students combined with the students at Stages 3 and 4 means that 91% of year 2 students are able to at least identify numbers in the range 0 to 100.

Table 4.10: Percentage of year 2 students by stage on the Numeral Identification domain at the start and end of the project and the contrast year 3 students

	Initial NID	Final NID	Year 3 Initial NID
n=	6014	6014	6887
Not Assessed	12%	27%	24%
0: Emergent	3%	1%	1%
1: To 10	14%	3%	2%
2: To 20	14%	5%	4%
3: To 100	42%	28%	25%
4: To 1000	15%	36%	32%

The NumPA includes questions on the grouping domain for students who are tested using Forms B and C. Students who were assessed using Form A were not assessed in this domain. As shown by Table 4.11 there was considerable improvement in the students' knowledge of groupings. The proportion of students assessed as using groupings of numbers increased from 32% to 79% over the project. The final results of year 2 students were considerably better than the contrast initial results of year 3 students.

Table 4.11: Percentage of year 2 students by stage on the Grouping domain at the start and end of the project and the contrast year 3 students

	Initial Group	Final Group	Year 3 Initial Group
n=	6014	6014	6887
Not Assessed	1%	1%	2%
0-1: Non-Grouping	66%	20%	36%
2-3: With 5s and within 10	23%	31%	30%
4: With 10s	9%	40%	28%
5: 10s in 100	0%	7%	4%
6: 10s and 100s	0%	1%	0%
7: 10s, 100s and 1000s	0%	0%	0%

Students who were interviewed using Forms B and C of the NumPA were assessed against the fractions domain of the framework. As shown by Table 4.12 the proportion of students who were assessed in this domain increased from 12% to 39% from the initial to final interview. The percentage of students assessed as being able to assign unit fractions (Stage 4) increased from 1% to 15%. This is considerably higher than the contrast year 3 initial group of whom only 8% were rated at this stage.

Table 4.12: Percentage of year 2 students by stage on the Fractions domain at the start and end of the project and the contrast year 3 students

	Initial fractions	Final Fractions	Year 3 Initial fractions
n=	6014	6014	6887
Not Assessed	88%	61%	52%
2-3: Non fractions	11%	16%	38%
4: Assigned unit fractions	1%	15%	8%
5: Ordered unit fractions	0%	7%	2%
6: Co-ordinated numerators/denominators	0%	1%	0%
7: Equivalent fractions		0%	0%

The Relationship Between Strategies and Knowledge

This section examines the relationship between the students' use of strategies and their number knowledge. More specifically, it compares the knowledge profile of those students who progressed to Stage 4 (advanced counting) at the final assessment with those who remained at Stage 2 or 3 (count from one). It is clear from Table 4.13 that these two groups differed in the four knowledge domains displayed.

Table 4.13: Comparing the number knowledge of initially count from one students (Stage 2 or 3) who became advanced counters (Stage 4+) with those who remained count from one

	Remained count from one	Became advanced counters
n=	1681	1669
FNWS		
0: Emergent	2%	0%
1: Initial to 10	1%	0%
2: To 10	6%	1%
3: To 20	24%	6%
4: To 100	63%	65%
5: To 1000	4%	27%
6: To 1000000	0%	1%
BNWS		
0: Emergent	2%	0%
1: Initial to 10	4%	0%
2: To 10	17%	3%
3: To 20	28%	11%
4: To 100	45%	62%
5: To 1000	3%	22%
6: To 1000000	0%	1%
NID		
Not Assessed	2%	35%
0: Emergent	1%	0%
1: To 10	3%	0%
2: To 20	5%	1%
3: To 100	49%	17%
4: To 1000	40%	46%
Group		
Not Assessed	2%	0%
0-1: Non-Grouping	30%	7%
2-3: With 5s and within 10	42%	31%
4: With 10s	25%	54%
5: 10s in 100	0%	6%
6: 10s and 100s	0%	0%

As with the year 0-1 students, a far higher proportion of those students who progressed (93%) knew their forward number word sequence to 100 (Stage 4) compared to those who remained at Stage 3 (67%). They were also stronger in their backward number word sequence and their understanding of groupings.

Chapter Five: Progress Made by Year 3 Students

The findings presented in this chapter are organised in a similar way to the previous two chapters. The first section presents an overview of the findings in terms of the percentages of students reaching the different stages on the Strategy Section of the Number Framework. The second section again focuses on the students' number strategies but looks at the progress of students with reference to their initial stage on the Number Framework. The progress of students in relation to the variables of gender, ethnicity, school region and school decile are examined. The third section details the progress that students made on the knowledge domains of the Number Framework. The fourth section explores the relationships that exist between the strategy and knowledge aspects of the Number Framework in relation to the transition to part-whole strategies.

Impact of the Project on Students' Number Strategies

A large proportion of year 3 students made progress over the duration of the project in their use of number strategies. As shown by Table 5.1 the percentage of students at the upper stages on the framework for additive strategies increased while those at the lower stages decreased. The additive domain of the framework details the strategies that students use to add and subtract numbers mentally.

Table 5.1: Percentage of year 3 students by stage on the additive domain at the start and end of the project.

	Initial additive	Final additive	Year 4 Initial additive
n=	6887	6887	9040
0: Emergent	2%	1%	1%
1: One to one counting	6%	1%	2%
2: Counting from one on materials	22%	8%	8%
3: Counting from one by imaging	12%	10%	4%
4: Advanced counting	46%	42%	49%
5: Early additive part-whole	12%	33%	32%
6: Advanced Additive part whole	1%	5%	5%

At the start of the project 44% of the year 3 students were at Stage 3 or below. These students are assessed as using a count from one strategy to solve addition or subtraction problems. By the end of the project only 20% of the year 3 students were still unable to count-on to solve addition or subtraction problems. At the higher end of the framework the percentage of students who demonstrated use of part-whole strategies (Stage 5 and 6) to solve addition or subtraction problems had increased from 13% to 38%.

Although there was no control group as such, the year 4 students at the start of the project provide a contrast for the year 3 students at the end the project. Although the year 4 students are still about 6 months older at the start of the project than the year 3 students, at the end they performed at a similar level. The third column of results in Table 5.1 presents the initial results of the year 4 students.

Appendix E details the percentages of year 3 students by stage on the various domains of the number framework in terms of the subgroups studied. Tables 5.2 to 5.4 present the mean stage of the subgroups at the beginning and end of the project as an indication of where differences exist.

Table 5.2: Mean additive stage at the start and end of the project by gender for year 3 students

Gender	N	Initial additive	Final additive
Female	3405	3.3	4.0
Male	3482	3.4	4.1
Total	6887	3.3	4.1

Table 5.3: Mean additive stage at the start and end of the project by decile band for year 3 students

Decile group	N	Initial additive	Final additive
None given	140	3.6	4.5
Low	2399	3.0	3.8
Medium	2631	3.4	4.1
High	1717	3.6	4.4
Total	6887	3.3	4.1

Table 5.4: Mean additive stage at the start and end of the project by ethnicity for year 3 students

Ethnicity	N	Initial additive	Final additive
NZ European	4159	3.5	4.2
Māori	1481	3.0	3.8
Pasifika	663	3.8	3.6
Asian	304	3.8	4.4
Other	280	3.3	4.1
Total	6887	3.3	4.1

The multiplicative domain of the framework looks at the strategies that students use to multiply or divide numbers mentally. This domain includes stages where the students use counting strategies in order to solve a multiplication problem. For example, if a student skip counts they are assessed as being at Stage 4. True multiplicative strategies, which involve working with groups of numbers, begin at Stage 6. Table 5.5 shows the proportions of year 3 students at each stage of the multiplicative domain. The year 4 initial results are also included as a way of contrasting the progress that would have been made by students prior to the implementation of the Numeracy Project. In a similar finding to that reported in Table 5.1, the year 3 students at the end of the project are similar in performance to the older contrast group.

Table 5.5: Percentage of year 3 students by stage on the multiplicative domains at the start and end of the project.

	Initial multiplicative	Final multiplicative	Year 4 Initial Multiplicative
n=	6887	6887	9040
Not Rated	54%	29%	21%
2-3: Counting from one	17%	9%	16%
4: Advanced counting	23%	38%	41%
5: Early additive part-whole	5%	17%	15%
6: Advanced Additive part whole	1%	6%	6%
7: Advanced multiplicative part-whole	0%	1%	1%

Less than half of year 3 students were assessed on the proportional domain of the Number Framework in their initial assessment. This had increased to 70% by their final assessment. Fifty-seven percent of those assessed initially and 86% of those assessed at the end of the project were able to share a region or set as a strategy (Stage 2-4) to solve number problems.

Table 5.6: Percentage of year 3 students by stage on the proportional domains at the start and end of the project.

	Initial proportional	Final proportional	Year 4 Initial Proportional
n=	6887	6887	9040
Not Rated	55%	30%	21%
1: Unequal sharing	20%	10%	26%
2-4: Equal Sharing	23%	44%	40%
5: Early additive part-whole	3%	13%	10%
6: Advanced additive part-whole	0%	3%	3%
7: Advanced multiplicative part-whole	0%	0%	0%
8: Advanced proportional part-whole		0%	0%

Considerably more students (33%) were assessed as early additive part-whole on the additive domain than on the multiplicative (17%) or proportional (13%) domains.

Patterns of Progress on the Number Framework (Strategies)

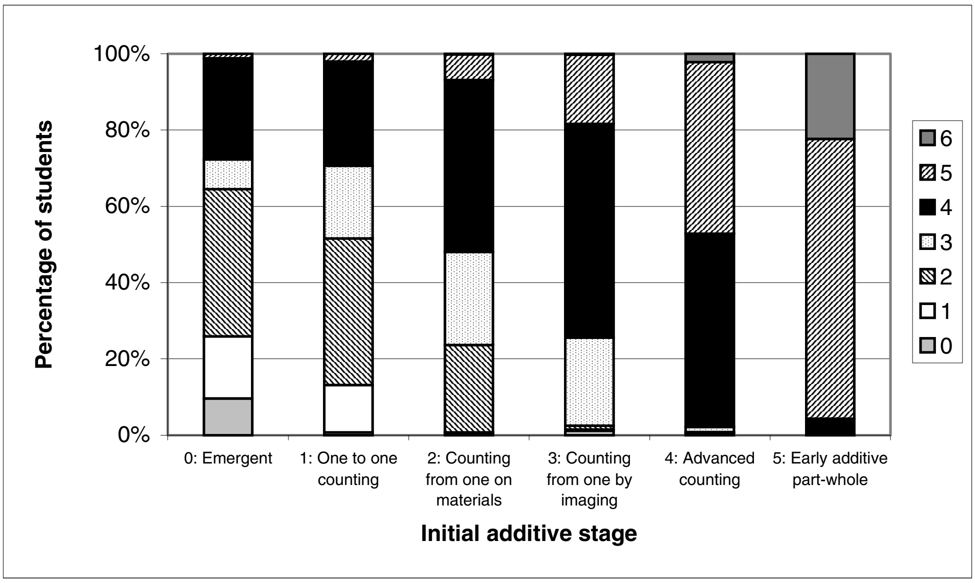
This section examines progress in relation to starting points on the framework as a function of a number of subgroups.

Table 5.6 shows the percentage of students at each stage of the additive domain at the end of the project in relation to their starting stage. The percentages in bold are the students who remained at the same stage throughout the project. In italics are the small percentages of students whose final stage is lower than their initial stage. Consistent with the year 0-2 findings the students who began the project at the lower levels of the framework moved up the highest number of stages. It is of concern to note that 10% of the year 3 students who were emergent at the start of the project were still emergent at the end. However, these students as a total represent less than half a percent of the year 3 students. On the other hand, 28% of the emergent students progressed at least 4 stages to advanced counting or early additive. Of the students who began the project as advanced counters 46% made the transition to part-whole strategies (Stage 5 or 6). The bars in Figure 5.1 present the same data.

Table 5.7: Final additive stage by initial additive stage for year 3 students

	Initial additive stage							
Final additive stage	0	1	2	3	4	5	6	Total
0:Emergent	**10%**	*1%*	*0%*	*1%*	*1%*	*0%*		1%
1:One to one counting	16%	**12%**	*0%*	*0%*				1%
2:Counting from one on materials	39%	38%	**23%**	*1%*	*0%*			8%
3:Counting from one by imaging	8%	19%	24%	**23%**	*1%*			10%
4:Advanced counting	27%	27%	45%	56%	**51%**	*4%*		42%
5:Early additive part-whole	1%	2%	7%	18%	45%	**73%**	*3%*	33%
6:Advanced Additive part whole			0%	0%	2%	22%	**97%**	5%
Total	166	388	1504	794	3140	828	67	6887

Figure 5.1: Final additive stage by initial additive stage for year 3 students



Figures 5.2 to 5.5 illustrate the final additive stage of the various subgroups of year 3 as a function of their starting stage on the additive domain of the framework. Figure 5.2 presents the pattern of progress by gender and shows that year 3 boys make greater progress at the higher stages than the girls.

Figure 5.2: Final additive stage by initial additive stage and gender for year 3 students

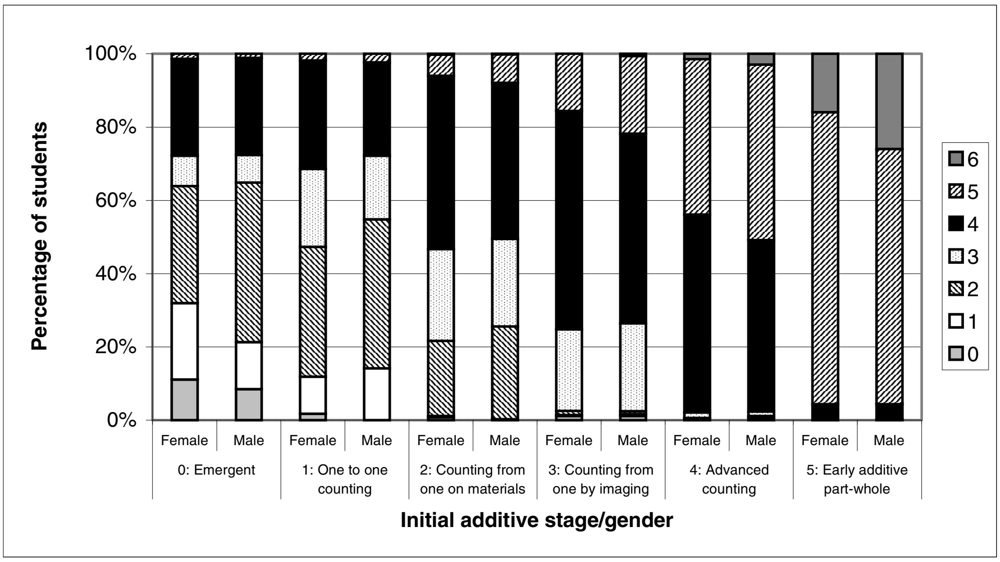


Figure 5.3 presents the final additive stages of the year 3 students when the results are analysed according to ethnicity and starting stage. Consistent with the findings of the previous two chapters, the gains made by Māori and Pasifika students were generally lower than the gains made by the other groups. The two exceptions are at the emergent and early additive part-whole stages. At the emergent stage the Pasifika students make greater progress than the NZ European, Māori and Asian students. At the early additive part-whole stage only 5% of the Pasifika students make the transition to advanced additive part-whole, while Māori students made similar progress to NZ European and Asian students with approximately 23% making the transition.

Figure 5.3: Final additive stage by initial additive stage and ethnicity for year 3 students

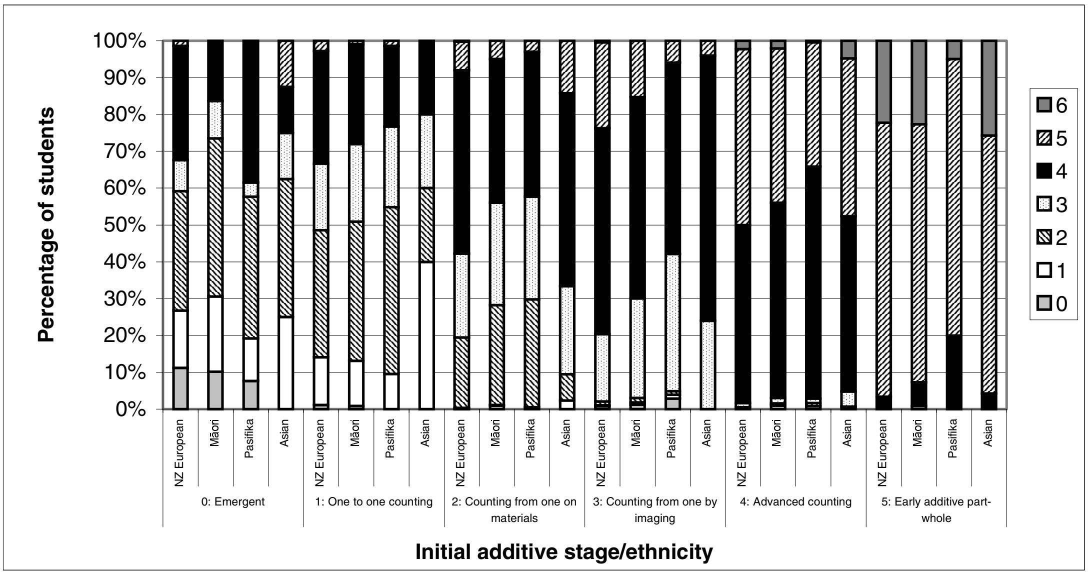
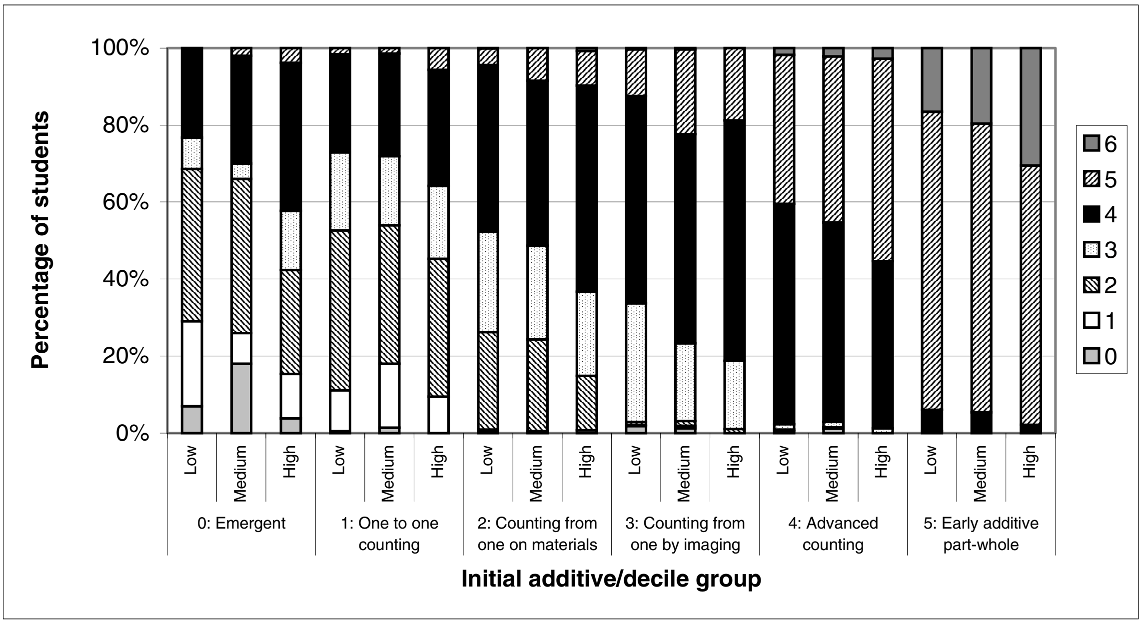


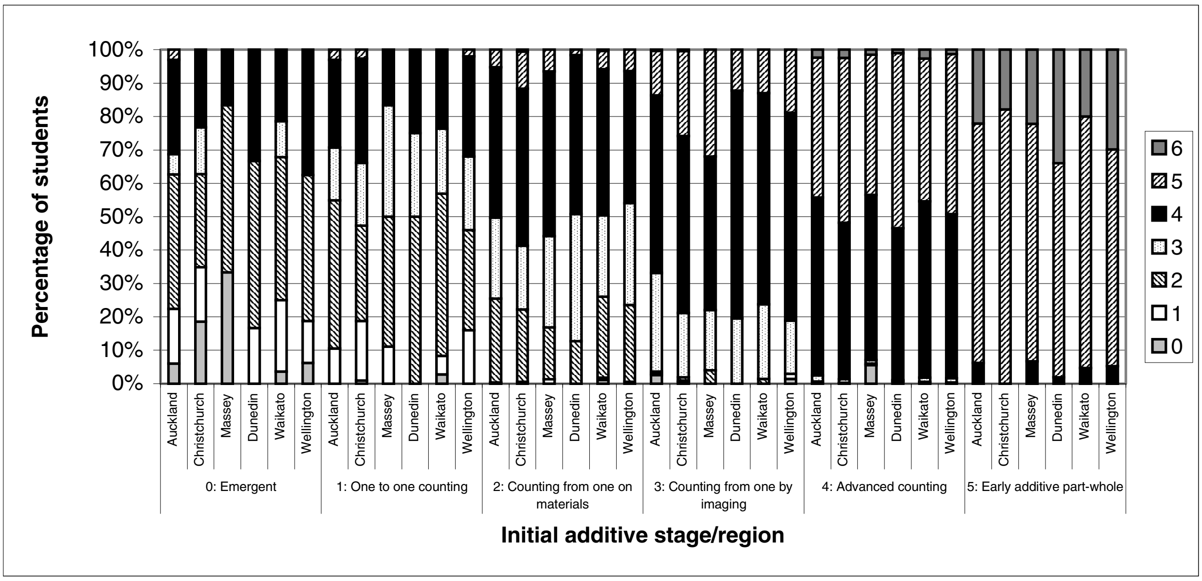
Figure 5.4 presents the final additive stage of the year 3 students when their progress is analysed according to the decile level of their school and starting stage. As shown, there is a consistent advantage to the higher decile schools.

Figure 5.4: Final additive stage by initial additive stage and decile group for year 3 students



As presented by Figure 5.5 there are fluctuations in the performance of students when progress is analysed by region and in relation to their starting stage. One of the fluctuations occurs with students who began the project at Stage 3. Between 12% and 19% of the students in Auckland, Dunedin, Waikato and Wellington moved 2 stages to become early additive part-whole. This is in contrast to the 32% of students from Massey and 26% of students from Christchurch.

Figure 5.5: Final additive stage by initial additive stage and region for year 3 students



Impact of the Project on Students' Number Knowledge

This section examines the progress that students make on five of the six knowledge domains as assessed by the NumPA. As only 3% of the year 3 students were assessed on the decimal domain, these results are not reported in this section. Tables 5.8 to 5.12 present the proportion of year 3 students at each stage on the framework at the initial and final assessments. The initial results for the year 4 students who form the contrast group are also shown.

Table 5.8: Percentage of year 3 students by stage on the FNWS domain at the start and end of the project and the contrast year 4 students

	Initial FNWS	Final FNWS	Year 4 Initial FNWS
n=	6887	6887	9040
0: Emergent	1%	1%	1%
1: Initial to 10	2%	0%	0%
2: To 10	6%	1%	2%
3: To 20	14%	5%	5%
4: To 100	54%	37%	39%
5: To 1000	22%	45%	47%
6: To 1000000	2%	11%	6%

Table 5.8 shows that at the start of the project 78% of the year 3 students were able to identify the number after a given number in at least the range 0 to 100. By the end of the project this had increased to 93%.

Table 5.9 shows the similar pattern of improvement for the backward number word sequence. By the end of the project, 88% of the students knew their BNWS to at least 100 compared to 66% at the start of the project.

Table 5.9: Percentage of year 3 students by stage on the BNWS domain at the start and end of the project and the contrast year 4 students

	Initial BNWS	Final BNWS	Year 4 Initial BNWS
n=	6887	6887	9040
0: Emergent	3%	1%	1%
1: Initial to 10	4%	1%	1%
2: To 10	12%	3%	4%
3: To 20	15%	7%	7%
4: To 100	44%	37%	37%
5: To 1000	20%	42%	44%
6: To 1000000	2%	9%	6%

Table 5.10 illustrates the positive impact that the project has on the students' ability to identify numbers. As discussed in Chapter Two the NumPA interview has three forms. Students who are assessed using Forms B and C are not asked numeral identification questions. By the end of the project the proportion of year 3 students not assessed in this domain had increased to 63%. These students combined with the students at Stage 4, means that 89% of year 3 students are able to at least identify numbers in the range 0 to 1000.

Table 5.10: Percentage of year 3 students by stage on the Numeral Identification domain at the start and end of the project and the contrast year 4 students

	Initial NID	Final NID	Year 4 Initial NID
n=	6887	6887	9040
Not Assessed	46%	63%	79%
0: Emergent	1%	0%	1%
1: To 10	2%	1%	1%
2: To 20	5%	1%	1%
3: To 100	23%	9%	6%
4: To 1000	22%	26%	12%

The NumPA includes questions on the grouping domain for students who are tested using Forms B and C. Students who were assessed using Form A were not assessed in this domain. As shown by Table 5.11 there was considerable improvement in the students' knowledge of groupings. The proportion of students assessed as knowing groupings with at least 5s had increased from 62% to 91% over the project.

Table 5.11: Percentage of year 3 students by stage on the grouping domain at the start and end of the project and the contrast year 4 students

	Initial Group	Final Group	Year 4 Initial Group
n=	6887	6887	9040
Not Assessed	2%	1%	1%
0-1: Non-Grouping	36%	7%	15%
2-3: With 5s and within 10	30%	20%	27%
4: With 10s	28%	50%	43%
5: 10s in 100	4%	19%	12%
6: 10s and 100s	0%	2%	1%
7: 10s 100s and 1000s	0%	0%	0%
8: Tenths hundredths and thousandths		0%	0%

Students who were interviewed using Forms B and C of the NumPA were assessed against the fractions domain of framework. As shown by Table 5.12 the proportion of students who were assessed in this domain increased from 48% to 74% from the initial to final interview. Of those students assessed, the percentage able to at least assign unit fractions to regions or sets increased from 22% to 74%.

Table 5.12: Percentage of year 3 students by stage on the fractions domain at the start and end of the project and the contrast year 4 students

	Initial fractions	Final Fractions	Year 4 initial fractions
n=	6887	6887	9040
Not Assessed	52%	26%	20%
2-3: Non fractions	38%	20%	51%
4: Assigned unit fractions	8%	28%	21%
5: Ordered unit fractions	2%	22%	6%
6: Co-ordinated numerators/denominators	0%	4%	2%
7: Equivalent fractions	0%	0%	0%
8: Orders fractions		0%	0%

The Relationship Between Strategies and Knowledge

This section examines the relationship between the students' use of strategies and their number knowledge. More specifically, it compares the knowledge profile of students who progressed to Stage 5 (early additive part-whole) at the final assessment with those who remained at Stage 4 (advanced counting). It is clear from Table 5.13 that these two groups differed in the five knowledge domains displayed.

Table 5.13: Comparing the number knowledge of initially advanced counting students (Stage 4) who became part-whole thinkers (Stage 5+) with those who remained advanced counters

	Remained advanced counting	Became part-whole
n=	1659	1481
FNWS		
0: Emergent	1%	
1: Initial to 10		
2: To 10	0%	
3: To 20	2%	0%
4: To 100	43%	12%
5: To 1000	47%	70%
6: To 1000000	6%	18%
BNWS		
0: Emergent	1%	
1: Initial to 10		
2: To 10	1%	0%
3: To 20	4%	1%
4: To 100	47%	14%
5: To 1000	43%	70%
6: To 1000000	5%	15%
NID		
Not Assessed	78%	87%
0: Emergent	0%	0%
1: To 10	0%	
2: To 20	0%	0%
3: To 100	4%	0%
4: To 1000	18%	12%
Group		
Not Assessed	1%	
0-1: Non-Grouping	4%	1%
2-3: With 5s and within 10	21%	7%
4: With 10s	59%	55%
5: 10s in 100	14%	34%
6: 10s and 100s	0%	3%
7: 10s 100s and 1000s		0%
Fraction		
Not Assessed	14%	1%
2-3: Non fractions	26%	18%
4: Assigned unit fractions	36%	37%
5: Ordered unit fractions	23%	37%
6: Co-ordinated numerators/denominators	1%	8%
7: Equivalent fractions		0%
8: Orders fractions		0%

Eighty-eight percent of those students who progressed knew their forward number word sequence to 1000 compared to 53% of those who remained advanced counting. The proportions who knew their backward number word sequence to 1000 were similar (85% compared to 48%). Thirty-seven percent of those who progressed knew at least groupings of 10s in 100, compared to 14% of those who did not.

Chapter Six: Evidence for Expectations

> *In thinking about assessing student progress, it is one thing to ask the question: Has the student made progress? … But the harder question that goes to the heart of school improvement, is: How well should (or could) the students be doing?* (McMahon, 2001, page 3)

Amendments to the Education Act in October 2001 positions "planning for, and reporting on, improved student achievement at the top of the schools' agenda" (McMahon, 2002, page 6). Schools' charters will have a section which describes the long-term strategic goals for students' achievement. The charter also has an annually updated part in which schools outline annual targets so that the long-term goals are obtained. While the Government has decided that schools will choose their priority areas for themselves, they are expected to take cognisance of national priorities as set out in the National Administrative Guidelines. Improved literacy and numeracy and improved outcomes for Māori and Pasifika students are such priorities.

This chapter presents the NumPA data with the aim of providing reference information for strategic planning and target-setting at the classroom and school level. The data collected from around 70,000 year 0-3 students since 2000 has the potential to help teachers develop realistic expectations of what their students can achieve. Schools should also be able to use the information to set targets for improving student outcomes.

Cautionary Note

> *Low expectations can result in low achievement, thus "confirming" teachers' initial expectations. Too many children and young people have been caught up in this cycle of low expectation – in wider society as well as at school.* (Pitches et al, 2002, page 6)

While it is important to establish realistic and informed expectations, we need to acknowledge that expectations can also be self-fulfilling.

This chapter takes the results of the three previous chapters and presents them as evidence for setting expectations for achievement. The danger with this is that the evidence may be "tarnished" by existing expectations. Generally, we believe that all children can achieve in numeracy and if they aren't achieving as well as others, especially if this lower achievement is confined to particular subgroups, then further questions must be asked. One such question relates to the effectiveness of the teaching that the students receive as raised in the following quote by Pitches et al (2002) in relation to literacy underachievement.

> *Instead of blaming the children's backgrounds, we need to acknowledge that we don't do as good a job as we could with particular children and that we may need to modify our existing*

teaching practices in response. (Pitches, Thompson and Watson, 2002, page 6)

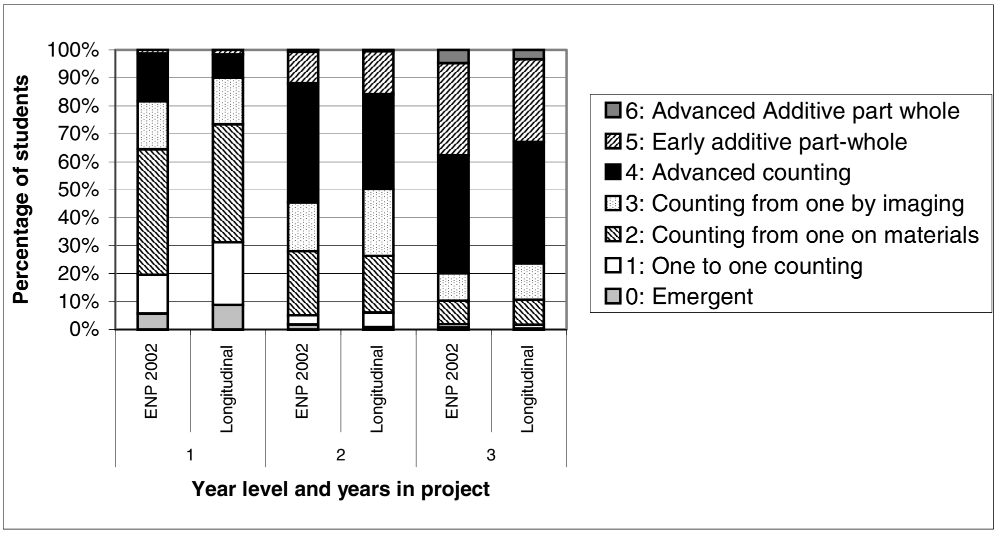
With these cautions and concerns in mind the following two sections use the evidence to establish expectations.

How High to Aim

This section details the levels of achievement, as measured by stages on the Number Framework, that can be obtained by students as a result of having a strong emphasis on numeracy in classroom mathematics programmes. The following section outlines the progress that students are likely to make towards these levels of achievement as a function of their starting points on the Number Framework. These starting points, as well as the degrees of progress, have been shown to be a function of four variables: gender, year level, school decile and ethnicity.

Figure 6.1 presents the end of year results for the 18,392 year 0-3 students who participated in 2002 and the 3169 students from schools which first participated in the Numeracy Projects in either 2000 or 2001. These schools are referred to as the longitudinal schools in Figure 6.1 and the following discussion. The bars show that the ENP 2002 students are at the same or slightly higher levels than those students in the longitudinal schools. This is inconsistent with the findings of the 2001 ENP evaluation where it was found that "schools that had been involved in the numeracy project for two years had slightly higher mean scores on the SEAL than those who had been involved for one year" (Thomas and Ward, 2001, page 24). As discussed in Chapter Seven, this is also inconsistent with the teachers' perceptions about the improvements in their students.

Figure 6.1: Additive Strategies by year level and number of years in the project



After one year in the project 80% of the year 0-1 students are able to at least solve simple addition and subtraction problems by counting from one on materials. This compares to 69% of the students from the longitudinal schools. Eighteen percent of the ENP 2002 year 0-1 students are able to use advanced counting strategies (count-on or count-back) compared to 10% of the longitudinal schools.

While the bars in Figure 6.1 are more similar for the year 2 and year 3 students than for year 1 students, there are some interesting differences between the ENP 2002 and longitudinal students. After one year in the project, 11% of the year 2 students are early additive in their use of number strategies compared to 15% in the longitudinal schools. With this greater percentage of students at the higher level you would expect a correspondingly lower percentage at the lower levels but this is not the case. Fifty percent of the year 2 longitudinal students are in the lowest 4 stages compared to 45% of the 2002 students. The biggest difference is at the advanced counting stage with 43% of the 2002 year 2 students compared to 35% of the longitudinal students. Thirty-four percent of the year 3 students in the longitudinal schools were early or advanced additive. This is 4% less than in the ENP 2002 schools.

Table 6.1 presents a summary of the percentage of students who may be expected to achieve the given strategy levels by the end of each school year. These percentages are rounded to the nearest 5% and present the average of the ENP 2002 and longitudinal results. This table is presented as a general guide and is not intended as a benchmark that schools should accept without consideration of their own results. The three count from one stages have been combined.

Table 6.1: Evidence to expectations: additive strategy stages by year level

Additive stage	Year 1	Year 2	Year 3
Emergent (stage 0)	5%		
Count from 1 (stage 1, 2 and 3)	80%	45%	20%
Advanced counting (stage 4)	15%	40%	45%
Early additive (stage 5)		15%	35%

The percentage of students who made the transition from counting to additive part-whole strategies is similar to that of the CMIT project for 2000 and the ENP in 2001. Table 6.2 compares the percentages of students in the two broad categories of counting and part-whole strategies. At the conclusion of the ENP 2002 project, 18% of the students were using part-whole strategies compared to 23% for ENP 2001 and 30% for the CMIT project. It should be noted that the number framework used for the CMIT 2000 project was substantially different to that used in subsequent years and that this may have affected the proportions of students assessed as being part-whole in their strategy use.

Table 6.2: A comparison between ENP 2002, ENP 2001 and CMIT 2000 results – percentages of students using counting and part-whole strategies

		Counting strategies	Part-whole strategies
CMIT 2000	Initial	90%	10%
	Final	70%	30%
ENP 2001	Initial	93%	7%
	Final	77%	23%
ENP 2002	Initial	95%	5%
	Final	82%	18%

Tables 6.3 to 6.5 present a summary of the percentage of students who may be expected to achieve the given stages within the domains of the knowledge section of the Number Framework. These percentages are rounded to the nearest 5% and are derived from the ENP 2002 results. The 'Initial to 10' stage has been combined with the 'To 10' stage for both FNWS and BNWS.

Table 6.3: Evidence to expectations: Forward number sequences by year level

Forward Number Sequence	Year 1	Year 2	Year 3
Emergent	5%		
To 10	20%	5%	
To 20	30%	15%	5%
To 100	40%	55%	40%
To 1000	5%	20%	45%
To 1000000		5%	10%

Table 6.4: Evidence to expectations: Backward number sequences by year level

Backward Number Sequence	Year 1	Year 2	Year 3
Emergent	5%		
To 10	40%	15%	5%
To 20	25%	15%	5%
To 100	30%	50%	40%
To 1000		20%	40%
To 1000000			10%

Table 6.5: Evidence to expectations: Groupings by year level

Grouping	Year 1	Year 2	Year 3
Non-Grouping	50%	20%	5%
With 5s and within 10	30%	30%	20%
With 10s	20%	40%	50%
10s in 100		10%	20%
10s and 100s			5%
10s 100s and 1000s			

How Much Progress to Expect

Patterns of progress in the current and 2001 evaluation were explored as a function of the students' initial stage on the starting framework. A key finding from the analysis of the data is that a student's initial stage on the Number Framework needs to be taken into account when setting expectations for their short-term attainment on the framework. The analysis shows that the progress that students make is dependent on their year level, their ethnicity and the decile level of the school. The 2002 results also suggest an advantage to boys at the higher levels of the framework.

Figure 6.2 presents the patterns of progress for students who began the project at each of the initial stages. At the lower end of the framework (Stage 0 and 1) more than 80% of students go up at least one stage and many progress two or three stages. At the upper end of the framework just over 20% move up the single stage from early to advanced additive. Forty percent of the students made the transition from advanced counting (Stage 4) to additive part-whole strategies (Stage 5 and 6).

Figure 6.2: Final additive stage as a function of initial additive stage

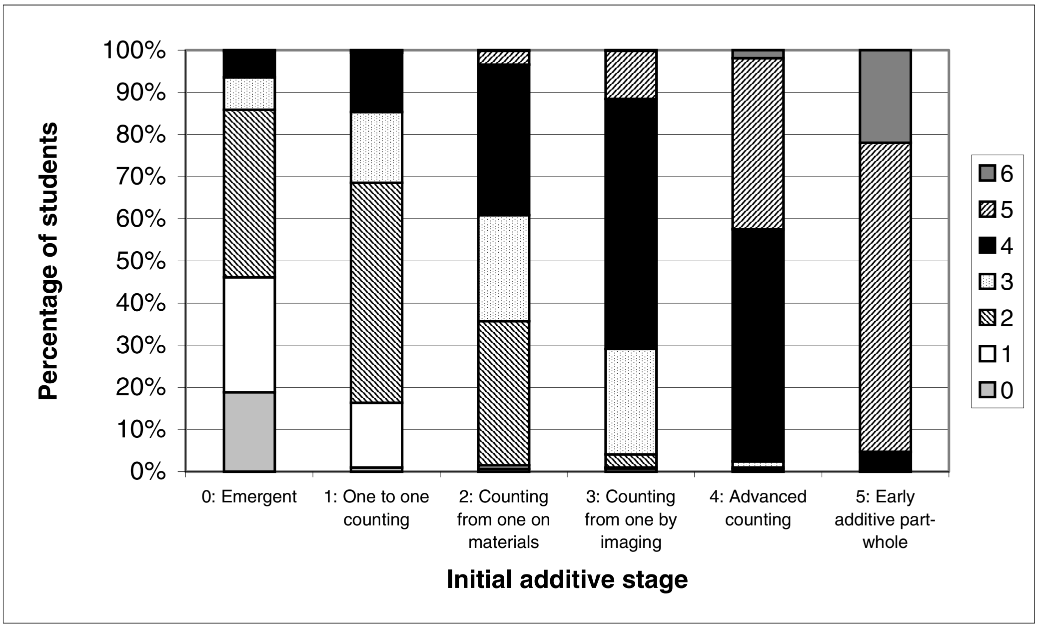


Table 6.6 presents a summary of the progress expected from given starting points on the Number Framework as a result of having a strong focus on numeracy in the mathematics programme. These percentages are collated from the 2002 ENP results and have been rounded to the nearest 5%.

Table 6.6: Expectations of progress as a function of initial stage

	Remain same stage	Move one stage	Move two or more stages
0: Emergent	20%	25%	55%
1: One to one counting	15%	55%	30%
2: Counting from one on materials	35%	25%	40%
3: Counting from one by imaging	30%	60%	10%
4: Advanced counting	60%	40%	
5: Early additive part-whole	80%	20%	

When the patterns of progress are analysed by year level, the results show that the older students made greater progress than the younger students who were positioned at the same initial stage. The bars in Figure 6.3 illustrate that the older students are less likely to remain at the same stage and consequently are more likely to move up one or more stages. For example, 57% of the year 0-1 students who began the project at Stage 3 remain there. This compares to 27% of year 2 and 26% of year 3 students. Eighteen percent of the year 3 students move up 2 stages to become early additive, compared to 8% of the year 2 and 4% of the year 0-1's.

Figure 6.3: Final additive stage as a function of initial additive stage and year level

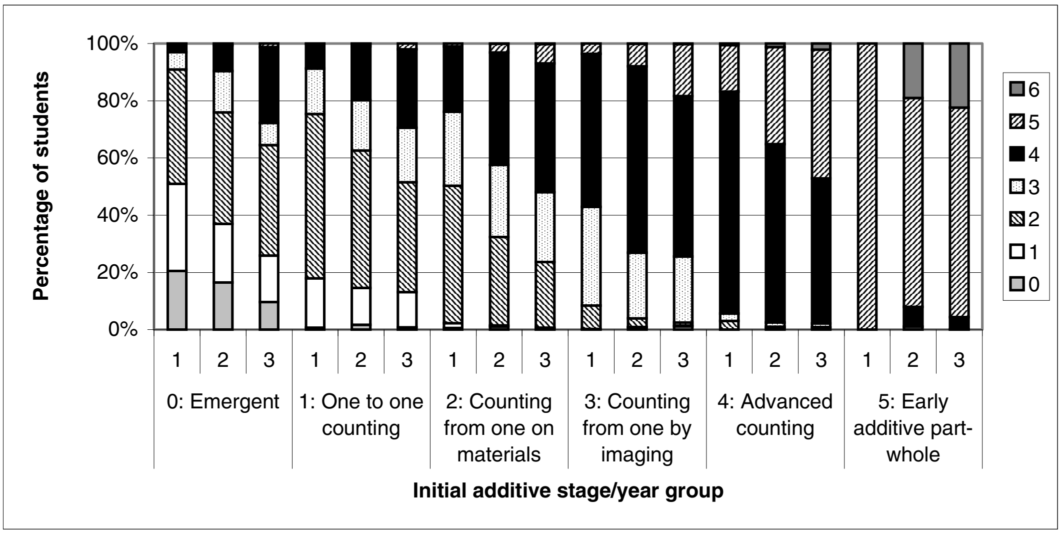


Figure 6.4 illustrates the consistent impact of school decile level on the progress that students make. Irrespective of starting point students in higher decile schools make greater progress. For example, 35% of students in low decile schools make the transition from advanced counting (Stage 4) to early additive part-whole (Stage 5). This is substantially lower than the 42% from the medium decile schools and 50% from high decile schools.

Figure 6.4: Final additive stage as a function of initial additive stage and decile level of school.

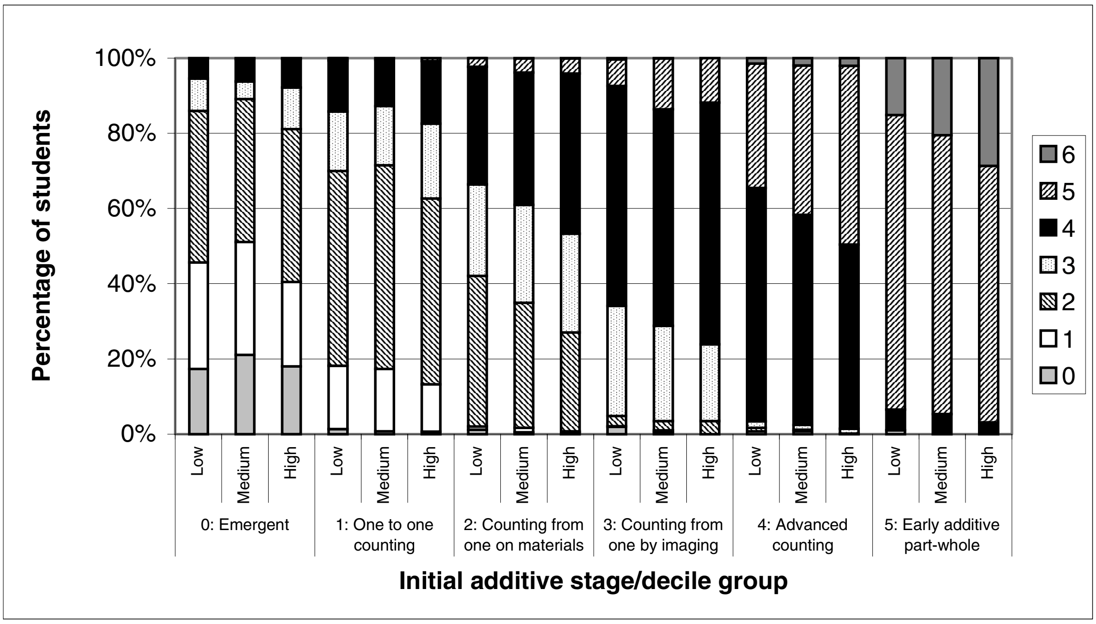
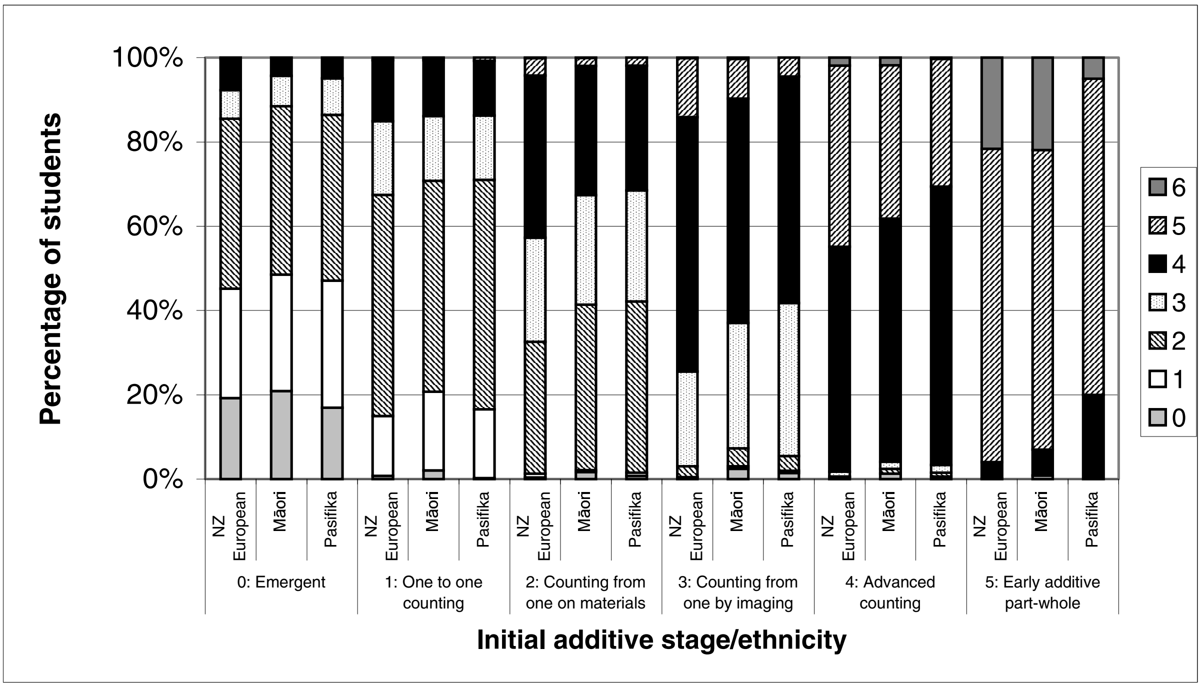


Figure 6.5 presents the patterns of progress made by students when reported by ethnicity. The progress made from the lowest two stages (stage 0 or 1) is similar between the three groups reported. However, consistent with the findings of the previous evaluations, (Thomas and Ward, 2001, 2002) the progress made by Pasifika students at the higher stages is consistently lower than that made by Māori and NZ European students. At the highest stage (Stage 5) the progress made by Māori students is similar to that made by NZ European students. The NZ European students who began the project in Stages 2, 3 or 4 made greater progress than both the Māori and Pasifika students. Similarly, the Māori students who began in these stages made better progress than the Pasifika students.

Figure 6.5: Final additive stage as a function of initial additive stage and ethnicity



Concluding comment

The revised Education Act sharpens the focus on student outcomes by requiring that schools comment explicitly on the progress that they are making towards long-term goals for student achievement. Using up-to-date externally referenced information is seen as a key element in the process of setting goals for student achievement (Ministry of Education, 2002b). The Numeracy Project is rich in data that can be used for this purpose.

The data also highlights the issue of the relative underachievement of Māori and Pasifika students and students from lower decile schools.

Chapter Seven: Lesson Study Group

> *An effective approach to professional development must acknowledge that teacher development takes time and needs to be accompanied by sustained support. It should build on teachers' current pedagogical knowledge and allow for action, reflection and evaluation.* (Parsons 2001, page 14)

This is the first of two chapters examining the sustainability of the ENP. It outlines a trial of a professional development initiative based on the Japanese concept of the lesson study group.

The following chapter looks at sustainability issues in schools that have been involved in the numeracy project for two or three years and also the progress of the students' number strategies.

Rationale for Approach

This section of the report describes a case study involving two city schools. These schools started their involvement in the Numeracy Project in 2000 with CMIT and continued in 2001 with the ANP. One of the schools has continued involvement in 2002 with the NEST programme operating in their school and one beginning teacher receiving some facilitator support. This school has a decile of 10 and a roll of 278 students. The second school, also a decile 10, with a roll of 277 students has finished all training with no ongoing facilitator support.

The concept under trial was a method of professional development reportedly used extensively in Japan, the lesson study group. The Japanese model of the lesson study group has been outlined and discussed extensively in the literature since the Third International Mathematics and Science Survey in 1997 (Lewis and Tsuchida, 1998; Shimahara and Saki, 1995; Stigler and Hiebert, 1997) and has been an area of interest for those involved in professional development programmes.

The lesson study group approach has been described as a "built-in" system for teaching improvement in Japan (Stigler and Hiebert, 1997) and is a formalised part of teachers' professional development programmes in some areas (Lewis and Tsuchida, 1998). The approach is best described by Stigler and Hiebert, (1997):

> *During their careers, Japanese teachers engage in a relentless, continuous process of improving their lessons to improve students' opportunities to achieve the learning goals. A key part of this process is their participation in "lesson study groups". Small groups of teachers meet regularly, once a week for about an hour, to collaboratively plan, implement, evaluate and revise lessons. Many*

> *groups focus on only a few lessons over the course of the year with the aim of perfecting these.* (page. 14)

Recognising that teaching is a cultural activity as it is surrounded by the social, economic and political forces in society, elements of one system cannot be transplanted to another environment with the same results expected (Stigler and Hiebert, 1997). Therefore, the approach was to adapt elements of the lesson study group system for use by teachers of the case study schools.

Comparing the approach taken by the lesson study group teachers and that described by Lewis and Tsuchida (1998), key similarities and differences can be noted. In both approaches the teachers involved decided the theme and frequency of the meetings and the teaching plans were carefully developed in collaboration with other teachers. The meetings were focused, and teacher development was around an area of curriculum change which in the current project was the ENP. The key differences between the approaches were that the trial group lacked any observation of the teaching meetings by other teachers and the lessons were generally not recorded in any way, with the exception of one teacher who video recorded two lessons for her own reflection and on several occasions the teachers made written notes on the lesson they had just taught. These were used for further reflection and as the basis for sharing at future meetings.

The Lesson Study Group

The initiative started with invitations extended to the principals, the year 0-3 teachers and the Boards of Trustees of the two schools concerned. Accompanying the invitations were copies of reading material on the Japanese lesson study group approach (Lewis and Tsuchida, 1998; Stigler and Hiebert, 1997). Once these schools had accepted the invitation to be involved, an initial meeting was planned with the agenda including a discussion of how the approach would best fit the current situation.

It was decided that the seven teachers from the two schools would meet together regularly, at the local college of education, to discuss ideas and collaboratively plan lessons. The teachers felt that the meetings would be more productive if they were held away from their schools where they were likely to be distracted by interruptions. The researchers would facilitate and record the discussion in the form of minutes to be circulated to all group members and the principals of the schools involved. In addition to this, the meetings were audio-recorded with portions of the tapes transcribed for later analysis. It was planned that the lesson plans produced by this group would be communicated with other teachers via the nzmaths website. The lessons to be planned were to focus around the use of key pieces of equipment in the ENP, in particular the tens frame. A lesson was defined to include the stage of the number framework students were working at, the learning outcomes, the pre-requisite knowledge and skills, the resources required and a teaching sequence which may be extended over several days.

In all, the group met a total of twelve times over the second and third terms. Meetings were scheduled for an hour and this time-frame was adhered to. The meetings were used to plan and review lesson plans based around the use of tens frames, focusing on how the equipment could be utilised at different stages of the number framework for different purposes. Between meetings, teachers trialed the lessons, with feedback being brought to the following meeting.

Six lesson plans were developed collaboratively, with each one being reviewed before the next one was started. This initial planning took five meetings and the following six meetings were used for further review of the lesson plans with further trials of the lessons, both by the teachers involved and by colleagues.

The discussion at meetings was largely focused on the planning and revision task but a variety of other more general topics arose from this and were discussed. These included issues such as the forms of written recording that might be appropriate in different contexts, the sharing of useful resource material and the importance of accurate assessment information. The minutes of the meetings and the updated lesson plans were circulated to the group between meetings.

The nature of the discussion at the meetings was collaborative with all participants sharing ideas and contributing to the ongoing planning task. Discussion became quite detailed with issues such as the appropriate wording for questions, the best sequence for ideas to be introduced in a lesson and the most appropriate context for a particular problem being discussed. The participants did not always agree and this created further opportunities to discuss issues in an open and accepting environment.

The outcomes from this group were two-fold. Firstly, the lesson plans provide a resource to be shared with other teachers. Secondly, the teachers provided valuable feedback about the use of the lesson study group approach and its effectiveness as a professional development tool.

Outcomes of the Lesson Study Group

Polished Lessons

Six lesson plans were written, each focusing on the use of tens frames for a different purpose. These were:

1. one to one counting
2. to build the basic facts of five
3. to describe patterns to ten
4. to build the addition and subtraction facts to ten
5. "teen" numbers
6. the strategy of bridging to ten

A full copy of the lessons planned is included as Appendix F.

The lesson plans varied substantially from the initial draft with revisions including the addition of a section outlining the importance of each teaching activity, various changes to the wording of questioning or the resources used, the addition of explanatory notes and in one case the re-writing of the entire lesson.

Professional Development

Information about the success of the lesson study group as a professional development tool was obtained in two ways. Firstly, the teachers were encouraged to reflect on the process as it progressed with the researchers asking questions during the meetings about the process itself. As the meetings were recorded on audiotape these comments were later transcribed for analysis. Secondly, all participants were provided with a questionnaire prior to the final session. These were returned for analysis and also provided the basis of some discussion at the final meeting. See Appendix I for a copy of the questionnaire.

The teachers involved were all relatively experienced, with the number of years of classroom teaching experience ranging from ten to thirty years. They had all completed training for CMIT in their current schools and prior to this initiative had been involved in a variety of professional development programmes in mathematics. These included the training associated with the Beginning School Mathematics programme, Advanced Studies in Teaching papers and several other workshops and mathematics problem solving courses.

Six of the seven teachers involved believed the lesson study group had been either very significant or moderately significant in developing their own professional knowledge in mathematics. The teachers commented that the process had given them a clearer understanding of the uses of the tens frame, and enabled them to clarify their own thinking and internalise the material covered.

> *I think that instead of going back to the book all the time it's become more a mental thing. You can pull out what you want, when you want it, without having to search through that book.*

They also found it a useful way to consolidate existing ideas:

> *It has reinforced and refined knowledge gained through the Early Numeracy Professional Development.*

The teachers believed their classroom mathematics programmes had changed as a result of the lesson study group with four teachers believing their programmes had changed moderately while two teachers believed their programmes had changed considerably. The most common change listed was an increased use of the tens frame, while teachers also commented they felt their questioning had become more focused. Two teachers felt they had a greater appreciation of the need for small, incremental steps in the children's understanding, with one commenting:

> *I have pulled back in my approach to number and now feel that much smaller steps is better than trying to plan too much...consolidation is a focus rather than moving too quickly on and upwards.*

All of the teachers found focusing on one aspect of teaching in great detail to be a useful feature of the lesson study group approach. Three teachers found it useful to "reflect on the details of lessons" and have the opportunity to "fine tune" these.

> *The beauty of it is you can come, talk about it, go away, try it and come back again and say "Hey, that worked" or "I tried this..." and that's been really good.*

The focus on one area that the approach employs also allowed the teachers to develop an understanding of a teaching progression based around one piece of equipment and the majority of teachers found this useful.

> *It was good to see how the tens frame could be used in a flexible way at various stages.*

As well as finding the focus on one specific area helpful, the teachers commented that a variety of approaches to professional development was needed to achieve different purposes.

> *...a variety of things [make a successful professional development programme]. It depends what you're presenting as to what's going to suit that purpose. Sometimes it might be a workshop, a presentation or a discussion.*

The one approach to professional development that the teachers identified as particularly effective was the use of demonstration lessons in the ENP. All of the teachers had found this to be a "very powerful" way to learn and appreciated someone with skills in a particular area coming to work with them.

The majority of the group felt it would not be possible to use the lesson study group approach to professional development in their own school without outside facilitation. The teachers felt the objectivity of the facilitator was useful to guide the group and keep it on track while the professional knowledge offered was extremely valuable. Two teachers felt there would be too many interruptions in the school environment to sustain the approach independently.

> *I feel the professional leadership of [Researcher] was vital to keeping us on track and steering us towards sound plans and outcomes.*

Concluding comment

In conclusion, the teachers found the lesson study group to be a useful professional development approach, sharing the view of Lewis and Tsuchida, (1998), that it allowed them to explore conflicting ideas:

> *..by looking at one aspect we could go into depth and it also brought up a variety of issues.*

The teachers who participated in the lesson study group were concerned about their ability to sustain the developments that had occurred as a result of their involvement in the ENP. In contrast to Japanese teachers who “assume that improvement will come through a steady, gradual, cumulative process” (Stigler and Hiebert, 1997, page 15), the study group teachers saw the role of the facilitator as crucial to successful development.

> *It’s having someone come in who’s done the donkey work, who’s been through the resources, who’s got wonderful examples and brings it in and you think- yes, I could use that. Because we’re teaching maths, science...the curriculum’s huge.*

They held particular concerns about the size of the curriculum and their ability to deliver all areas. These concerns highlight the differences between the “shared, frugal curriculum” in Japan, (Lewis and Tsuchida, 1998, page 50) and the situation in New Zealand.

Chapter Eight: Sustaining the Project Over Time

This is the second of two chapters examining the sustainability of the Numeracy Project. The focus of this chapter is the sustainability of the developments over time.

As described in Chapter Two, nineteen schools (eight from CMIT 2000 and eleven from ENP 2001) participated in the longitudinal component of the research. The selected schools provided information on student achievement by submitting the strategy stages of their students as described by the NumPA to the project website by November 1. Teachers also completed a questionnaire focusing on the sustainability of the project. See Appendix H for a copy of the questionnaire. In addition to this, researchers visited the CMIT 2000 schools twice, once in June and once in October to meet with the principal and/or the lead teacher for the project and gather data on student achievement using measures other than the diagnostic interview. Their perceptions on the sustainability of the project were also discussed.

Questionnaire responses were received from all eight CMIT 2000 schools, with a total of 84 questionnaires returned. Eight ENP 2001 schools returned questionnaires, with 57 completed forms being received. The questionnaire forms were sealed in individual envelopes for confidentiality, with the school responses being collected for postage in one larger envelope.

The questionnaire asked teachers to respond using a five-point likert scale, with written comments for further elaboration. Responses will be reported firstly as percentages, using the scale results, with the nature of the comments received being used to illustrate the responses further. Note that where percentages do not total one hundred this is due to respondents not answering all questions. The quotes used are typical of the comments given and are taken directly from the questionnaires.

Table 8.1 shows the year levels taught by respondent teachers. Note that in ENP 2001 schools only teachers of year 0-3 students received questionnaires as the other teachers were participating in the ANP in 2002.

Table 8.1: Year level of respondent teachers

Year Level currently taught	CMIT 2000 (%)	ENP 2001 (%)
0 – 3	45	100
3 – 6	42	
6 – 8	12	

Table 8.2 outlines the type of training teachers in the project received. The great majority of teachers (over 80%) completed the Numeracy Project in their current school. Fifteen percent of teachers have incomplete training experiences and are either receiving

professional development during the 2002 school year or have not had an opportunity to take part in the professional development programme.

Table 8.2: Completion of the numeracy project professional development programme

	CMIT 2000	ENP 2001
Programme completed in current school	86%	82%
Programme completed in different school	1%	3%
Currently undergoing programme	8%	5%
No programme received	5%	10%

The impacts of the project over the longer term will be looked at in three ways. Firstly, the impact on student achievement will be analysed, both by examining teacher perceptions of achievement and by measuring student progress on the Number Framework. Secondly, the impact on teachers will be discussed using data gained from questionnaire responses and on-site interviews and thirdly, school-wide sustainability will be examined, using information gathered in on-site interviews.

Impact on Student Achievement

The effect of the project on student achievement over the longer term was analysed in two ways. Firstly, teachers and principals gave their perceptions in questionnaires and interviews. Secondly, student progress on the Number Framework was analysed.

<u>Teacher Perception</u>

In general, questionnaire respondents believe the project has had a positive impact on students' ability in number with 82% describing this impact as very positive or highly positive. A positive impact on students' mathematics in general was also noted, with 73% of respondents describing this impact as very positive or highly positive.

Comments describe students' ability at solving mental number problems as faster and more accurate.

> *Huge improvements in children's knowledge – very fast at adding numbers in their head. They can now see links between numbers.*

The reasons identified for this increased ability indicate that the students' increased enjoyment and confidence due to a feeling of success and increased teacher expectation are key contributors.

> *Nearly all approach maths sessions confidently and enthusiastically because they all feel successful using and showing what they know.*

The children hate to miss maths. They are confident, love a challenge and will talk about maths problems enthusiastically.

I believe my expectations of what children can do has increased and because of this the children are achieving at higher levels.

Big numbers aren't only for big people. It's all about smart thinking.

Teachers noted that the increase in number knowledge has led to increased success in other areas of mathematics, with skills being transferable across the strands of the curriculum.

Sound number knowledge provides a basis for concepts in other strands, especially measurement and algebra.

It empowers children by giving strategies that can be used in other areas of maths.

On-site interviews supported the views expressed in the questionnaires, with lead teachers generally feeling students are achieving at a higher level since the implementation of the numeracy project.

Researcher	*Have you noticed any difference in the children's maths [apart from Numeracy project results]?*
Teacher One	*The children are far more advanced.*
Researcher	*How are you so sure that they're doing better?*
Teacher One	*I've never taught higher [curriculum levels] in year 3 before in my life.*
Researcher	*What are you teaching at a higher level?*
Teacher One	*Operations, decimal numbers. I do a lot of fractions and decimals now. I wouldn't be the only one who taught higher levels now.*
Teacher Two	*No, that's true. And we are really cautious about how we interpret that. I mean I think our statements about improvement are informed.*
Teacher Two	*We have contact with X [facilitator] and he said several times that he has really noticed a big difference.*

During the interviews, the participants were asked for evidence of the students' increased achievement.

Researcher	*You said earlier that children in your class were operating at a higher level. Do you have any written evidence of this?*
Teacher One	*You mean like notes or tests?*
Researcher	*Anything to support your belief. I believe you, but in terms of demonstrating improvement, I wondered if you had anything recorded.*
Teacher One	*I have evidence in my planning maybe. If I looked back and compared AO's [achievement objectives] they are higher now*

in my planning.

Four of the eight CMIT 2000 schools had data from Progressive Aptitude Tests (PAT) showing trends in student achievement since the introduction of the Numeracy Project. Three of the four showed an increase in student achievement, with one remaining constant. Further follow-up of these trends would be interesting.

Reseacher	*Can you show me the PAT information you have?*
Principal	*Yes, certainly. The year 4 children you know it's the first year they do it and so we have got no comparative data. We were fairly pleased that there were so few children in the lower levels. Remembering that it's the first year that year 4s do a PAT test it's hard for them for the first time. That's quite an interesting picture I think with the bulk that way [pointed towards upper stanines] rather than that way [lower stanines].*
Researcher	*Very positive looking information.*
Principal	*This result here for 2001 there's still quite good medians the median was 5. 22 and this year its 5. 34. There is a few down there [pointing towards lower stanine] which we weren't happy about and again getting more into that end [pointing to upper stanines].*
Researcher	*Again that's quite positive.*
Principal	*It is. Year six the children would have had only last year [that their teachers and classes were involved in the project].*
Teacher	*Half a year.*
Principal	*But the results blew me away really. They've gone from 5. 55, which is reasonably good anyway to 6. 12 median, and there it is [pointing to graph] I thought that the year six result was amazing.*
Researcher	*Very impressive.*
Principal	*Our year seven's a big group, there's 94 of them. They were in a year 4, 5 and 6 composite, they've gone from a 4. 95 to a 5. 79 so that's a pretty impressive move.*

The reasons given by principals for this increased achievement included increased teacher expectation.

Principal	*My observations of the junior kids – I taught juniors before I became a school principal. The knowledge and the things that they can do are amazing, you know, to see little five year olds doubling and doing all this different stuff. Teachers got that in the old maths programmes where you couldn't teach them to write numbers, and all the nonsense that went on, but the things that they can do and their brains.*
Teacher	*I think that's been a big thing – the raising of teachers' expectations about what the children can actually do.*
Principal	*And the levels you can get them to by the time they're seven basically.*
Teacher	*That's made a huge difference to what you can do with them in the*

classroom.

When discussing student achievement, on-site interviews also covered students who were failing to progress as expected. One of the CMIT 2000 schools identified a possible reason for this lack of progress:

Researcher	*Are all students successful?*
Teacher One	*There are still a group of children who are struggling and I haven't quite worked out why. But I think it is because parents intervene by pulling them into private programmes [names programme], and it seems to stunt their growth and I can't move them at the moment.*
Researcher	*Why do you think it stunts their growth?*
Teacher One	*I think it it's back to rote and rules.*
Teacher Two	*And they don't want to take risks.*
Teacher One	*Yes, I think that's it. They won't have a go.*

One of the CMIT 2000 schools operated a maths recovery programme for 15 – 20 children from the year 4 to 6 classes. Included in the maths recovery programme were children who were assessed as being at the initial stages [Stages 0-3] of the Number Framework.

The children were in three groups with each group receiving 30 minutes of extra teaching per day from the numeracy lead teacher in the school. They had received this since the middle of Term one. The teacher who took the sessions was released for reading recovery in the mornings and then did maths recovery in the afternoons. The following excerpt describes the maths recovery programme and its perceived success.

Researcher	*How has your maths recovery groups been going?*
Teacher	*Well I have the three groups which I take for half an hour each day, mostly for the number strand, but I use the other strands to make it more practical. Like we use the number strategies for working out measurement problems. Well, one of the teachers said to me yesterday "You know the girl you have got from my class, well she is doing better than everyone else in some of the strands, so you know, can I swap her for one of the other children?*
Researcher	*That's wonderful. How did you select the children for the recovery programme?*
Teacher	*These are year 4 to 6 kids, who were right at the bottom of the framework. The girl I talked about couldn't even tell me 5 plus 7 and now she is deriving from her multiplication facts. And yesterday I gave them a problem where I have got this many blocks in a box and how many blocks to fill up the whole box. How many blocks are in the box already and how many extra do I need to fill up the box? Well she just did it like that [snaps fingers] and that's two multiplications and a subtraction, she was so quick and so proud of herself.*
Researcher	*So what do you think had made these large shifts?*

Teacher	*Well basically I give them time to take risks.*
Researcher	*How do you do that?*
Teacher	*I give extra time to figure it out and I don't jump on them if they are wrong. I just have a blank face and ask them to explain how they got the answer.*
Researcher	*Yes*
Teacher	*And then I might ask if anyone else could tell me what they do. And so we share them all and then together we focus on some ways together. No one feels pressured. They have a ball.*
Researcher	*You have mentioned risk-taking, sharing and then guidance. Is that how you have always taught maths?*
Teacher	*Probably in bits, but not something I thought about, like I do now. Taking time to listen and share strategies works so well.*
Researcher	*How will they fit back into their regular class?*
Teacher	*Well they aren't behind anymore and a lot were turned off maths in a big way and didn't enjoy it. Well now they are waiting for me to arrive. I never have to chase up any of them and at the start they really were the reluctant ones.*
Researcher	*Are they all doing well?*
Teacher	*There is only one from the three groups that I am really concerned about. She is still struggling to count, but all the rest are making great progress.*

<u>Results on the Number Framework</u>

The eight CMIT 2000 schools and eight of the eleven ENP 2001 schools submitted the final additive strategy of their year 0-3 students to the project website. The CMIT schools are labeled C1 to C8 with the ENP 2001 schools labeled E1 to E8. The 2001 additive strategy results were also available from all but three of the longitudinal schools. Table 8.3 presents the mean additive stage by age. The mean strategy stage is consistently lower in 2002 than 2001 across all age groups in thirteen of the schools. Just three of the schools (C4, C7, and E2) have higher means in 2002 compared to 2001 although C7 only returned results for 7 year olds in 2002. Schools E6 and E7 returned higher results for 5 year olds, but lower results for 6 and 7 year olds, which is interesting as it is the 6 and 7 year olds who have had two years exposure to the Numeracy Project.

Taken generally, these results are surprising given the positive comments made about student progress in both the questionnaires and on site interviews. The difference between the perceptions of improvement and the actual results is an area that needs to be further investigated.

Table 8.3: Mean additive stage students from longitudinal schools as a function of age and years in project.

	Age	5		6		7	
	Year	2001	2002	2001	2002	2001	2002
School	C1	3.2	2.0	4.5	3.1	5.1	4.2
	C2	1.5	0.8	3.7	2.3	4.1	3.6
	C3	N/A	1.5	N/A	2.8	N/A	3.9
	C4	2.0	2.3	2.9	4.2	3.7	4.6
	C5	N/A	1.6	N/A	2.6	N/A	3.9
	C6	N/A	1.6	N/A	1.6	N/A	2.2
	C7	1.1	N/A	2.7	N/A	3.3	3.6
	C8	N/A	1.2	5.0	2.6	4.6	3.6
	E1	1.9	1.2	2.9	2.2	3.8	3.2
	E2	1.0	2.2	3.1	3.9	4.4	4.7
	E3	3.0	1.6	4.4	3.1	4.9	3.9
	E4	2.3	1.7	3.6	2.4	4.0	3.3
	E5	2.0	1.4	4.1	2.3	4.6	3.9
	E6	1.9	3.0	3.6	2.8	5.0	3.4
	E7	1.8	2.0	2.1	2.0	3.6	2.7
	E8	2.9	1.5	3.9	2.7	4.4	4.0

Impact on Teachers

Attitudes to Mathematics

Respondents were asked to rank their attitude towards mathematics both before and after the project. Teachers in both CMIT 2000 and ENP 2001 schools had similar attitudes towards teaching mathematics prior to the project with 45% of teachers describing themselves as moderately enthusiastic and 46% as very or highly enthusiastic.

Forty-nine percent of teachers reported feeling more positive towards mathematics since participating in the project while 40% of teachers believed their attitudes towards mathematics had not changed. Seven percent of teachers identified a more negative attitude since completing the project.

Further comments contained three themes. Some respondents report an unchanged attitude towards teaching mathematics, explaining:

> *I've always loved maths*
>
> *I have always tried to be enthusiastic in maths teaching and make it challenging and enjoyable for the kids.*

A group of respondents found teaching mathematics difficult prior to participating in the project due to a lack of personal knowledge:

> *Little mathematical skills myself so little enthusiasm.*
>
> *[Maths was] not my favourite area due to my own experience and lack of knowledge.*

Another group of teachers reported that they are enjoying using the activities linked to the project, with comments such as those below more commonly noted by 2000 teachers than 2001 teachers.

> *Maths is so much more exciting for me and the children with the equipment and ideas we can incorporate into our teaching.*
>
> *I get hooked on new activities. I love it!*

Comments from the on-site interviews support the view that in general there is more enthusiasm about teaching and learning mathematics.

Teacher	*In the junior school now the children really like maths. They are much more aware now of what maths really is, because it's become much more explicit to them - I think because the lessons have become much more structured and that numeracy warm up is a component of every mathematics lesson, whatever the strand is. So, I think it's defined it more clearly for both the teachers and students actually.*
Principal	*Mmm I think so.*
Teacher	*I think the children have really enjoyed the emphasis on the counting and the games and all that skip counting and all that sort of thing and that's been very effective. . .*
Teacher	*. . . I think I've noticed in the junior school that maths is no longer one of the things that would be missed if there was a visiting production or things like that. I think the reason for that is that the lessons are more clearly defined and people feel secure about it. It's not a huge amount of work because for the children it's all become automatic.*

<u>Development of Professional Knowledge</u>

Ninety six percent of respondents found the project to be moderately to highly significant in developing their professional knowledge in mathematics. Trends among the responses mentioned an increased awareness of the stages of development students are likely to follow and a greater focus on the strategies students are using to solve number problems:

> *It gave me a very clear sequence of learning.*

I have a firm grasp on the Numeracy Project strategy stages and a child's progression within these.

I feel much more knowledgeable about maths "thinking" and "strategising". I am more focused in maths sessions to listening to students' responses.

Respondents also describe an increased focus on students' individual needs, with more accurate planning and assessment taking place:

It was really worthwhile having the test as it gave me a reference point to work from. Listening to the children gave me a clear indication of where I needed to "go" or what I needed to concentrate on.

It gave me a very clear idea of what each child needs.

The comments of teachers during interviews support the view that the numeracy project has led to significant development of the professional knowledge of teachers:

Teacher One	*It is amazing to me that maths is no longer done by rote. It is virtually done by problem solving and natural thinking. But you have to free up your own thinking as a teacher to be able to do it in the first place.*
Researcher	*How do you feel about this in relation to the teachers in your school?*
Teacher Two	*At the end of 2000 I made a point of asking individually, each of the teachers what they thought. I expected a range of opinions but not one person was anything other than positive. And that was really major for me, because we are talking about an experienced staff. You have got a lot of people who have been teaching maths for years and they are good teachers. I wondered about how easy it would have been for some to have made that shift. And they did!*
Researcher	*That's good*
Teacher Two	*Another interesting one is that we had a parents' meeting last year and X [facilitator] came and spoke at our invitation and it was the best attended parents' meeting I have seen at this school or probably just about anywhere.*
Teacher One	*And that meeting went so well and people had the opportunity to ask questions and they all came away saying "well that was great and thank you". I mean that in itself is a real affirmation of the programme.*
Teacher Two	*The overall impression is that we are happy with and enthusiastic about our teaching methods. I think our attitudes are definitely better.*

The Professional Development Programme

Respondents were asked to describe the relative usefulness of elements of the professional development programme. Teachers in CMIT 2000 schools were reflecting on a programme delivered 2 years ago, while ENP 2001 teachers were commenting on last year's professional development programme. Table 8.4 summarises these results.

Table 8.4: Usefulness of elements of the professional development programme

	Not or Slightly Useful	Moderately Useful	Very or Highly Useful
Teaching Materials	6%	18%	72%
Diagnostic Interview	2%	11%	81%
Facilitator	16%	17%	62%
Facilitator Demonstrations	17%	17%	61%
Project Meetings	24%	27%	39%

Note: Some percentages given do not add to 100 as not all respondents completed all questions.

The two elements of the professional development programme respondents identified as most useful were the teaching materials and the diagnostic interview. Seventy-two percent of respondents found the teaching materials to be a very or highly useful element of the professional development programme, with 18% believing them to be moderately useful. Some teachers found the time taken to make resources demanding, with comments such as those below more commonly found among responses from the CMIT 2000 teachers than the ENP 2001 teachers.

> *Time constraints and large amounts of organisation going through making up activities.*
>
> *Difficult for teachers to make all the equipment.*

Principals and teachers leading developments in CMIT 2000 schools supported this view in on-site interviews. One principal and lead teacher commented on the need to make up new diagnostic interview kits to address the changes made in the NumPA since 2000. The principal was also concerned about the time required to interview the children individually.

Teacher	*A lot of people haven't even made up their kit yet. It's another whole kit to make. It took me two hours to make up my testing kit.*
Principal	*It's difficult to give the teachers the amount of release time, because even the testing takes so long it's really difficult financially to give the teachers enough release time to actually test effectively.*

In addition, the CMIT 2000 teachers working with students in years 5 to 8 believe the resources are not appropriate for this age level and there is a need for more resources in this area.

> *Lots of activities became redundant very quickly as children progressed quickly. We were left with few resources at part-whole and advanced additive levels.*
>
> *What happens to the bright children who've been at the top for several years? BORING. We need extension.*
>
> *Some activities in the higher levels are quite "babyish".*

Eighty-one percent of respondents described the diagnostic interview as very or highly useful, while 11% found the interview to be moderately useful. Further comments from 2001 teachers describe some frustration with the time taken to carry out the testing fully.

> *I am concerned about the time the testing will take. Will there be any funding provided to schools so that teachers can be released to test, to help decrease the pressure of an already crammed classroom programme?*

Responses received regarding the usefulness of both the facilitator and the teaching demonstrations they provided were similar. Sixty-two percent of respondents describe the facilitator themselves as very or highly useful and 61% regard the facilitator demonstrations as very or highly useful. Sixteen percent of respondents believe the facilitator to be not useful or slightly useful with 17% describing the teaching demonstrations as not useful or slightly useful. In general, comments regarding the facilitator were polarised:

> *Excellent presentation from facilitator. Classroom demonstrations really motivated us.*
>
> *Very rushed delivery of instruction [from the facilitator]. Could not explain clearly. Just kept saying "We'll come to that".*

Principals and teachers supported these views in on-site interviews, with one school in particular feeling the facilitator's input was worthwhile, but that more time for discussion would have been beneficial.

Teacher	*One of the things that people have said is that we didn't have sufficient on-site opportunity to talk with the people responsible for their professional development. We went to big meetings and we saw the person on-site two or three times.*
Teacher	*Yes*
Principal	*I think that was the feedback that I got too. That the P. D. was great, but sometimes it was like a show and then they were gone and you would've liked to have unpeeled it for a little while, worked alongside that person and had them watch you work.*

Respondents report a mixed response to the project workshops. Thirty-nine percent rank them as very or highly useful, while 27% believe them to be moderately useful, and 24% describe them as useful or slightly useful. Comments collected reflect the diversity of views about the meetings, although they tended to be negative rather than positive in nature.

> *The project meetings have too much content and not enough time to process or discuss it.*

> *The workshops can sometimes be drawn out and self-explanatory.*

<u>Ongoing Use of Numeracy Project Materials</u>

Fifteen percent of respondents report their classroom mathematics programme has incorporated Numeracy Project ideas and materials to a moderate extent with 80% describing this incorporation as considerable or full. Further comments indicate that teachers are incorporating aspects of the Numeracy Project into other areas of their teaching, with number being integrated into other mathematics strands, other curriculum areas and daily routines.

> *Within the strands other than number we identify, discuss and practise strategies already covered.*

> *We make use of numeracy project materials in other subject areas.*

> *More anecdotal maths done at opportune times.*

Respondent teachers believe they will continue to incorporate Numeracy Project ideas and materials into their classroom mathematics programmes, with 16% describing this process as moderate and 82% identifying that they will continue to use the materials considerably or fully.

> *I wouldn't dream of not using it.*

<u>Teachers with Incomplete Training</u>

Three trends are evident among the responses of the 15% of teachers who have incomplete professional development experiences; that is, teachers who are receiving professional development during the 2002 school year or have not taken part in the professional development programme.

Firstly, these teachers express a desire to receive the complete training and support of the Numeracy Project professional development programme.

> *Some quality PD would help those who missed the initial implementation last year.*

I would like to think the project course is available to those who haven't yet completed it.

Secondly, this group of teachers expressed a more negative attitude change than those teachers who had completed the professional development programme. Table 8.5 summarises these differences.

Table 8.5: Attitude changes among teachers receiving incomplete training

Attitude to teaching mathematics	Training Incomplete	Training Complete
No change	33%	42%
Became more positive	33%	53%
Become more negative	34%	3%

It can be noted that a larger proportion of teachers whose training was incomplete became more negative since becoming involved with the Numeracy Project (34%) than of those whose training was complete (3%).

Comments suggest that this more negative attitude has led to a decreased motivation for teaching mathematics, with teachers indicating that this may improve as they become more skilled with numeracy programme ideas and materials.

I don't know where I'm going, what I'm doing… hopefully, this will improve with experience.

I'm sure attitudes will change as we come to terms with the material.

As I am gaining more confidence with the programme I am using more and more resources.

The third trend noted among teachers with an incomplete professional development experience was their differing perception of the impact of the programme on student ability. In general, they believe the programme is having less effect than teachers with complete professional development experiences believe. Tables 8.6 and 8.7 outline these differences.

Table 8.6: Perception of impact on students' ability in number

	Incomplete training	Complete training
Negative impact or no impact	2%	0%
Slightly positive impact	38%	12%
Very or highly positive impact	60%	88%

Table 8.7: Perception of impact on students' ability in maths in general

	Incomplete training	Complete training
Negative impact or no impact	0%	1%
Slightly positive impact	64%	21%
Very or highly positive impact	36%	78%

School-Wide Sustainability

On-site interviews focused on issues of sustainability and sought to identify how schools were sustaining the Numeracy Project developments within their practice over time. Schools had developed a number of different strategies to help ensure progress was maintained.

One of the CMIT 2000 schools had compiled resources to support new and existing staff and reviewed recent issues at staff meetings:

Researcher	*Do you have new teachers in your school who have had the numeracy professional development?*
Teacher One	*Yes, one this year.*
Researcher	*How is she or he managing?*
Teacher One	*She is being supported by a folder at this stage.*
Researcher	*What is in the folder?*
Teacher One	*The core. What you need to get up and running.*
Researcher	*Did you put together the folder?*
Teacher One	*Well it's mostly from Count Me In Too but then I added bits from the booklets from last year.*
Researcher	*OK. How about the other teachers, have you had to do anything to sustain the momentum of the programme in your school?*
Teacher One	*I produced a booklet for maths teachers. I also put together another folder with all the blackline masters. And we do staff meetings, so if there are any updates we share them then.*

Several of the schools had developed systems for monitoring the teaching and learning process. These included writing numeracy objectives into long-term planning documents and setting benchmarks for student achievement.

Principal	*I think our crowd are pretty enthusiastic about it, it's just a matter of keeping the impetus up once this years training is up. The final lot will be trained this year and then it will be our job as the senior management team too say which block of numeracy we will do again, as well as the bits to be done each term.*
Teacher	*I think we have already written that in our curriculum plan.*
Principal	*Yes we have a statement that covers that.*

Researcher	*Do you have any issues with sustaining the project in your school?*
Principal	*The maths team have updated the school math's scheme to include the ENP and ANP stages. Now we are setting some targets about what strategies they need to have by year 1, year 2 and so on.*
Researcher	*As benchmarks?*
Principal	*Yes*
Researcher	*What have you decided on?*
Principal	*When we looked at your thing [ENP 2001 evaluation] we upped the ante a bit, so by the end of year 2 they need to be starting on advanced counting.*
Researcher	*What about the higher year levels?*
Principal	*Expect part-whole by end of year 4.*

Researcher	*What about teachers' enthusiasm for the programme?*
Principal	*You need to keep up the interest – while they had to report back [to facilitator and website] the interest was there.*
Researcher	*Yes*
Principal	*But now there is no deadline for giving information, some are letting it slide.*
Researcher	*Do you have any solutions?*
Principal	*I guess we need to set deadlines within the school now. But I had hoped they would do it.*

Principals discussed the types of teachers who may need more support to sustain the changes made with one principal commenting:

Principal	*We have been talking about having another push on numeracy next year. Getting teachers to share what they are doing. Going and teach or watching an expert and have her watching their rooms, just like X [facilitator] did. I am going to do the same release to let that happen.*
Researcher	*To what extent do you think teachers will sustain the changes they made?*
Principal	*Some will completely. For some others we will need to keep encouraging.*
Researcher	*Which ones need encouraging?*
Principal	*I think maybe those who were either very confident with how they used to teach or not so confident. I think maybe the ones in the middle were more willing and able to change and keep going. Now that would be a good research topic for you.*

Schools face particular sustainability issues when new staff arrive and several schools felt they would be able to rely on the skills of their existing staff to help train new and inexperienced staff.

Researcher	*Within the school if you didn't have access to facilitators or advisors would you be able to train new staff members?*
Principal	*I think so. I think there's people within the school that have been through the training twice. You know, because they happened to be with early numeracy and advanced numeracy [the school operates composite classrooms]. Certainly, in the training we're doing at the moment we've got one teacher who's done early numeracy last year and she's doing advanced this year. She's got so much more knowledge than the rest of us at the meetings and we've got enough in the school that have had that sort of training.*

One school felt it would be possible for school staff to work alongside external facilitators to support the training of new staff members.

Researcher	*In maintaining the project how will your school deal with changes of staffing?*
Principal	*How are we going to keep those people up to speed? I don't know. It hasn't been thought about; how the training is going to happen for those people.*
Researcher	*No quick solutions come to mind?*
Principal	*The expertise is there… the expertise is there on the staff now with the people that we've got. I guess there is no reason why those people then couldn't help in-school facilitators and they could monitor and work alongside those staff.* *For instance, if it was a beginning teacher coming in they are going to have a tutor teacher, and the chances are the tutor teacher is familiar with what's going on any way so therefore its going to be picked up and worked during that time so that's not going to be an issue. If it's an experienced teacher that comes in then there's going to be a syndicate leader and the syndicate leader is going to be there to support and help in that way. As well as that we've got X [teachers name] and as well as that we've got someone of the calibre of Y [teachers name] in the junior school so we've got people with strengths.*

Concluding comment

This chapter has reviewed the sustainability of the Numeracy Project over time, looking at the impacts on student achievement, teachers and school-wide sustainability. In general, teachers perceive that the Numeracy Project has had a positive impact on student ability, both in number and in mathematics. However, these perceptions were not supported by the data on students' progress, measured by mean strategy stages on the New Zealand Number Framework. Some schools have evidence to suggest improved student outcomes in mathematics more generally.

Teachers indicated their classroom programmes have incorporated Numeracy Project ideas and materials and will continue to do so. Schools had developed a number of different strategies to help ensure progress was maintained and these included the collation of resource material, the development of systems to monitor the teaching and learning process and the use of staff meetings to review progress and communicate further developments.

Conclusion

This report describes the impact of the ENP on just over 18,000 year 0-3 students who participated in the project in 2002. It also reports on the longer-term sustainability of the project as investigated in 18 schools that first participated in the project in 2000 or 2001.

Overall, the project had a positive impact on students' number strategies at all year levels with the percentage of students at the higher stages of the Number Framework increasing over the duration of the project. This improvement was greater than that which would have been expected prior to the implementation of the project.

The quantitative data collected provides a valuable source of information on which to establish realistic expectations of achievement. Expectations that have been established from the data include 80% of year 0-1 students being able to count from one to solve addition and subtraction problems with a further 15% able to count-on or count-back to solve such problems. By the end of year 2, 40% of the students are able to use advanced counting strategies with 15% using early additive part-whole strategies to solve addition and subtraction problems. Thirty-five percent of year 3 students are able to use at least one part-whole strategy to solve problems involving addition or subtraction.

The progress that students make on the Number Framework is linked to year level, gender, ethnicity and the decile level of the school. In general, older students make greater progress than younger students from all stages on the Number Framework with boys tending to make greater progress than girls from the higher stages of the framework. Students in higher decile schools make greater progress than students in lower decile schools and the progress made by Pasifika students from the higher stages of the Number Framework is consistently lower than that made by Māori and NZ European students.

Student ability across the knowledge and strategy domains appears to be linked. Students who make the transition from counting from one to advanced counting strategies are stronger in their knowledge of the forward and backward number word sequence to 100. Students who make the transition from counting to part-whole strategies are stronger in their knowledge of the sequence of numbers to 1000 and are more likely to know their groupings of 10s.

The trial of the Japanese lesson study group professional development model produced six lesson plans, each focusing on the use of tens frames at a different level of the Number Framework. These provide a valuable resource that will be communicated to teachers via the NZMaths website.

The lesson study group teachers also provided valuable feedback about the use of the approach and its effectiveness as a professional development tool. They found it to be significant in developing their own professional knowledge in mathematics, giving them a clearer understanding of the uses of the tens frame, enabling them to clarify their own thinking and consolidate existing ideas. They changed their classroom mathematics programmes as a result of the group, increasing their use of the tens frame and becoming more focused with a greater appreciation of the need for small, incremental steps in the children's understanding.

The ability to focus on one aspect of teaching in great detail was found to be a useful feature of the lesson study group approach as it allowed the teachers to develop an understanding of a teaching progression based around one piece of equipment. However, the group felt it would not be possible to use the approach in their own school without outside facilitation as the facilitator offered invaluable direction and knowledge.

The longitudinal component of the evaluation examined the sustainability of the developments over time. In general, teachers believe the project has had a positive impact on students' ability, both in number, and in mathematics in general. The reasons identified for this include students' increased enjoyment and confidence, due to a feeling of success, and increased teacher expectation. The teachers also believed that the increase in number knowledge has led to increased success in other areas of mathematics, with skills being transferable across the strands of the curriculum.

In a puzzling finding the students progress, as measured against the Number Framework, did not support the teachers' perceptions of improved numeracy outcomes. The mean strategy stage of the students by age was consistently lower in 2002 than in 2001 in the majority (88%) of longitudinal schools.

Teachers in the longitudinal study found the project to be significant in developing their professional knowledge in mathematics, increasing their awareness of the stages of development students are likely to follow and enlarging their focus on the strategies students are using to solve number problems. Teachers also reported an increased focus on students' individual needs, with more accurate planning and assessment taking place. In general, teachers reported feeling more positive towards mathematics since participating in the project; however, those teachers who had not received the full training associated with the programme were more negative than teachers with complete training experiences.

Schools in the longitudinal study had developed a number of different strategies to help ensure progress was maintained. These included the compilation of resource material to support new and existing staff, the use of staff meetings to review recent issues and the development of systems to monitor the teaching and learning process. Schools felt they would rely on the skills of their existing staff to help train new and inexperienced staff, possibly working alongside external facilitators for support.

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www.nzmaths.co.nz/Numeracy/project_material.htm

Appendix A: Stages of the Number Framework

Stage Zero: Emergent
Students at this stage are unable to consistently count a given number of objects because they lack knowledge of counting sequences and/or the ability to match things in one-to-one correspondence.

Stage One: One-to-one Counting
This stage is characterised by students who can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets, like 4 + 3.

Stage Two: Counting from One on Materials
Given a joining or separating of sets problem, students at this stage rely on counting physical materials, like their fingers. They count all the objects in both sets to find an answer, as in "Five lollies and three more lollies. How many lollies is that altogether?"

Stage Three: Counting from One by Imaging
This stage is also characterised by students counting all of the objects in simple joining and separating problems. Students at this stage are able to image visual patterns of the objects in their mind and count them.

Stage Four: Advanced Counting (Counting-On)
Students at this stage understand that the end number in a counting sequence measures the whole set and can relate the addition or subtraction of objects to the forward and backward number sequences by ones, tens, etc. For example, instead of counting all objects to solve 6 + 5, the student recognises that "6" represents all six objects and counts on from there: "7, 8, 9, 10, 11."
Students at this stage also have the ability to co-ordinate equivalent counts, such as "10, 20, 30, 40, 50," to get $50 in $10 notes. This is the beginning of grouping to solve multiplication and division problems.

Stage Five: Early Additive Part-Whole
At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and combined. This is called *part-whole thinking*. A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or teen numbers.

Stage Six: Advanced Additive Part-Whole
Students at the advanced additive stage are learning to choose appropriately from a repertoire of part-whole strategies to estimate answers and solve addition and subtraction problems. They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining. Simultaneously, the efficiency of these students in addition and subtraction is reflected in their ability to derive multiplication answers from known facts. These students can also solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6.

Stage Seven: Advanced Multiplicative Part-Whole
Students at the Advanced Multiplicative stage are learning to choose appropriately from a range of part-whole strategies to estimate answers and solve problems involving multiplication and division. Some writers describe this stage as "operating on the operator". This means than one or more of the numbers involved in a multiplication or division is partitioned and then recombined.

For example, to solve 27 x 6, 27 might be split into 20 + 7 and these parts multiplied then recombined, as in 20 x 6 = 120, 7 x 6 = 42, 120 + 42 = 162. This strategy uses the distributive property.

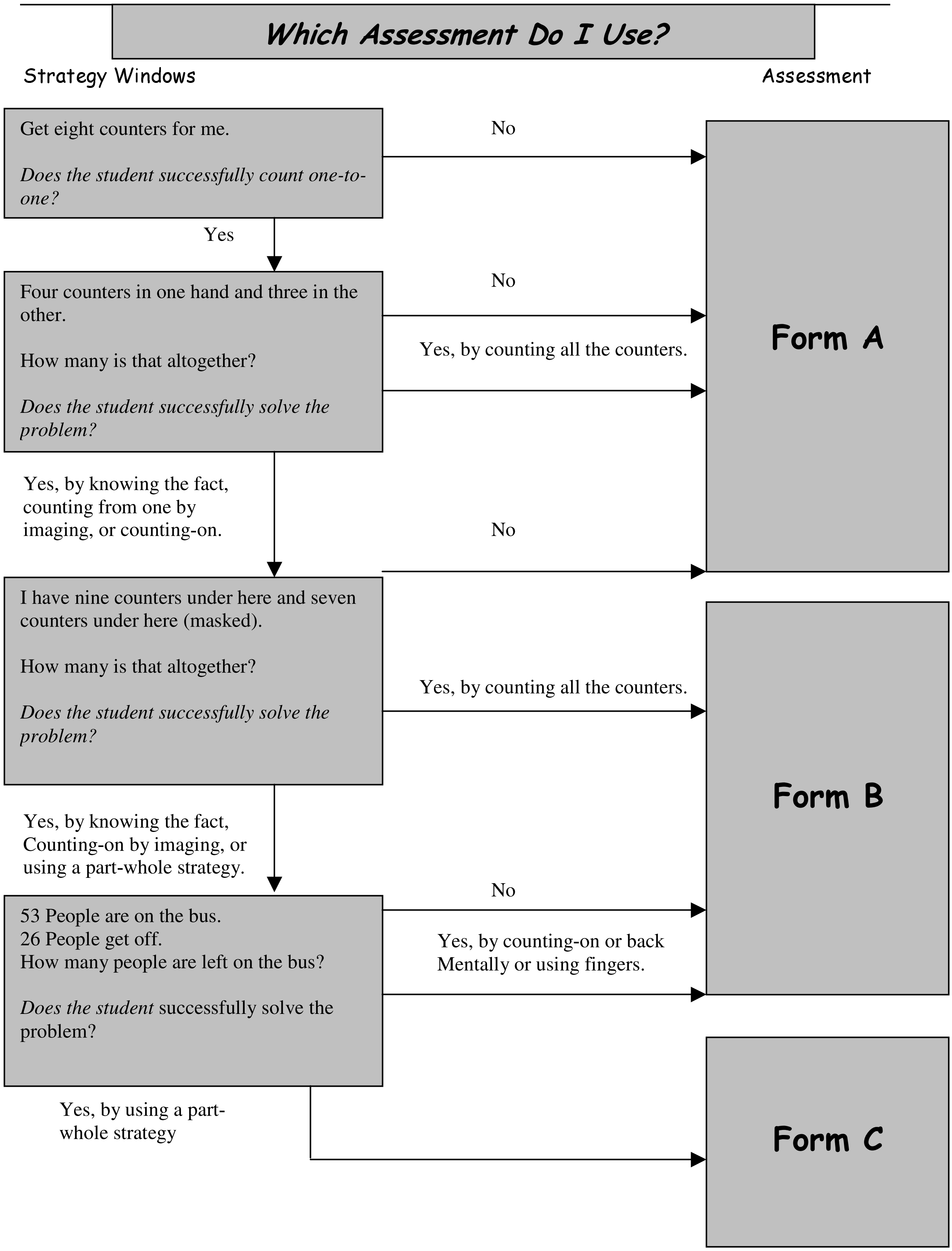
A critical development at this stage is the use of reversibility, in particular, solving division problems using multiplication. Advanced Multiplicative Part-Whole students are also able to estimate answers and solve problems with fractions using multiplication and division.

Stage Eight: Advanced Proportional Part-Whole
Students at the Advanced Proportional stage are learning to select from a repertoire of part-whole strategies to estimate answers and solve problems involving fractions, proportions, and ratios. This includes strategies for the multiplication of decimals and the calculation of percentages.

These students are able to find the multiplicative relationship between quantities of two different measures. This can be thought of as a mapping. For example, consider this problem: "You can make 21 glasses of lemonade from 28 lemons. How many glasses can you make using 8 lemons?"

To solve the problem, students need to find a relationship between the number of lemons and the number of glasses. This involves the creation of a new measure, glasses per lemon. The relationship is that the number of glasses is three-quarters the number of lemons. This could be recorded as: 21:28 as □ : 8, 21 is ¾ of 28, ¾ of 8 is 6.

Appendix B: NumPA Strategy Window



Appendix C: Percentage of Year 0-1 students by demographics and domains

Table C.1: Percentage of year 0-1 students at each additive stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	5%	13%	47%	17%	17%	2%		1802
Christchurch	8%	15%	43%	19%	14%	1%		1474
Massey	5%	16%	43%	19%	16%	2%		227
Dunedin	2%	11%	41%	17%	28%	2%	0%	175
Waikato	5%	15%	46%	16%	17%	1%		1014
Wellington	7%	11%	43%	14%	23%	1%		548
Total	6%	14%	45%	17%	17%	1%	0%	5491

Table C.2: Percentage of year 0-1 students at each additive stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	5%	12%	43%	18%	20%	2%	0%	3365
Māori	8%	17%	49%	16%	10%	0%		1103
Pasifika	8%	21%	50%	11%	9%	1%		565
Asian	3%	10%	40%	25%	21%	2%		269
Other	7%	17%	39%	16%	19%	1%		189
Total	6%	14%	45%	17%	17%	1%	0%	5491

Table C.3: Percentage of year 0-1 students at each additive stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	5%	13%	48%	18%	15%	1%	0%	2690
Male	6%	15%	42%	17%	19%	2%		2801
Total	6%	14%	45%	17%	17%	1%	0%	5491

Table C.4: Percentage of year 0-1 students at each additive stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given	6%	2%	64%	11%	15%	1%		124
Low	6%	17%	50%	14%	12%	0%		1717
Medium	6%	15%	45%	17%	15%	1%		1966
High	5%	10%	38%	21%	24%	2%	0%	1684
Total	6%	14%	45%	17%	17%	1%	0%	5491

Table C.5: Percentage of year 0-1 students at each multiplicative stage by region

Region	Not rated	2 to 3	4	5	6	Total
Auckland	93%	2%	5%	0%	0%	1984
Christchurch	95%	2%	3%	0%		1474
Massey	92%	4%	5%			227
Dunedin	85%	4%	9%	2%		244
Waikato	91%	2%	6%	0%		1014
Wellington	88%	5%	6%	1%		548
Total	92%	2%	5%	0%	0%	5491

Table C.6: Percentage of year 0-1 students at each multiplicative stage by ethnicity group

Ethnicity	Not rated	2 to 3	4	5	6	Total
NZ European	90%	3%	6%	1%		3365
Māori	96%	1%	3%	0%		1103
Pasifika	96%	2%	2%		0%	565
Asian	91%	4%	4%	0%		269
Other	93%	1%	6%			189
Total	92%	2%	5%	0%	0%	5491

Table C.7: Percentage of year 0-1 students at each multiplicative stage by gender

Gender	Not rated	2 to 3	4	5	6	Total
Female	94%	2%	4%	0%		2690
Male	91%	3%	6%	1%	0%	2801
Total	92%	2%	5%	0%	0%	5491

Table C.8: Percentage of year 0-1 students at each multiplicative stage by decile group

Decile group	Not rated	2 to 3	4	5	6	Total
None given	85%	8%	6%	1%		124
Low	94%	2%	4%	0%		1717
Medium	94%	2%	4%	0%		1966
High	90%	3%	7%	1%	0%	1684
Total	92%	2%	5%	0%	0%	5491

Table C.9: Percentage of year 0-1 students at each FNWS stage by decile group

Region	0	1	2	3	4	5	6	Total
Auckland	1%	7%	19%	32%	38%	2%	0%	1984
Christchurch	2%	5%	13%	33%	45%	2%	0%	1474
Dunedin	0%	5%	11%	30%	46%	7%	0%	227
Massey	3%	6%	19%	34%	35%	3%		244
Waikato	2%	8%	18%	34%	35%	3%	0%	1014
Wellington	2%	7%	15%	29%	41%	6%	0%	548
Total	2%	7%	16%	32%	40%	3%	0%	5491

Table C.10: Percentage of year 0-1 students at each FNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	1%	5%	14%	32%	44%	4%	0%	3365
Māori	3%	10%	21%	34%	31%	1%	0%	1103
Pasifika	2%	13%	23%	30%	30%	1%		565
Asian	1%	6%	14%	27%	48%	4%	1%	269
Other	2%	6%	15%	31%	43%	2%	1%	189
Total	2%	7%	16%	32%	40%	3%	0%	5491

Table C.11: Percentage of year 0-1 students at each FNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	1%	6%	16%	35%	39%	2%	0%	2690
Male	2%	7%	17%	29%	40%	4%	0%	2801
Total	2%	7%	16%	32%	40%	3%	0%	5491

Table C.12: Percentage of year 0-1 students at each FNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given		1%	15%	33%	49%	1%	1%	124
Low	2%	9%	20%	32%	35%	2%	0%	1717
Medium	2%	7%	17%	33%	37%	3%	0%	1966
High	1%	4%	11%	32%	47%	4%	0%	1684
Total	2%	7%	16%	32%	40%	3%	0%	5491

Table C.13: Percentage of year 0-1 students at each BNWS stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	6%	13%	29%	25%	25%	2%	0%	1984
Christchurch	5%	9%	25%	25%	33%	2%	0%	1474
Dunedin	2%	9%	23%	27%	33%	6%		227
Massey	7%	13%	33%	19%	25%	3%		244
Waikato	8%	14%	28%	23%	25%	2%	0%	1014
Wellington	7%	13%	25%	27%	24%	4%		548
Total	6%	12%	27%	25%	27%	2%	0%	5491

Table C.14: Percentage of year 0-1 students at each BNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	4%	10%	26%	25%	32%	3%	0%	3365
Māori	9%	17%	31%	25%	18%	1%	0%	1103
Pasifika	14%	18%	30%	22%	15%	1%		565
Asian	4%	9%	23%	27%	33%	3%	1%	269
Other	9%	12%	23%	24%	30%	2%	1%	189
Total	6%	12%	27%	25%	27%	2%	0%	5491

Table C.15: Percentage of year 0-1 students at each BNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	5%	12%	30%	27%	25%	1%	0%	2690
Male	7%	12%	25%	23%	29%	4%	0%	2801
Total	6%	12%	27%	25%	27%	2%	0%	5491

Table C.16: Percentage of year 0-1 students at each BNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given		10%	18%	36%	35%	2%		124
Low	9%	16%	28%	25%	20%	1%		1717
Medium	6%	12%	30%	23%	27%	2%	0%	1966
High	4%	8%	23%	25%	36%	4%	0%	1684
Total	6%	12%	27%	25%	27%	2%	0%	5491

Table C.17: Percentage of year 0-1 students at each NID stage by region

Region	N/A	0	1	2	3	4	Total
Auckland	3%	4%	16%	16%	47%	14%	1984
Christchurch	2%	3%	14%	14%	46%	22%	1474
Dunedin	13%	1%	8%	14%	44%	20%	227
Massey	5%	5%	22%	20%	33%	14%	244
Waikato	6%	4%	20%	17%	42%	11%	1014
Wellington	6%	4%	17%	12%	38%	22%	548
Total	4%	3%	16%	15%	44%	17%	5491

Table C.18: Percentage of year 0-1 students at each NID stage by ethnicity

Ethnicity	N/A	0	1	2	3	4	Total
NZ European	6%	3%	14%	15%	44%	19%	3365
Māori	2%	5%	22%	19%	39%	13%	1103
Pasifika	1%	7%	19%	16%	43%	13%	565
Asian	5%	0%	12%	11%	54%	17%	269
Other	4%	3%	12%	12%	52%	17%	189
Total	4%	3%	16%	15%	44%	17%	5491

Table C.19: Percentage of year 0-1 students at each NID stage by gender

Gender	N/A	0	1	2	3	4	Total
Female	4%	3%	16%	17%	48%	13%	2690
Male	5%	4%	16%	14%	40%	20%	2801
Total	4%	3%	16%	15%	44%	17%	5491

Table C.20: Percentage of year 0-1 students at each NID stage by decile group

Decile group	N/A	0	1	2	3	4	Total
None given	13%	1%	2%	18%	55%	11%	124
Low	3%	5%	20%	17%	42%	14%	1717
Medium	4%	4%	17%	16%	42%	17%	1966
High	6%	2%	12%	13%	47%	19%	1684
Total	4%	3%	16%	15%	44%	17%	5491

Table C.21: Percentage of year 0-1 students at each grouping stage by region

Region	N/A	0 to 1	2 to 3	4	5	6	Total
Auckland	1%	51%	31%	17%	1%		1984
Christchurch	1%	40%	36%	22%	1%	0%	1474
Dunedin	2%	36%	35%	25%	2%		227
Massey	3%	47%	34%	17%			244
Waikato	2%	54%	29%	14%	2%	0%	1014
Wellington	1%	51%	30%	17%	1%		548
Total	1%	48%	32%	18%	1%	0%	5491

Table C.22: Percentage of year 0-1 students at each grouping stage by region

Ethnicity	N/A	0 to 1	2 to 3	4	5	6	Total
NZ European	1%	43%	34%	21%	1%	0%	3365
Māori	2%	57%	29%	11%	1%		1103
Pasifika	1%	60%	29%	9%	0%		565
Asian	1%	38%	33%	27%	1%		269
Other	3%	46%	31%	20%	1%		189
Total	1%	48%	32%	18%	1%	0%	5491

Table C.23: Percentage of year 0-1 students at each grouping stage by gender

Gender	N/A	0 to 1	2 to 3	4	5	6	Total
Female	1%	48%	34%	16%	0%		2690
Male	1%	47%	30%	20%	1%	0%	2801
Total	1%	48%	32%	18%	1%	0%	5491

Table C.24: Percentage of year 0-1 students at each grouping stage by decile group

Decile group	N/A	0 to 1	2 to 3	4	5	6	Total
None given		48%	30%	22%	1%		124
Low	1%	54%	31%	13%	1%		1717
Medium	2%	51%	31%	16%	1%		1966
High	1%	37%	35%	26%	1%	0%	1684
Total	1%	48%	32%	18%	1%	0%	5491

Appendix D: Percentages of Year 2 students by demographics and domains

Table D.1: Percentage of year 2 students at each additive stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	1%	3%	24%	17%	42%	12%	1%	2181
Christchurch	1%	4%	24%	20%	38%	12%	1%	1531
Dunedin	1%	1%	10%	12%	60%	16%	1%	389
Massey	2%	8%	16%	19%	47%	7%	2%	193
Waikato	4%	3%	22%	20%	41%	9%	1%	1055
Wellington	2%	3%	26%	14%	44%	10%	1%	665
Total	2%	3%	23%	17%	43%	11%	1%	6014

Table D.2: Percentage of year 2 students at each additive stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	1%	3%	19%	16%	46%	14%	1%	3605
Māori	5%	5%	30%	19%	36%	5%	0%	1200
Pasifika	2%	5%	33%	23%	33%	4%		683
Asian		1%	14%	16%	49%	18%	2%	286
Other	1%	2%	23%	15%	49%	9%	1%	240
Total	2%	3%	23%	17%	43%	11%	1%	6014

Table D.3: Percentage of year 2 students at each additive stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	2%	3%	24%	18%	44%	8%	0%	2953
Male	2%	3%	22%	17%	41%	14%	1%	3061
Total	2%	3%	23%	17%	43%	11%	1%	6014

Table D.4: Percentage of year 2 students at each additive stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given		1%	17%	13%	48%	21%		112
Low	4%	4%	30%	19%	37%	5%	0%	2091
Medium	1%	3%	21%	18%	44%	12%	1%	2160
High	1%	2%	16%	16%	48%	18%	1%	1651
Total	2%	3%	23%	17%	43%	11%	1%	6014

Table D.5: Percentage of year 2 students at each multiplicative stage by region

Region	Not rated	2 to 3	4	5	6	7	Total
Auckland	65%	7%	23%	4%	1%	0%	2181
Christchurch	65%	9%	20%	4%	1%	0%	1531
Dunedin	34%	12%	43%	10%	1%		389
Massey	54%	10%	30%	4%	2%		193
Waikato	67%	5%	22%	5%	1%		1055
Wellington	63%	9%	21%	5%	2%	0%	665
Total	63%	8%	23%	5%	1%	0%	6014

Table D.6: Percentage of year 2 students at each multiplicative stage by ethnicity

Ethnicity	Not rated	2 to 3	4	5	6	7	Total
NZ European	56%	9%	28%	6%	1%	0%	3605
Māori	75%	6%	17%	3%	0%		1200
Pasifika	80%	7%	11%	2%			683
Asian	57%	9%	25%	7%	2%	0%	286
Other	58%	10%	26%	5%	1%		240
Total	63%	8%	23%	5%	1%	0%	6014

Table D.7: Percentage of year 2 students at each multiplicative stage by gender

Gender	Not rated	2 to 3	4	5	6	7	Total
Female	65%	8%	23%	4%	0%	0%	2953
Male	61%	8%	24%	6%	2%	0%	3061
Total	63%	8%	23%	5%	1%	0%	6014

Table D.8: Percentage of year 2 students at each multiplicative stage by decile group

Decile group	Not rated	2 to 3	4	5	6	7	Total
None given	36%	12%	46%	4%	3%		112
Low	74%	7%	16%	3%	0%	0%	2091
Medium	61%	10%	23%	5%	1%	0%	2160
High	53%	7%	32%	7%	2%	0%	1651
Total	63%	8%	23%	5%	1%	0%	6014

Table D.9: Percentage of year 2 students at each FNWS stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	1%	1%	5%	15%	58%	19%	1%	2181
Christchurch	0%	1%	4%	14%	55%	22%	3%	1531
Dunedin	0%	1%	4%	11%	50%	33%	2%	389
Massey	1%	2%	4%	18%	44%	25%	7%	193
Waikato	3%	1%	4%	15%	56%	18%	2%	1055
Wellington	1%	1%	6%	15%	51%	22%	4%	665
Total	1%	1%	5%	15%	55%	21%	2%	6014

Table D.10: Percentage of year 2 students at each FNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	0%	1%	4%	13%	54%	26%	3%	3605
Māori	4%	2%	6%	19%	54%	15%	1%	1200
Pasifika	1%	2%	8%	21%	58%	9%	0%	683
Asian	0%	0%	2%	7%	62%	24%	4%	286
Other	0%	1%	4%	14%	58%	20%	3%	240
Total	1%	1%	5%	15%	55%	21%	2%	6014

Table D.11: Percentage of year 2 students at each FNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	1%	1%	5%	15%	59%	17%	1%	2953
Male	1%	2%	4%	14%	51%	25%	4%	3061
Total	1%	1%	5%	15%	55%	21%	2%	6014

Table D.12: Percentage of year 2 students at each FNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given			1%	4%	70%	26%		112
Low	2%	2%	6%	19%	54%	15%	2%	2091
Medium	0%	1%	4%	14%	56%	23%	2%	2160
High	0%	1%	4%	10%	53%	28%	4%	1651
Total	1%	1%	5%	15%	55%	21%	2%	6014

Table D.13: Percentage of year 2 students at each BNWS stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	3%	4%	11%	19%	47%	16%	1%	2181
Christchurch	1%	3%	11%	16%	48%	20%	2%	1531
Dunedin	1%	2%	9%	12%	50%	26%	1%	389
Massey	2%	6%	8%	13%	50%	17%	5%	193
Waikato	4%	3%	12%	16%	47%	16%	2%	1055
Wellington	2%	3%	13%	18%	40%	20%	3%	665
Total	2%	3%	11%	17%	47%	18%	2%	6014

Table D.14: Percentage of year 2 students at each BNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	1%	2%	10%	15%	48%	22%	2%	3605
Māori	5%	4%	14%	21%	44%	11%	1%	1200
Pasifika	4%	6%	15%	23%	45%	7%	0%	683
Asian	1%	3%	7%	13%	51%	22%	3%	286
Other	2%	4%	10%	12%	49%	21%	2%	240
Total	2%	3%	11%	17%	47%	18%	2%	6014

Table D.15: Percentage of year 2 students at each BNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	2%	3%	11%	18%	50%	14%	1%	2953
Male	2%	3%	11%	16%	44%	22%	3%	3061
Total	2%	3%	11%	17%	47%	18%	2%	6014

Table D.16: Percentage of year 2 students at each BNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given		1%	5%	12%	61%	21%		112
Low	5%	5%	14%	21%	44%	11%	1%	2091
Medium	1%	3%	10%	16%	48%	20%	2%	2160
High	1%	2%	9%	13%	49%	24%	3%	1651
Total	2%	3%	11%	17%	47%	18%	2%	6014

Table D.17: Percentage of year 2 students at each NID stage by region

Region	N/A	0	1	2	3	4	Total
Auckland	30%	1%	3%	5%	29%	33%	2181
Christchurch	22%	0%	2%	6%	27%	43%	1531
Dunedin	49%	0%	2%	3%	20%	26%	389
Massey	20%	1%	4%	6%	27%	42%	193
Waikato	23%	2%	3%	5%	31%	36%	1055
Wellington	26%	1%	4%	4%	28%	37%	665
Total	27%	1%	3%	5%	28%	36%	6014

Table D.18: Percentage of year 2 students at each NID stage by ethnicity

Ethnicity	N/A	0	1	2	3	4	Total
NZ European	32%	0%	2%	4%	25%	37%	3605
Māori	18%	2%	4%	7%	33%	36%	1200
Pasifika	11%	1%	5%	7%	42%	35%	683
Asian	35%	0%	1%	2%	25%	36%	286
Other	30%	0%	2%	5%	24%	39%	240
Total	27%	1%	3%	5%	28%	36%	6014

Table D.19: Percentage of year 2 students at each NID stage by gender

Gender	N/A	0	1	2	3	4	Total
Female	26%	1%	3%	4%	32%	34%	2953
Male	28%	1%	3%	5%	25%	38%	3061
Total	27%	1%	3%	5%	28%	36%	6014

Table D.20: Percentage of year 2 students at each NID stage by decile group

Decile group	N/A	0	1	2	3	4	Total
None given	46%			2%	21%	31%	112
Low	18%	2%	3%	7%	36%	35%	2091
Medium	26%	0%	3%	4%	27%	39%	2160
High	38%	1%	2%	3%	21%	35%	1651
Total	27%	1%	3%	5%	28%	36%	6014

Table D.21: Percentage of year 2 students at each grouping stage by region

Region	N/A	0 to 1	2 to 3	4	5	6	7	Total
Auckland	2%	23%	31%	38%	6%	0%		2181
Christchurch	0%	16%	31%	43%	8%	1%	0%	1531
Dunedin	0%	13%	28%	50%	8%	1%		389
Massey	1%	21%	35%	40%	3%			193
Waikato	3%	20%	33%	38%	5%	1%		1055
Wellington	1%	23%	30%	38%	9%	0%		665
Total	1%	20%	31%	40%	7%	1%	0%	6014

Table D.22: Percentage of year 2 students at each grouping stage by ethnicity

Ethnicity	N/A	0 to 1	2 to 3	4	5	6	7	Total
NZ European	0%	16%	30%	45%	8%	1%	0%	3605
Māori	4%	27%	31%	33%	4%	1%		1200
Pasifika	3%	33%	34%	28%	2%			683
Asian	1%	9%	35%	46%	8%	1%		286
Other	0%	21%	33%	39%	7%			240
Total	1%	20%	31%	40%	7%	1%	0%	6014

Table D.23: Percentage of year 2 students at each grouping stage by gender

Gender	N/A	0 to 1	2 to 3	4	5	6	7	Total
Female	1%	20%	33%	40%	5%	0%		2953
Male	1%	20%	30%	40%	8%	1%	0%	3061
Total	1%	20%	31%	40%	7%	1%	0%	6014

Table D.24: Percentage of year 2 students at each grouping stage by decile group

Decile group	N/A	0 to 1	2 to 3	4	5	6	7	Total
None given		9%	31%	54%	4%	1%		112
Low	3%	28%	31%	33%	4%	0%		2091
Medium	0%	17%	34%	43%	6%	0%	0%	2160
High	1%	13%	28%	46%	10%	1%		1651
Total	1%	20%	31%	40%	7%	1%	0%	6014

Appendix E: Percentages of Year 3 students by demographics and domains

Table E.1: Percentage of year 3 students at each additive stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	1%	1%	9%	10%	43%	31%	5%	2553
Christchurch	1%	2%	8%	9%	41%	36%	4%	1602
Dunedin		0%	4%	9%	41%	39%	8%	378
Massey	3%	1%	6%	10%	42%	34%	4%	400
Waikato	1%	1%	9%	10%	43%	32%	4%	1254
Wellington	0%	2%	9%	11%	41%	32%	4%	700
Total	1%	1%	8%	10%	42%	33%	5%	6887

Table E.2: Percentage of year 3 students at each additive stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	1%	1%	6%	8%	42%	38%	5%	4159
Māori	1%	2%	12%	13%	42%	26%	3%	1481
Pasifika	1%	2%	16%	17%	47%	16%	0%	663
Asian		2%	3%	8%	38%	39%	11%	304
Other	1%	1%	13%	6%	41%	32%	6%	280
Total	1%	1%	8%	10%	42%	33%	5%	6887

Table E.3: Percentage of year 3 students at each additive stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	1%	1%	7%	10%	47%	31%	3%	3405
Male	1%	1%	9%	9%	37%	35%	7%	3482
Total	1%	1%	8%	10%	42%	33%	5%	6887

Table E.4: Percentage of year 3 students at each additive stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given	0%	1%	6%	1%	34%	56%	3%	140
Low	1%	2%	12%	13%	45%	24%	2%	2399
Medium	1%	1%	8%	9%	42%	34%	4%	2631
High	0%	0%	4%	7%	39%	42%	8%	1717
Total	1%	1%	8%	10%	42%	33%	5%	6887

Table E.5: Percentage of year 3 students at each multiplicative stage by region

Region	Not rated	2 to 3	4	5	6	7	Total
Auckland	30%	8%	40%	15%	5%	1%	2553
Christchurch	36%	9%	35%	15%	5%	0%	1602
Dunedin	10%	10%	45%	24%	9%	1%	378
Massey	18%	12%	46%	19%	6%	1%	400
Waikato	28%	10%	38%	19%	5%	0%	1254
Wellington	30%	11%	31%	19%	7%	2%	700
Total	29%	9%	38%	17%	6%	1%	6887

Table E.6: Percentage of year 3 students at each multiplicative stage by ethnicity

Ethnicity	Not rated	2 to 3	4	5	6	7	Total
NZ European	24%	8%	40%	19%	7%	1%	4159
Māori	38%	10%	35%	14%	3%	0%	1481
Pasifika	48%	13%	30%	7%	2%	0%	663
Asian	20%	9%	40%	19%	11%	2%	304
Other	30%	10%	36%	14%	9%	1%	280
Total	29%	9%	38%	17%	6%	1%	6887

Table E.7: Percentage of year 3 students at each multiplicative stage by gender

Gender	Not rated	2 to 3	4	5	6	7	Total
Female	30%	10%	41%	16%	4%	0%	3405
Male	29%	9%	35%	18%	8%	1%	3482
Total	29%	9%	38%	17%	6%	1%	6887

Table E.8: Percentage of year 3 students at each multiplicative stage by decile group

Decile group	Not rated	2 to 3	4	5	6	7	Total
None given	16%	7%	41%	22%	13%	0%	140
Low	39%	11%	35%	13%	3%	0%	2399
Medium	27%	10%	40%	17%	6%	1%	2631
High	21%	7%	40%	22%	9%	2%	1717
Total	29%	9%	38%	17%	6%	1%	6887

Table E.9: Percentage of year 3 students at each proportional stage by region

Region	N/A	1	2 to 4	5	6	7	8	Total
Auckland	31%	11%	42%	13%	3%	0%		2553
Christchurch	36%	7%	41%	13%	2%	0%		1602
Dunedin	9%	9%	57%	18%	6%	2%		378
Massey	17%	11%	56%	14%	3%		0%	400
Waikato	29%	11%	43%	13%	3%	0%		1254
Wellington	31%	8%	41%	12%	6%	1%	0%	700
Total	30%	10%	44%	13%	3%	0%	0%	6887

Table E.10: Percentage of year 3 students at each proportional stage by ethnicity

Ethnicity	N/A	1	2 to 4	5	6	7	8	Total
NZ European	24%	9%	47%	15%	4%	1%	0%	4159
Māori	38%	11%	39%	10%	2%	0%		1481
Pasifika	48%	12%	34%	5%	1%			663
Asian	21%	11%	43%	17%	6%	1%		304
Other	30%	12%	41%	13%	3%	1%		280
Total	30%	10%	44%	13%	3%	0%	0%	6887

Table E.11: Percentage of year 3 students at each proportional stage by gender

Gender	N/A	1	2 to 4	5	6	7	8	Total
Female	30%	10%	46%	12%	3%	0%		3405
Male	30%	9%	41%	14%	4%	1%	0%	3482
Total	30%	10%	44%	13%	3%	0%	0%	6887

Table E.12: Percentage of year 3 students at each proportional stage by decile group

Decile group	N/A	1	2 to 4	5	6	7	8	Total
None given	18%	9%	46%	21%	6%	1%		140
Low	39%	11%	39%	9%	2%	0%		2399
Medium	27%	9%	47%	13%	3%	0%		2631
High	21%	8%	45%	19%	6%	1%	0%	1717
Total	30%	10%	44%	13%	3%	0%	0%	6887

Table E.13: Percentage of year 3 students at each FNWS stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	0%	0%	2%	6%	41%	43%	8%	2553
Christchurch	0%	0%	1%	4%	33%	49%	12%	1602
Dunedin			0%	4%	29%	58%	9%	378
Massey	3%		1%	3%	34%	45%	15%	400
Waikato	0%	0%	1%	5%	38%	43%	13%	1254
Wellington	0%	0%	1%	7%	39%	39%	14%	700
Total	1%	0%	1%	5%	37%	45%	11%	6887

Table E.14: Percentage of year 3 students at each FNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	0%	0%	1%	4%	32%	50%	13%	4159
Māori	1%	0%	1%	6%	46%	38%	8%	1481
Pasifika	0%	0%	3%	12%	54%	28%	3%	663
Asian		0%	0%	6%	30%	48%	16%	304
Other	0%	0%	1%	6%	40%	42%	10%	280
Total	1%	0%	1%	5%	37%	45%	11%	6887

Table E.15: Percentage of year 3 students at each FNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	1%	0%	1%	5%	42%	44%	7%	3405
Male	0%	0%	1%	5%	33%	46%	14%	3482
Total	1%	0%	1%	5%	37%	45%	11%	6887

Table E.16: Percentage of year 3 students at each FNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given				5%	25%	49%	21%	140
Low	0%	1%	2%	7%	45%	37%	8%	2399
Medium	1%	0%	1%	5%	36%	46%	11%	2631
High	0%	0%	1%	2%	29%	54%	14%	1717
Total	1%	0%	1%	5%	37%	45%	11%	6887

Table E.17: Percentage of year 3 students at each BNWS stage by region

Region	0	1	2	3	4	5	6	Total
Auckland	1%	1%	4%	9%	39%	40%	7%	2553
Christchurch	1%	1%	3%	6%	34%	46%	10%	1602
Dunedin	1%	1%	3%	4%	31%	54%	7%	378
Massey	3%		2%	5%	35%	40%	15%	400
Waikato	1%	0%	2%	8%	38%	39%	12%	1254
Wellington	0%	0%	3%	9%	37%	38%	11%	700
Total	1%	1%	3%	7%	37%	42%	9%	6887

Table E.18: Percentage of year 3 students at each BNWS stage by ethnicity

Ethnicity	0	1	2	3	4	5	6	Total
NZ European	1%	0%	2%	5%	33%	47%	11%	4159
Māori	2%	1%	3%	11%	42%	35%	7%	1481
Pasifika	1%	2%	5%	16%	51%	23%	2%	663
Asian	0%		2%	5%	33%	45%	15%	304
Other	1%		4%	7%	39%	42%	8%	280
Total	1%	1%	3%	7%	37%	42%	9%	6887

Table E.19: Percentage of year 3 students at each BNWS stage by gender

Gender	0	1	2	3	4	5	6	Total
Female	1%	1%	2%	8%	42%	40%	6%	3405
Male	1%	1%	4%	7%	32%	43%	12%	3482
Total	1%	1%	3%	7%	37%	42%	9%	6887

Table E.20: Percentage of year 3 students at each BNWS stage by decile group

Decile group	0	1	2	3	4	5	6	Total
None given		1%	1%	8%	30%	45%	15%	140
Low	1%	1%	4%	11%	43%	33%	8%	2399
Medium	1%	0%	3%	7%	36%	43%	9%	2631
High	0%	0%	1%	3%	30%	53%	12%	1717
Total	1%	1%	3%	7%	37%	42%	9%	6887

Table E.21: Percentage of year 3 students at each NID stage by region

Region	N/A	0	1	2	3	4	Total
Auckland	66%	0%	1%	2%	10%	21%	2553
Christchurch	62%	0%	1%	1%	8%	28%	1602
Dunedin	79%		1%	1%	4%	15%	378
Massey	64%	0%	0%	2%	5%	29%	400
Waikato	56%	0%	1%	1%	12%	31%	1254
Wellington	57%	0%		1%	8%	34%	700
Total	63%	0%	1%	1%	9%	26%	6887

Table E.22: Percentage of year 3 students at each NID stage by ethnicity

Ethnicity	N/A	0	1	2	3	4	Total
NZ European	68%	0%	1%	1%	7%	23%	4159
Māori	54%	0%	1%	1%	12%	32%	1481
Pasifika	43%	0%	1%	5%	18%	34%	663
Asian	73%			1%	6%	20%	304
Other	65%	1%		1%	9%	24%	280
Total	63%	0%	1%	1%	9%	26%	6887

Table E.23: Percentage of year 3 students at each NID stage by gender

Gender	N/A	0	1	2	3	4	Total
Female	62%	0%	0%	1%	10%	26%	3405
Male	63%	0%	1%	1%	9%	26%	3482
Total	63%	0%	1%	1%	9%	26%	6887

Table E.24: Percentage of year 3 students at each NID stage by decile group

Decile group	N/A	0	1	2	3	4	Total
None given	68%			1%	4%	28%	140
Low	51%	0%	1%	2%	13%	33%	2399
Medium	65%	0%	1%	1%	9%	24%	2631
High	75%	0%	0%	0%	5%	20%	1717
Total	63%	0%	1%	1%	9%	26%	6887

Table E.25: Percentage of year 3 students at each grouping stage by region

Region	N/A	0 to 1	2 to 3	4	5	6	7	8	Total
Auckland	0%	8%	23%	49%	17%	2%	0%	0%	2553
Christchurch	1%	6%	18%	50%	23%	3%	0%		1602
Dunedin		5%	13%	56%	24%	3%			378
Massey	4%	10%	16%	50%	17%	4%			400
Waikato	1%	8%	23%	46%	20%	2%	0%	0%	1254
Wellington	0%	7%	21%	52%	17%	2%	1%		700
Total	1%	7%	20%	50%	19%	2%	0%	0%	6887

Table E.26: Percentage of year 3 students at each grouping stage by ethnicity

Ethnicity	N/A	0 to 1	2 to 3	4	5	6	7	8	Total
NZ European	1%	6%	17%	51%	22%	3%	0%	0%	4159
Māori	1%	11%	24%	46%	16%	1%	0%		1481
Pasifika	0%	12%	32%	45%	10%	0%			663
Asian		3%	17%	55%	19%	6%	1%		304
Other	0%	9%	22%	47%	19%	3%			280
Total	1%	7%	20%	50%	19%	2%	0%	0%	6887

Table E.27: Percentage of year 3 students at each grouping stage by gender

Gender	N/A	0 to 1	2 to 3	4	5	6	7	8	Total
Female	1%	7%	23%	51%	17%	1%	0%		3405
Male	1%	8%	18%	48%	22%	3%	0%	0%	3482
Total	1%	7%	20%	50%	19%	2%	0%	0%	6887

Table E.28: Percentage of year 3 students at each grouping stage by decile group

Decile group	N/A	0 to 1	2 to 3	4	5	6	7	8	Total
None given		1%	19%	43%	31%	6%			140
Low	1%	11%	26%	45%	17%	1%	0%	0%	2399
Medium	1%	7%	19%	51%	19%	3%	0%		2631
High	0%	3%	16%	54%	23%	3%	1%	0%	1717
Total	1%	7%	20%	50%	19%	2%	0%	0%	6887

Table E.29: Percentage of year 3 students at each fractions stage by region

Region	N/A	2 to 3	4	5	6	7	8	Total
Auckland	28%	21%	28%	19%	4%	0%	0%	2553
Christchurch	22%	16%	27%	29%	6%	0%		1602
Dunedin	9%	17%	30%	37%	7%	0%		378
Massey	17%	17%	39%	23%	5%	0%	0%	400
Waikato	32%	23%	25%	18%	2%		0%	1254
Wellington	31%	19%	27%	20%	4%	0%	0%	700
Total	26%	20%	28%	22%	4%	0%	0%	6887

Table E.30: Percentage of year 3 students at each fractions stage by ethnicity

Ethnicity	N/A	2 to 3	4	5	6	7	8	Total
NZ European	20%	19%	29%	27%	6%	0%	0%	4159
Māori	36%	21%	27%	15%	1%			1481
Pasifika	46%	19%	24%	10%	1%			663
Asian	16%	22%	31%	25%	5%			304
Other	25%	21%	27%	21%	5%	0%		280
Total	26%	20%	28%	22%	4%	0%	0%	6887

Table E.31: Percentage of year 3 students at each fractions stage by gender

Gender	N/A	2 to 3	4	5	6	7	8	Total
Female	26%	19%	29%	23%	3%		0%	3405
Male	26%	20%	27%	22%	5%	0%	0%	3482
Total	26%	20%	28%	22%	4%	0%	0%	6887

Table E.32: Percentage of year 3 students at each fractions stage by decile group

Decile group	N/A	2 to 3	4	5	6	7	8	Total
None given	17%	16%	26%	35%	6%			140
Low	36%	20%	26%	15%	2%		0%	2399
Medium	24%	19%	29%	23%	5%	0%	0%	2631
High	15%	19%	29%	30%	7%	0%	0%	1717
Total	26%	20%	28%	22%	4%	0%	0%	6887

Table E.33: Percentage of year 3 students at each decimals stage by region

Region	N/A	2 to 3	4	5	6	7	Total
Auckland	97%	0%	2%	0%	0%	0%	2553
Christchurch	97%	0%	2%	1%	0%		1602
Dunedin	95%	1%	3%	1%	1%		378
Massey	98%	0%	1%	1%	0%		400
Waikato	98%	0%	1%	1%	0%		1254
Wellington	98%	0%	1%	1%	0%	0%	700
Total	97%	0%	2%	1%	0%	0%	6887

Table E.34: Percentage of year 3 students at each decimals stage by ethnicity

Ethnicity	N/A	2 to 3	4	5	6	7	Total
NZ European	97%	0%	2%	1%	0%	0%	4159
Māori	98%	0%	1%	0%	0%		1481
Pasifika	99%	0%	0%	0%			663
Asian	94%	1%	4%	1%			304
Other	97%		2%	0%	1%		280
Total	97%	0%	2%	1%	0%	0%	6887

Table E.35: Percentage of year 3 students at each decimals stage by gender

Gender	N/A	2 to 3	4	5	6	7	Total
Female	98%	0%	1%	0%	0%		3405
Male	96%	0%	2%	1%	0%	0%	3482
Total	97%	0%	2%	1%	0%	0%	6887

Table E.36: Percentage of year 3 students at each decimals stage by decile group

Decile group	N/A	2 to 3	4	5	6	7	Total
None given	99%		1%				140
Low	98%	0%	1%	1%	0%		2399
Medium	98%	0%	1%	0%	0%	0%	2631
High	95%	0%	3%	1%	0%	0%	1717
Total	97%	0%	2%	1%	0%	0%	6887

Appendix F: Lesson Plans from Lesson Study Group

General Introduction

The lesson plans that follow highlight some of the ways tens frames can be used to facilitate learning about number. They are not intended to be a sequential series of lessons, rather a description of how to use tens frames to achieve a variety of teaching and learning objectives. The lessons include a teaching sequence, which outlines activities and questions to help students' achieve the stated learning outcome. It is not intended that these lessons be taught for one session only. Students may need many opportunities to consolidate their understanding and the lessons may need to be repeated, modified or extended. Teachers should be guided by the needs of their students' when determining the frequency and length of the lessons.

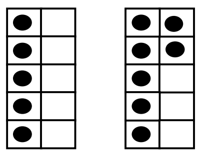
Using Tens Frames for One to One Counting

Why?
The purpose of this "unit" is to recognise the numbers 0 to 10 as represented using the quinary pattern on the tens frame. This develops the students' ability to visualise the quinary patterns and is fundamental to future work with tens frames and visualising number patterns.

Stage: One to one counting

Learning Outcome: Form the sets 0-10 on to an empty tens frame, using a quinary pattern (five in a row).

E.g.



Pre-requisite knowledge / skills: students need to be secure in one to one counting and the forming of sets to 10.

Resources:

- Teddies
- Counters
- Empty tens frames
- Tens frames for the numbers 0-5

Sequence:

1. Teacher creates interest, for example, by telling a story about teddies on a bus.
2. Teacher makes a group of five teddies and asks:

How many teddies have I got?

3. Teacher and students count the teddies together 1-1
4. Teacher demonstrates counting the five teddies on to the empty frame
5. Teacher spreads multiple copies of the tens frames (0-5) in front of the students and asks

   Who can find me a pattern that matches the teddies?
   How could you check that it's the same?
   Teacher encourages children to share their thinking in an open discussion. Possible solutions include count it with finger, match them – side by side or on top. Teacher establishes the link that 5 teddies have been replaced by 5 dots.
6. Repeat above with 1 – 4 teddies
7. For zero teddies: Teacher shows the children no teddies and asks them to find the matching tens frame.
8. Teacher gives each child their own set of tens frames to work with.
9. Teacher asks the children to put the tens frames 0-5 in order. Children work on this together, discussing the task as they carry it out

Note: This type of activity needs to be repeated as long as required with different materials and different contexts – until students are confident in this skill. This may take many weeks.

Additional Activities:
The following list of activities could form the basis for further development of one to one counting using tens frames.

- Repeat above teaching sequence for the numbers 6-10.
- Match numeral cards to tens frames (0-5) then 6-10.
- Children order their own set of tens frames (0-5) then 6-10.
- Children find the tens frame for a given number (0-5) then 6-10.
- Teacher flashes cards of tens frames to students and they chorus the number represented.
- Teacher flashes tens frames cards to individual students to identify.

Broadening activities:

- Have students make their own tens frame as a concrete to abstract link and retain them for future work.
- Ask students to colour in the correct number of dots on a tens frame for a given numeral.
- Have students make numbers on the tens frame on an OHP and then quickly turn the OHP on and off for others to identify the number they have made.
- Have students holding the tens frame cards for others to identify the number shown when the pattern is covered and only flashed at them for a short time. Work in large or small groups.
- Show students a tens frame with a number on it, cover the pattern and ask children to remember what it looks like and describe it.

- Have students break down numbers greater than five, (for example, seven is the same as five and two) making the connection between the five fingers on each of their hands and the two lines of five boxes on the tens frame.

Using Tens Frames to Build the Basic Facts of Five

<u>Why?</u>
The purpose of this “unit” is to introduce addition facts to five. It is important for children to have concrete experiences using a variety of contexts to build their knowledge of the basic facts of five. They need to become secure in this knowledge to enable them to build on these known facts in future learning.

Stage: Counting all from 1

Learning Outcome: build addition facts of five (and then ten) using a quinary frame.

Pre-requisite knowledge / skills: students need to be secure in combining sets up to five (and then ten) using concrete objects, then counting all to ascertain the total number.

Resources:

Counters
Tens frames for the numbers 0-5 (Option of giving one set to each child)

Sequence:

1. Teacher shows tens frame with three counters on it and says
   I have three teddies on my bus. Two more are waiting at the bus stop. Let’s put the teddies on to the bus and see how many we have.
2. Teacher puts two counters on to the tens frame and teacher and students count together.
3. Repeat using a different story for 3+2
4. Teacher gives each child a blank tens frame and 2 different colour counters
5. Children make up their own number stories about 3+2

Optional:
Teacher records on a chart once the sets have been combined for two or three examples.
3 and 2 is the same as 5
The notation *2+3=5* may be brought in after several days’ work on joining sets

Note: Students may find it difficult to create story contexts for given number sentences and may need to focus on this for some time to master it.

Additional Activities:
The following list of activities could form the basis for further development of the facts of five using tens frames.

- Repeat above teaching sequence with facts to ten, once facts to five are secure.
- Children work in pairs to complete different number stories for 3+2
- Compare the two ways of recording this addition: 3+2 and 2+3

Broadening Activities:

- Teacher poses a problem involving a difference to the children. For example, 3 teddies were on the bus, and some of their friends got on. If there are 5 on the bus now, how many friends got on?
- Children make up a number story and show it on the tens frame. They get others to solve it and write the number sentence.

Using Tens Frames to Describe Patterns to Ten

<u>Why?</u>
The purpose of this unit is to develop instant recognition of the quinary pattern of the numbers 5-10. This knowledge is important for building facts to ten.

Stage: Count from One by Imaging

Learning Outcome: recognising and describing patterns to ten on pre-made cards

Pre-requisite Knowledge / Skills: Able to form sets to ten

Resources:
Quinary tens frame flashcards for the numbers 5-10. (2 sets on different colour card ideal)

Sequence:
1. Teacher holds up one card at a time and asks
   How many dots are there?
   How do you know? [E.g. for seven "There's five and two more"]
   Children chorus back the number of dots and then the teacher goes around the group asking for individual responses.
2. Teacher changes the focus to quick recognition of the patterns, and repeats the above asking
   How many dots are there?
3. Repeat above asking
   How many gaps are there?
4. Repeat above asking
   How many more do we need to get ten?
   Initially, teacher also asks *How do you know?*
5. Teacher holds up cards one at a time and says:
   I'm looking at seven [for example], what do I see?

Broadening Activities:
Teacher spreads two sets of cards out face up. Students find two cards of different colours that "match", or make ten. This can then be developed into a memory game with the cards face down.

Note: tens frames are not the ideal piece of equipment for moving students towards the counting-on stage because the counters can still be seen and counted. Other equipment is more suitable.

Using Tens Frames to Build the Addition and Subtraction Facts to Ten

Why? The purpose of this unit is to build and consolidate the addition and subtraction facts to ten. These addition and subtraction facts are building blocks for part-whole thinking.

Stage: Advanced Counting (Counting-On) – Knowledge Building

Pre-requisite knowledge / skills: recognising and describing patterns to ten on pre-made cards

Resources:

Quinary tens frames with dot patterns for the numbers 0-10

A: Focus on Addition

Learning Outcome: form addition facts to ten using dot patterns on a quinary frame

Sequence:

1. Teacher holds up a tens frame and asks
   Tell me the number pattern?
   How many gaps are there?
   Teacher then states the appropriate number sentence e.g. 6 and 4 is 10.
2. Repeat several times with all cards to build facts to ten.
3. Teacher holds up a tens frame and asks
   Who can tell me the number sentence for this card?
4. Repeat several times, ensuring that all cards (0-10) have been covered up to this point
5. Teacher models the recording of the number sentences as "6 and 4 is ten", then "6+4=10"

B: Focus on Subtraction

Learning Outcome: form subtraction facts to ten using dot patterns on a quinary frame

Sequence:

1. Teacher explains to students that today:
   We are using the cards to work on take-away equations starting with ten.
2. Teacher holds up a tens frame and asks
   How many dots could I fit on this frame?
   How many have gone?
   How many are left?
   Who can give me a number sentence for this card?
   If necessary the teacher gives the appropriate number sentence e.g. 10 take away 6 is 4
3. Repeat several times with all cards to build facts to ten.

4. Teacher gives the number sentence and asks the children to find the appropriate card. For example:

I had ten dots and I took away 6, how many do I have left? Can you find the card for that? How many are left?

5. Teacher models the recording of the number sentences as “10 take away 6 is 4”, then “10-6=4”

Broadening activities:

- Teacher demonstrates that the facts can be placed in a pattern i.e. 0 + 10 = 10, 1 + 9 = 10, 2 + 8 =10. After the teacher has given some of the pattern students can work independently to find the rest.
- Teacher gives students the number sentence and they find the matching dot pattern.

Independent Activities:

- Students make their own dot patterns using blank tens frames and counters. They record these using + /or - and =. (Two sided counters ideal for this activity.)
- Students work on sheets of empty tens frames to fill in dots and record the corresponding number sentence.
- Given one card students record either two complimentary number sentences or the entire family of facts.
- Students match tens frames to appropriate number sentence on cards

Notes:

- There are many ways to record the work being done and what is appropriate depends on the children in the group. Options include:

  Teacher records
  Draw a picture
  Number story e.g. 3 and 4 make 7
  Number sentence or equation e.g. 3 + 4 = 7
  Use pre-made equations and match these to the appropriate card
  Use digit labels

Using Tens Frames for "Teen" Numbers

Why?
The purpose of this unit is to recognise and understand teen numbers. This is an essential piece of knowledge for the part-whole strategy of bridging to ten.

Stage: Advanced Counting (Counting-On) – Knowledge Building

Pre-requisite knowledge / skills: form addition facts to ten using dot patterns on a quinary frame

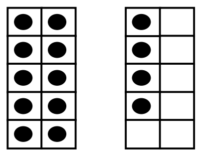
Learning Outcome: recognise and understand "teen" numbers, 11-20.

Resources:

Quinary tens frames with dot patterns for the numbers 0-10

Sequence

1. Teacher hold up a full tens frame and asks
   How many have I got?
2. Teacher holds up another card, alongside the full tens frame, and asks
   How many have I got now?



3. Repeat several times ensuring that the numbers 11 – 20 have been covered.
4. Teacher repeats above but this time shows the students the full tens frame has been turned over so the dots are not visible.
5. Teacher models 11 on the tens frame.
   Let's look at the number 11. How would we write that?
   Records: 10 + 1 = 11 then works through 10 + 2 = 12, 10 + 3 = 13 etc similarly to show the pattern.
6. Teacher asks students to solve problems such as 10 + 4, 10 + 9 with a focus on quick recall.

Extension Activities

- Teacher models one of the teen numbers on a tens frame. Students record the written form "15" while one child uses the expanded numeral cards to record the number. Once the recording is finished teacher asks:
  What does the 1 mean? [1 group of ten]
  What does the 5 mean? [5 ones]
- Teacher adds further blank tens frames to the model, for example 21: two tens and a 1.

Note: the context of a train with one full carriage and one partly full carriage can be used in this teaching sequence.

Using Tens Frames for the Strategy of Bridging to Ten

Why?
The purpose of this unit is to introduce the part-whole strategy of bridging to ten.

Stage: Early Part Whole

Pre-requisite knowledge / skills: Knowledge and recognition of the quinary patterns, knowledge of the facts to ten and the teen facts.

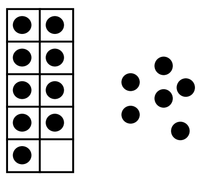
Learning Outcome: Bridging to ten to solve number problems

Resources:

Tens frames with 9 fixed dots
Blank frames
Counters

Sequence:

1. The teacher poses a problem.
   Josh has 9 oranges and 6 apples. How many does he have altogether?
2. Teacher records 9 + 6 on the board.
3. Teacher shows a fixed frame of 9 and asks the students to say where the six more should go without touching the tens frame.

   Teacher invites a student who answers that the answer is 15 to come and demonstrate how they got the answer.
   Teacher records the answer on the board.

4. If the students "count-on" teacher encourages them to think about adding one and then five by asking the follow up questions:
   Can we work it out without counting-on?
   Or *Is there a faster way?*
5. Repeat with 9 + 7, 9 + 6 with the students demonstrating using either the teacher's model or individual equipment. Teacher encourages students to explain their thinking.
6. Teacher focuses student attention on the importance of making a ten
   Why do you make a ten?

Note: future lessons could be extended to include 8 + □ and then 18 + □ with the pace depending on the students.

Appendix G: Interview Framework – Longitudinal

Aims of initial interview:

To work with each school to determine how we might measure the impact of the Numeracy Project on:

1. Students
    a. achievement in number
        - How are the students progressing on the number framework? What changes have you noted over time?
        - Talk about the entering information on all students' strategies by the end of October.

    b. achievement in maths
        - What changes have you noticed in maths achievement?
        - What "evidence" do you have for these changes?
        - PAT? (Gather information on year 4-6 from 1999 to 2002 if possible)
        - Other assessment tools used by the school?

    c. attitudes
        - teacher comments (student profiles)

2. Teachers
    a. Short questionnaire to all teachers in September.
    b. Interview with principal and/or lead teacher in June & October
        - What changes have you noticed in maths programmes
        - What changes in attitudes to maths teaching?

To discuss with each school their plans for sustaining the momentum of the project now that the facilitator has left.

1. What are the problems with sustaining the momentum in your school now that the facilitator has left?
    a. New teachers in school?
    b. Reluctance from some teachers?

2. What ideas (or actions) do you think would overcome these problems?

Appendix H: Teacher Questionnaire – Longitudinal

Early Numeracy Project, Longitudinal Study 2002

All responses to this evaluation will be kept confidential. Information collected will be used solely for the purpose of evaluating the effectiveness of the Early Numeracy Project and to inform future developments.

Please complete and enclose in the blank envelope provided. Pass to the principal / lead teacher for mailing.

> Circle the word(s) that best describes your response to the stated question. Use the space below each question to elaborate on your response. If you need more space, please continue overleaf.

1. What year class do you currently teach? ___________________

2. Did you complete the Numeracy Professional Development Project? Yes No

   If yes, was it in your current school or a different school? Current school Different school

3. Prior to participating in the Numeracy Project how would you describe your attitude towards teaching mathematics?

   Unenthusiastic *Slightly enthusiastic* *Moderately enthusiastic* *Very enthusiastic* *Highly enthusiastic*

4. Since participating in the Numeracy Project how would you describe your attitude towards teaching mathematics?

   Unenthusiastic *Slightly enthusiastic* *Moderately enthusiastic* *Very enthusiastic* *Highly enthusiastic*

5. How significant has the Numeracy Project been in developing your own professional knowledge in mathematics?

   Not significant *Slightly significant* *Moderately significant* *Very significant* *Highly significant*

6. How useful did you find:

	Not useful	*Slightly useful*	*Moderately useful*	*Very useful*	*Highly useful*
The teaching materials	1	2	3	4	5
The diagnostic interview	1	2	3	4	5
Facilitator demonstrations	1	2	3	4	5
The facilitator	1	2	3	4	5
Project meetings	1	2	3	4	5

7. What impact do you believe the project has had on students' ability in number?

Negative impact *No impact* *Slightly positive impact* *Very positive impact* *Highly positive impact*

8. What impact do you believe the project has had on students' mathematics in general?

Negative impact *No impact* *Slightly positive impact* *Very positive impact* *Highly positive impact*

9. To what extent has your classroom mathematics programme incorporated Numeracy Project ideas and materials?

Not at all *Slightly* *Moderately* *Considerably* *Fully*

10. To what extent will you continue to incorporate Numeracy Project ideas and materials into classroom mathematics programmes?

Not at all *Slightly* *Moderately* *Considerably* *Fully*

11. Are there any other comments you would like to make about the Numeracy Project?

Appendix I: Teacher Questionnaire – Lesson Study Group

Early Numeracy Project, Lesson Study Group 2002

All responses to this evaluation will be kept confidential. Information collected will be used solely for the purpose of evaluating the effectiveness of the Early Numeracy Project and to inform future developments.

Please complete, enclose in the envelope provided and bring to the final meeting on 12 September.

Circle the word(s) that best describes your response to the stated question. Use the space below each question to elaborate on your response.

12. What year level(s) do you currently teach? ___________________

13. Did you complete the Numeracy Professional Development Project? Yes No

    If yes, was it in your current school or a different school? Current school Different school

14. How many years teaching experience do you have? _________________

15. What other professional development initiatives in mathematics have you been involved in?

16. How significant has the Lesson Study Group been in developing your own professional knowledge in mathematics?

    Not significant *Slightly significant* *Moderately significant* *Very significant* *Highly significant*

17. To what extent has your classroom mathematics programme changed as a result of the Lesson Study Group?
    Not at all　　*Slightly*　　*Moderately*　　*Considerably*　　*Fully*

18. Do you believe it would be possible to use this type of professional development in your own school without outside facilitation?
    Yes　　*No*

19. The Lesson Study Group approach of professional development focuses on one aspect of teaching in great detail. Did you find this useful?
    Yes　　*No*

20. Are there any other comments you would like to make about the Lesson Study Group?