An Evaluation of the Advanced Numeracy Project 2003

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with Linda Bonne and Karen Fraser

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Executive Summary

In 2003, the Ministry of Education continued to offer New Zealand schools an opportunity to improve the teaching and learning of number concepts and skills through the Advanced Numeracy Project, a professional development programme for teachers.

The overall aim of the Advanced Numeracy Project was to develop teachers' knowledge of number concepts, student strategies and instructional practice in order to improve the achievement of students in years 4 to 6. A key part of the project was presenting teachers with a framework of broad stages that describe students' mathematical thinking. Each stage is characterised by the range of strategies that students use to solve problems. Teachers were involved in using a diagnostic interview to assess the stages of their students' thinking. The results of the diagnostic interview formed the basis of students' instructional groups. Teachers were also introduced to problem-solving strategies, activities and equipment to use when working with children. There was a particular emphasis on ways of developing increasingly sophisticated strategies for solving number problems.

This report evaluates the impact of the Advanced Numeracy Project on approximately 35,000 children and 1,200 teachers across eight regions: Northland, Auckland, Waikato, Massey, Wellington, Canterbury, Otago and Southland. It identifies changes in student achievement and teacher knowledge and practice that can be attributed to the professional development provided by the Advanced Numeracy Project.

The research addressed the following main questions:

1. What progress do children make on the New Zealand Number Framework?
2. How is progress linked to age, ethnicity, gender, geographical region, or school decile level?
3. In what ways does the ANP impact on teachers' subject and pedagogical content knowledge?
4. How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?
5. What was it that the facilitators did that had the most impact on improving teachers' classroom practice?
6. What have schools done to sustain the project? Are there identifiable policies and practices that lead to the long-term sustainability of the changes arising from ANP?
7. How do schools use the student data?
8. How accurate were teacher judgements in the second interview?
9. What actions have schools taken to establish and use communities of practice for staff?
10. In what ways does the ANP impact on facilitators' subject and pedagogical content knowledge? To what extent is the experience of the facilitator a factor in effective facilitation?
11. What characterises "effective" facilitation in the ANP? What facilitator actions do teachers find most helpful in enhancing their classroom practice?
12. What actions of the facilitators have led to identifiable communities of practice amongst teachers and amongst facilitators?

Key Findings

Student Achievement

- For the third year running, the majority of students improved during their participation in the Advanced Numeracy Project. As in previous years, the figures show that the results in addition and subtraction across all year levels were slightly better than for the other operational domains.

- The patterns for each year group in addition and subtraction were similar to those in previous years, with the number of students who remained at the same stage at the end of the project increasing with each advancing stage. The patterns of results for multiplication and division followed a similar pattern to those in 2002. The percentages of students moving to the more advanced of these stages increased across the year levels and were greater than the figures for proportions and ratios but less than the figures for addition and subtraction.

- The students' shifts through the patterns of achievement mirrored the shifts seen in the previous year. The pattern of achievement for proportions and ratios was the same as 2002, with students starting at a lower stage than for the other domains. When students' starting place is taken into account, this pattern was consistent with that found for addition and subtraction and multiplication and division in that, with the increasing sophistication of stages, fewer students shifted, and this confirms the 2002 figures. The pattern of achievement in the knowledge aspects confirmed the 2002 results and was similar in terms of stages gained over the period of instruction.

- The shift from counting-based to part-whole strategies is an important marker in judging the progress of students participating in the Numeracy Project. For addition and subtraction, the percentage of students across year groups shifting to part-whole strategies was the same as for 2002, at 50% of students. These percentages are a drop from the 2001 figure of 63%. For multiplication and division, the percentage of students across all year groups who moved to part-whole strategies was 44% and for proportions and ratios 39%. These figures were similar to those for 2002. The percentage of students who initially used counting-based strategies and shifted to part-whole strategies increased at each year level for each operational domain. The proportion of those who shifted to more advanced stages increased at each year level. The percentages for each year group were similar to those from 2002.

- Compared to 2002, there was a big increase in addition and subtraction in the number of Year 4 students who became advanced additive in 2003 (from 2% in 2002 to 12% in 2003). The percentage of students who became advanced additive in multiplication and division across all year groups increased from 11% in 2002 to 14% in 2003. As for 2002 the greatest shift across all year groups and operational domains was to the early additive stage. Fewer students moved to the stages of advanced additive and advanced multiplicative part-whole, which represent more sophisticated part-whole thinking. Virtually no students shifted to advanced proportional part-whole.

- The effect of ethnicity on student progress in addition and subtraction appears to lessen as students move up the year levels. At Year 4 the range across the different ethnic groups for those remaining with counting-based strategies was notable, being 26 percentage points compared to 22 at Year 5 and 15 at Year 6. This compares to 24 for Year 4, 21 for Year 5, and 19 for Year 6 in 2002.

- When compared to the 2002 results there appears to be a decline in the percentage of students adopting part-whole strategies in addition and subtraction for most ethnic groups at Years 5 and 6. In Year 6 the percentages of those becoming part-whole declined for all ethnic groups except Pasifika students, with the students categorised as Other and Māori declining the most.

- More Asian students than those of any other ethnic group shifted to part-whole strategies at Year 4 (57% Asian compared to 47% New Zealand European) and at Year 5 (59% Asian compared to 53% New Zealand European).

- The effect of decile on the patterns of achievement was similar to that in previous years with increasing numbers of students shifting to part-whole strategies across the composite groups of low (deciles 1 to 3), middle (deciles 4 to 7) and high (deciles 8 to 10) decile rankings. More students adopted part-whole strategies in 2003 than in 2002 for each decile group. The middle deciles improved the most, from 45% in 2002 to 50% of students shifting to part-whole strategies in 2003.

- The levels of knowledge for those with part-whole strategies were consistently higher than for those with counting-based strategies. The results for 2003 in general were better than those for 2002.

Facilitation and Sustainability

- Effective facilitators centred their work with teachers on the Number Framework, the related diagnostic interview and the Teaching Model rather than on classroom activities.

- Participants transposed and extended the essence of the Framework and the Teaching Model to the settings in which they operated. Transformation of the structure over time appears to be a critical component in ensuring the sustainability of the project.

- Key components that helped schools to sustain the project were the initial innovation of classroom practice by individual teachers and the internalisation of the new practices, firstly at the individual teacher level, and then through the cooperative efforts of a group of staff.

- The role of the facilitator evolved during the work with a school. Initially the facilitators' work focused on individual teachers working collegially and later shifted to the lead teacher and principal. This shift in focus to a school-wide level helped support longer-term changes to the ways in which schools teach numeracy.

Recommendations

- Notwithstanding the improvements in the number of students using part-whole thinking, the central importance of this thinking needs to be emphasised by facilitators and teachers in order to further improve the strategic thinking of children.

- Continued work is needed on investigating ways of raising Māori and Pasifika students' achievement to reverse the trend identified in this evaluation.

- The findings of the investigation into facilitation and sustainability should be incorporated into a professional development package for facilitators.

Chapter 1: Introduction

The Numeracy Project has been heralded as an example of successful transformation of policy to practice through a dynamic approach to the policy process (Higgins with Parsons & Hyland, 2003). Major factors in the success of the implementation of this policy include ongoing evaluation, developing a research base from the findings, and promoting the concept of a learning community that includes Ministry of Education personnel, facilitators, researchers and teachers.

Evidence of raised student achievement (irrespective of students' age, gender, ethnicity, school region or decile ranking) and improved teacher knowledge has been reported for three consecutive years for the Early and Advanced Numeracy Projects (see Thomas & Ward, 2001 and 2002; Thomas, Tagg, & Ward, 2003; Higgins, 2001, 2002 and 2003). An important part of raising student achievement is the shift from counting-based to part-whole thinking. Māori and Pasifika student results continue to lag behind those from other ethnic groups.

Teachers' professional knowledge and practice has been enhanced through participation in the Advanced Numeracy Project. The Number Framework has provided teachers with a workable structure against which to plan and teach. Teachers have placed a greater emphasis on student explanations of number problems, with increased use of small groups for instructional work (Higgins, 2003). The Fraivillig et al. (1999) model informed this trend for some teachers. Use of the Teaching Model (Ministry of Education, 2004b) guided the progression in the representation of mathematical ideas from a physical model to the abstract form.

As the project has expanded over the last few years, so has the team of facilitators. Facilitators' mathematical and pedagogical content knowledge, as well as their knowledge of facilitation, underpins the success of the project. The introduction of models of effective teaching based on Hattie's (2002) work, and stages in teaching (Hughes, 2003), as well as the Number Framework (Ministry of Education, 2004a) and diagnostic interview and the Teaching Model (Ministry of Education, 2004b), structure facilitators' work with teachers. A dynamic project retains its integrity from various interpretations of core ideas – in the case of the Numeracy Project the core ideas are: increasing levels of abstraction in children's levels of understanding mathematical ideas, and representing those ideas. Ongoing analysis of the ways in which the manifestation of these ideas through the Number Framework and the Teaching Model (as well as other models that describe stages in the development of teachers) support facilitators in their work with teachers is important.

This report evaluates the impact of the Advanced Numeracy Project in 2003 on the participating students, teachers, principals and facilitators. The Government's Literacy and Numeracy Strategy, which aims to raise student achievement, develop teacher capability and increase community involvement in the areas of literacy and numeracy, provides the overall vision for the project.

Facilitators who have worked alongside teachers and schools have guided the development of teacher knowledge and practice. The key structures of The Number Framework (Ministry of Education, 2004a), and the Teaching Model (Ministry of Education, 2004b) underpin the development. The Number Framework represents stages of children's mathematical thought in terms of levels of abstraction in thinking about numbers. An important distinction in this thinking – between that based on counting and that which reflects numbers as parts of a whole – is represented by the dotted line in the diagram below.

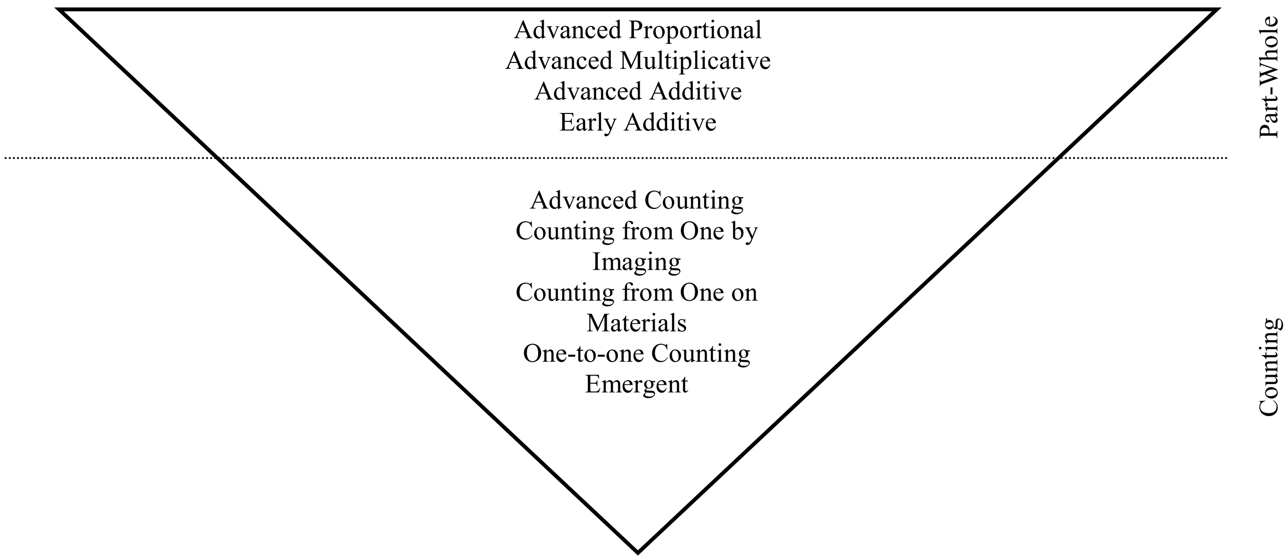

Figure 1-1: Stages of the Number Framework (Ministry of Education, 2004a)

The Teaching Model (Ministry of Education, 2004b), drawing on the work of Pirie and Kieren (1989) represents levels of abstraction in representations of mathematical ideas. Distinctions are drawn between representing mathematical ideas using materials such as a number frame, using children's images in their minds as an aid to solving problems, and the most abstract stage of using number properties to represent number ideas. The model suggests that a process of working from using materials to using number properties be followed when encountering new ideas, irrespective of a child's stage on the Number Framework.

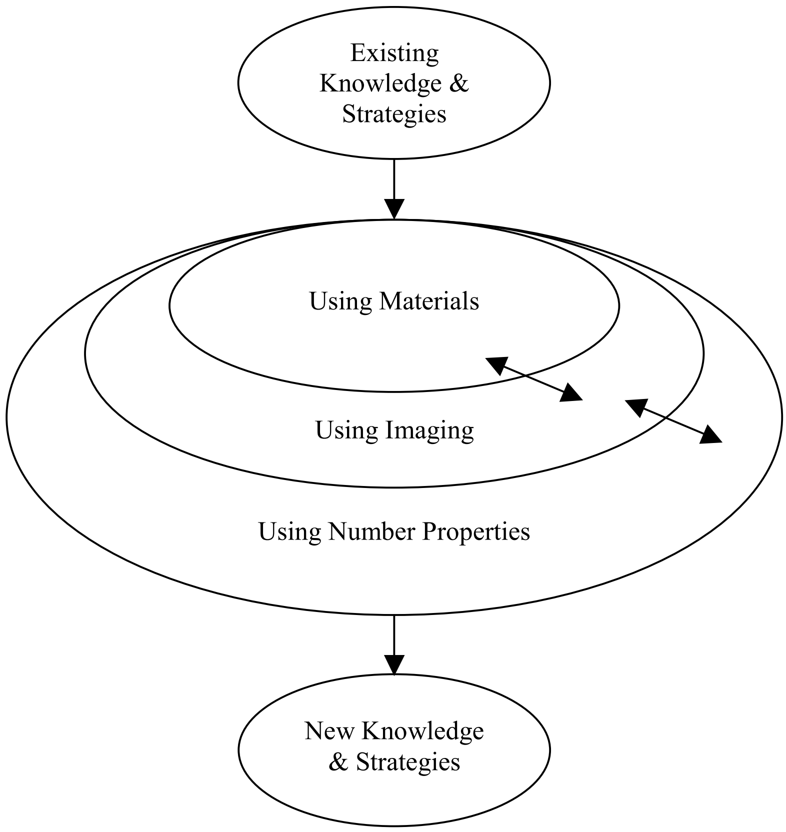

Figure 1-2: The Teaching Model (Ministry of Education, 2004b)

Over the period that the Numeracy Project has been implemented, knowledge about professional practice has been generated in a number of areas. Of relevance to this report are the models of effective practice and facilitation.

A useful guide to teaching strategies that promote children's thinking, which has been adopted by teachers and facilitators, is that of Fraivillig, Murphy and Fuson (1999). Three classes of teacher action identified as key to developing children's thinking are eliciting, supporting, and extending. The model is not only useful for teachers in planning the teaching sessions, but also has provided a useful structure against which teachers and facilitators can judge the actions taken in teaching sessions. The approach has highlighted the emphasis on encouraging children's explanations of their problem-solving strategies.

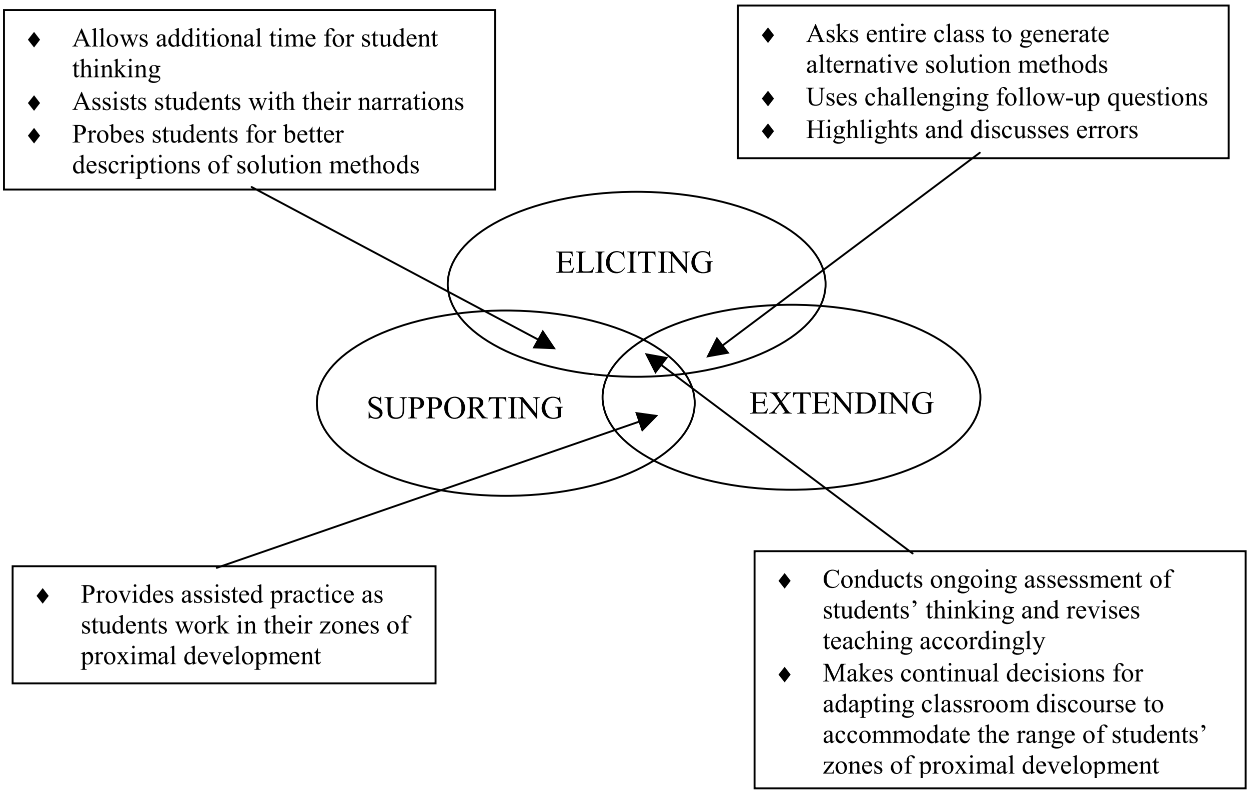

Figure 1-3:
Diagram of the Advancing Children's Thinking (ACT) framework and the teaching strategies that reside in the intersections between and among the instructional components of the ACT framework
from Fraivillig, Murphy and Fuson, 1999, p.7

Instructional components of ACT framework		
Eliciting	**Supporting**	**Extending**
Facilitates students' responding ♦ Elicits many solution methods for one problem from the entire class ♦ Waits for and listens to students' descriptions of solution methods ♦ Encourages elaboration of students' responses ♦ Conveys an accepting attitude towards students' errors and problem-solving efforts ♦ Promotes collaborative problem solving **Orchestrates classroom discussions** ♦ Uses students' explanations for lesson's content ♦ Monitors students' levels of engagement ♦ Decides which students need opportunities to speak publicly or which methods should be discussed	**Supports describer's thinking** ♦ Reminds students of conceptually similar problem situations ♦ Provides background knowledge ♦ Directs group help for an individual student ♦ Assists individual students in clarifying their own solution methods **Supports listeners' thinking** ♦ Provides teacher-led instant replays ♦ Demonstrates teacher-selected solution methods without endorsing the adoption of a particular method **Supports describer's and listeners' thinking** ♦ Records symbolic representation of each solution method on the board ♦ Asks a different student to explain a peer's method **Supports individuals in private help sessions** ♦ Encourages students to request assistance (only when needed)	**Maintains high standards and expectations for all students** ♦ Asks all students to attempt to solve difficult problems and to try various solution methods **Encourages mathematical reflection** ♦ Encourages students to analyse, compare, and generalise mathematical concepts ♦ Encourages students to consider and discuss interrelationships among concepts ♦ Lists all solution methods on the board to promote reflection **Goes beyond initial solution methods** ♦ Pushes individual students to try alternative solution methods for one problem situation ♦ Promotes use of more efficient solution methods for all students ♦ Uses students' responses, questions, and problems as core lesson **Cultivates love of challenge**

Table 1-1
Examples of instructional strategies employed to elicit, support and extend children's mathematical thinking
from Fraivillig, Murphy and Fuson, 1999, p.8

The model of facilitator practice (Higgins, 2001) represented the relationship between teachers' and facilitators' contexts of practice. (See Chapter 5 of this report for a fuller discussion.) It is based on Fennema and Franke's (1992) teacher model, which showed how a teacher's practice was shaped by their beliefs and attitudes, their content knowledge, their knowledge of how students learn mathematics, and their knowledge of how to teach mathematics (pedagogical content knowledge). The teacher's context of practice is only one dimension of a facilitator's context of practice. As well as factors related to the teacher with whom the facilitator is working, there are dimensions of practice that relate to the facilitator themselves, such as the beliefs and values that they bring to the facilitator role, their knowledge of mathematics, their knowledge of how to teach mathematics (pedagogical content knowledge), and their knowledge of teachers as learners. The complexity of the facilitator's tasks arises from the overlaying of the facilitator's dimensions (or context of practice) on the dimensions of practice of the teacher with whom they are working. This means a facilitator needs to consider multiple dimensions of practice. Further complexity arises from the dynamic of professional autonomy generated from the intersection of a teacher's and facilitator's context of practice. The dynamic between dimensions within the two respective contexts has the potential to be played out in multiple ways and presents special challenges in facilitator preparation and practice. Part of this report examines the structural elements that underpin a facilitator's work with teachers and schools.

As more schools are involved in the Numeracy Professional Development Projects each year, there is an increased focus on sustaining the development when the facilitator is no longer working in the school. Part of this report looks into the progression from facilitator-guided implementation to autonomous implementation by teachers and schools. Autonomous implementation by schools has implications for a reconfiguring of structures, policies and roles that support the long-term efficacy of the project in schools.

The Structure of this Report

Chapter 2 discusses the methodology. Chapter 3 presents the analysis of student results from the initial and final diagnostic interviews. Chapter 4 discusses the shift from counting-based to part-whole strategies. Chapter 5 examines how the structure of the project underpins its facilitation in schools. Chapter 6 discusses issues of sustainability in terms of the progression of establishing, internalising and authenticating the ideas of the project in schools.

Chapter 2: Methodology

Aim of the Investigation

This project complements the other evaluations of different aspects of the Numeracy Professional Development Projects. This investigation examined the impact of the Advanced Numeracy Project on participating teachers and students. The intended outcomes of the project were to increase the professional knowledge of teachers and the levels of numeracy of students. This investigation examined the sustainability of the project through school policies and practices and the facilitation of the professional development.

Design and Methodology

The data set comprises an overall evaluation and a multi-faceted case study. The overall evaluation draws on the quantification of student progress using the New Zealand Number Framework, and on questionnaires to teachers, principals and facilitators on the effectiveness of the teacher professional development programme. The evaluation examines the longitudinal impact of the ANP on student achievement, the impact of facilitation on classroom practice, and the sustainability of the project at the school level. The broad areas of the investigation are outlined below.

Before data was gathered, ethical approval was sought from the Wellington College of Education Ethics Committee, which operates under the NZARE Code of Ethics. The data gathering was guided by the principles of this code.

Students

The first broad area investigated the impact of the ANP on children's understanding of number concepts as detailed on the New Zealand Number Framework. The following research questions were addressed:

1. What progress do children make on the New Zealand Number Framework?
2. How is progress linked to age, ethnicity, gender, geographical region, or school decile level?

Teachers and Schools

The second broad area is that of investigating the impact of the ANP on teachers' subject and pedagogical content knowledge and classroom practice. Specific questions that were addressed are:

3. In what ways does the ANP impact on teachers' subject and pedagogical content knowledge?
4. How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?

5. What was it that the facilitators did that had the most impact on improving teachers' classroom practice?

6. What have schools done to sustain the project? Are there identifiable policies and practices that lead to the long-term sustainability of the changes arising from ANP?

7. How do schools use the student data?

8. How accurate were teacher judgements in the second interview?

9. What actions have schools taken to establish and use communities of practice for staff?

Facilitators

The third broad area investigated the characteristics of "effective" facilitation through looking at the ways in which the ANP impacts on facilitators' knowledge, and the effectiveness of the facilitator training. The project was designed to answer the following research questions:

10. In what ways does the ANP impact on facilitators' subject and pedagogical content knowledge? To what extent is the experience of the facilitator a factor in effective facilitation?

11. What characterises "effective" facilitation in the ANP? What facilitator actions do teachers find most helpful in enhancing their classroom practice?

12. What actions of the facilitators have led to identifiable communities of practice amongst teachers and amongst facilitators?

Summary of the Data-gathering Process

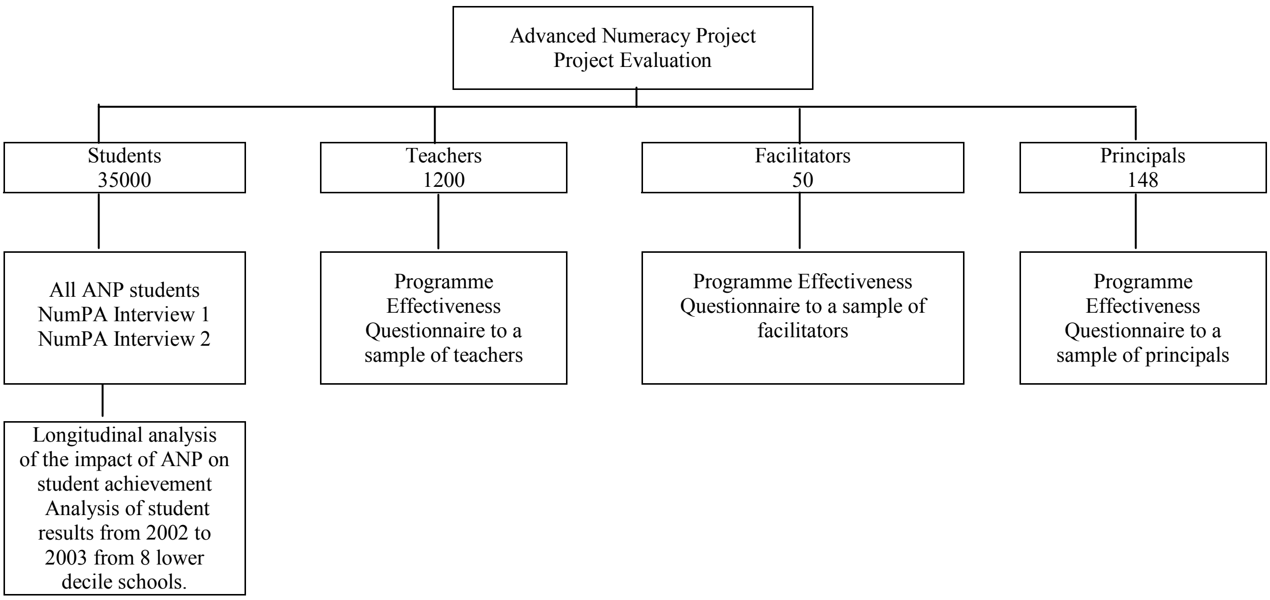

Figure 2-1: Evaluation of ANP – impact on participants

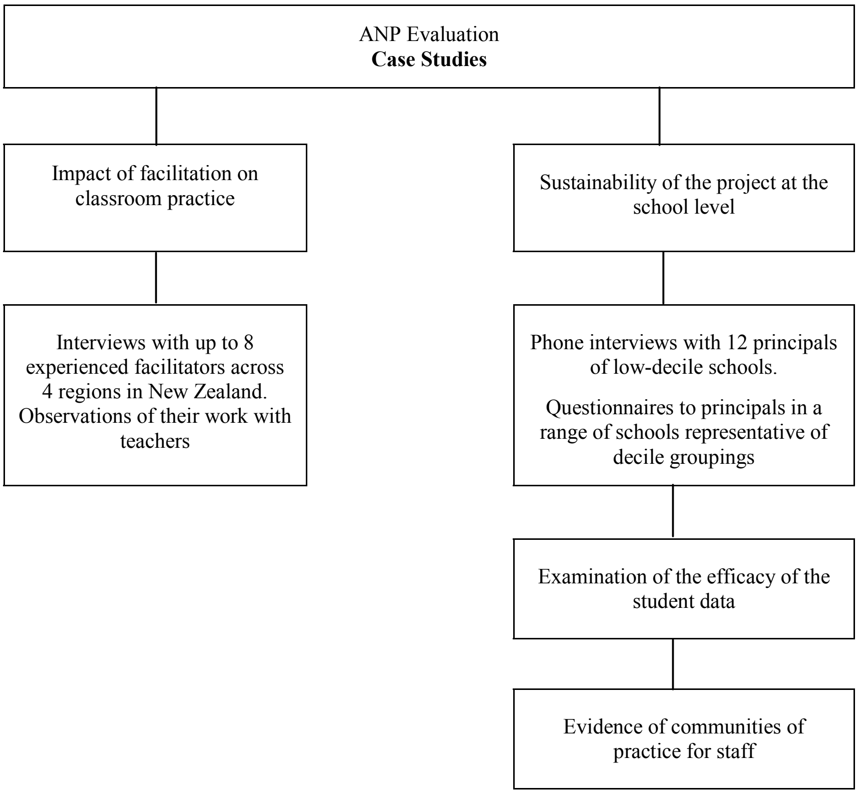

Figure 2-2:
Implementation of ANP in schools – an investigation of the impact of facilitation on classroom practice, the sustainability of the projects in 12 schools and a longitudinal analysis of the results in 10 schools

The Participants

Following on from 2002 the Ministry of Education continued to offer the ANP to schools throughout New Zealand. The ANP involved approximately 50 facilitators, 1200 teachers and 35000 students in 2003. A similar approach was used to that of previous years, with participating teachers organised into clusters comprising a few schools for after-school or whole-day meetings. Continuing the approach adopted in previous years, the teacher development had an initial focus on the Numeracy Project Assessment (NumPA) diagnostic tool. A similar process was followed to that of 2002, with teachers assessing students twice during the project, using the NumPA tool first at the start of the project after teachers had received their initial training, and then again after 15 weeks' teaching, at the completion of the project. On the basis of the initial results, facilitators worked with teachers to plan their instructional programme. These results were used to track the achievement of students using the web-site set up especially for the project. Teachers completed a profile for each student, including their date of birth, their year level and their ethnicity (the ethnic group with which they mostly identified). A longitudinal analysis of low-decile schools that participated in ANP in 2002 and 2003 is compared to the overall sample.

Implementation of ANP in Schools: Case Studies

Impact of Facilitation on Classroom Practice

The evaluation of ANP in 2001 and 2002 suggested that facilitators played a key role in acting as mediators between teachers' existing and new practices within individual teachers' contexts of practice. More detailed investigation of the actions of facilitators within communities of practice in 2003 provided deeper understanding of structures of support for teachers and facilitators engaged in professional development projects such as ANP. Eight experienced facilitators in four regions were interviewed and observed working with up to two teachers each.

Sustainability in Schools

An important aspect of any intervention in schools is the degree to which it is sustained in subsequent years. School structures, policies and practices are key to the long-term efficacy of an intervention. The case study identified critical actions taken by schools that appeared likely to sustain the project. Telephone interviews were conducted with the lead teacher and principal from each of 12 schools that had participated in the Advanced Numeracy Project in 2002. At least one school from each of the eight regions participated, with two or three schools involved in the regions with the greatest populations. Regional co-ordinators were asked to submit names of low-decile schools in their region that their facilitator judged were sustaining the development. From these lists, some of the lowest decile schools were asked to participate in the interviews. All of the schools agreed, so principals and lead teachers were posted an information sheet that outlined the purpose of the research, along with a consent form for them to sign and return. A copy of the interview questions was also included so that responses could be considered before the interviews took place. The interviews were tape-recorded for later transcription and took place in term 4 of 2003.

The ways in which schools and teachers used student data was investigated. The investigation included looking at the accuracy of teacher judgement in the second interview; the focus of the first interview being on teachers' professional development rather than the accuracy of judgement. Expert facilitators examined the judgements made by a small sample of teachers.

Overview of Student Participants

Ethnicity		**Decile**										**Total**
		1	2	3	4	5	6	7	8	9	10	
NZ European	N	259	821	1307	1167	2000	1341	1160	1921	1160	2458	13594
	%	1.9%	6.0%	9.6%	8.6%	14.7%	9.9%	8.5%	14.1%	8.5%	18.1%	100%
Māori	N	1491	1107	825	464	515	288	178	208	119	84	5279
	%	28.2%	21.0%	15.6%	8.8%	9.8%	5.5%	3.4%	3.9%	2.3%	1.6%	100%
Pasifika	N	611	382	240	60	123	63	66	54	20	23	1642
	%	37.2%	23.3%	14.6%	3.7%	7.5%	3.8%	4.0%	3.3%	1.2%	1.4%	100%
Asian	N	21	66	48	77	210	84	53	134	58	137	888
	%	2.4%	7.4%	5.4%	8.7%	23.6%	9.5%	6.0%	15.1%	6.5%	15.4%	100%
Other	N	44	130	64	65	238	72	58	96	47	100	914
	%	4.8%	14.2%	7.0%	7.1%	26.0%	7.9%	6.3%	10.5%	5.1%	10.9%	100%
Total	N	2426	2506	2484	1833	3086	1848	1515	2413	1404	2802	22317
	%	10.9%	11.2%	11.1%	8.2%	13.8%	8.3%	6.8%	10.8%	6.3%	12.6%	100%

Table 2-1: Numbers of students by ethnicity and decile

Gender		Region								Total
		Auckland	Canterbury	Massey	Northland	Otago	Southland	Waikato	Wellington	
Boys	N	2595	2520	1222	802	578	366	2138	825	11046
	%	23.5%	22.8%	11.1%	7.3%	5.2%	3.3%	19.4%	7.5%	100%
Girls	N	2453	2646	1283	764	612	356	2215	942	11271
	N	21.8%	23.5%	11.4%	6.8%	5.4%	3.2%	19.7%	8.4%	100%
Total	N	5048	5166	2505	1566	1190	722	4353	1767	22317
	%	22.6%	23.1%	11.2%	7.0%	5.3%	3.2%	19.5%	7.9%	100%

Table 2-2: Numbers of students by gender and region

Ethnicity		Region								Total
		Auckland	Canterbury	Massey	Northland	Otago	Southland	Waikato	Wellington	
NZ European	N	1995	4266	1548	531	1009	601	2611	1033	13594
	% within region	39.5%	82.6%	61.8%	33.9%	84.8%	83.2%	60.0%	58.5%	60.9%
Māori	N	920	488	851	969	116	95	1426	414	5279
	% within region	18.2%	9.4%	34.0%	61.9%	9.7%	13.2%	32.8%	23.4%	23.7%
Pasifika	N	1173	139	40	29	28	4	76	153	1642
	% within region	23.2%	2.7%	1.6%	1.9%	2.4%	.6%	1.7%	8.7%	7.4%
Asian	N	466	165	20	9	18	13	100	97	888
	% within region	9.2%	3.2%	.8%	.6%	1.5%	1.8%	2.3%	5.5%	4.0%
Other	N	494	108	46	28	19	9	140	70	914
	% within region	9.8%	2.1%	1.8%	1.8%	1.6%	1.2%	3.2%	4.0%	4.1%
Total	N	5048	5166	2505	1566	1190	722	4353	1767	22317
	% within region	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table 2-3: Numbers of students by ethnicity and region

Chapter 3: Patterns of Achievement within Year Groups

Overview of the Findings

For the third year running, the majority of students improved during their participation in the Advanced Numeracy Project. The patterns of achievement within year groups in 2003 confirm previous years' results (see Higgins, 2002 and 2003; Thomas & Ward, 2001 and 2002; Thomas, Tagg & Ward, 2003).

The focus on developing efficient strategies for solving problems is a key part of the Numeracy Project. The stages in the operational domains map increasing sophistication in the problem-solving strategies used. The most important shift is between counting-based thinking and part-whole thinking. Part-whole thinking allows students to develop more sophisticated solutions when problem-solving. The results are presented for each operational domain (addition and subtraction, multiplication and division, and proportions and ratios). Comparisons are drawn between year levels, previous years' results and the three operational domains. Both pre- and post-instruction and patterns of improvement are used as the basis for these comparisons. Pre- and post-instruction tracks progress through the change in the percentage of students at particular stages before and after the instructional period irrespective of their starting point, while patterns of development tracks achievement against the students' starting points.

It is also important to examine the patterns of development in aspects of knowledge because these underpin the improvement of students' strategic thinking. The knowledge and strategy relationship requires students to make progress in both domains. The knowledge aspects discussed are Forward Number Word Sequence (FNWS), Backward Number Word Sequence (BNWS), Number Identification, Grouping, Fractions, and Decimals and Percentages.

Addition and Subtraction

Across all year levels, as for previous years, the figures show that the results in addition and subtraction were slightly better than those in the other operational domains.

The majority of students in Year 4 continue to be at the advanced counting stage in addition and subtraction prior to instruction (see Table 3-1). This means that the students' problem-solving strategies are based on various forms of counting. The percentage of students using part-whole strategies prior to instruction was 36% and by the end of the project the percentage had increased to 62%. These percentages were almost identical to those in 2002.

The achievement patterns of Year 5 students, when tracked over the course of the project, are similar to those of 2002. The results in addition and subtraction were slightly better than the other operational domains, as is reported for Year 4 (see Tables 3-2 and 3-3). About the same number of students initially used counting-based strategies (47%) as those who used part-whole strategies (52%). By the end of the project, the percentage of those using part-whole strategies had increased to 75% in addition and subtraction. This was 74% for 2002. These results are markedly better than those of Year 4 students, of whom 36% initially used part-whole strategies, which increased to 62% after instruction (see Table 3-1).

The majority of Year 6 students were at the advanced counting stage or above prior to instruction, with the results in addition and subtraction being slightly better than in the other operational domains. Just under half (48%) of the students were at the early additive stage after instruction and just over a third (35%) were at the advanced counting stage. Nearly two-thirds of the students (63%) were using part-whole strategies before instruction. These figures are similar to those in 2002. This rose to 83% after instruction, compared to 75% for Year 5 and 62% for Year 4 students.

Stage	Initial		Final	
Emergent	1%	(69)	0%	(29)
One-to-one counting	1%	(114)	0%	(30)
Counting from one on materials	8%	(630)	2%	(158)
Counting from one by imaging	5%	(416)	3%	(240)
Advanced counting	49%	(3958)	32%	(2623)
Early additive part-whole	32%	(2630)	50%	(4028)
Advanced additive part-whole	4%	(298)	12%	(1007)
Total	100% [1]	(8115)[2]	100%	(8115)

Table 3-1: Year 4: Addition and subtraction – students' initial and final assessment

Stage	Initial		Final	
Emergent	1%	(64)	0%	(27)
One-to-one counting	1%	(63)	0%	(19)
Counting from one on materials	3%	(191)	1%	(57)
Counting from one by imaging	2%	(174)	1%	(83)
Advanced counting	40%	(2840)	23%	(1587)
Early additive part-whole	44%	(3119)	53%	(3716)
Advanced additive part-whole	8%	(547)	22%	(1509)
Total	100%	(6998)	100%	(6998)

Table 3-2: Year 5: Addition and subtraction – students' initial and final assessment

Stage	Initial		Final	
Emergent	1%	(58)	0%	(27)
One-to-one counting	1%	(38)	0%	(5)
Counting from one on materials	2%	(116)	1%	(52)
Counting from one by imaging	1%	(74)	1%	(51)
Advanced counting	34%	(2444)	16%	(1115)
Early additive part-whole	47%	(3356)	48%	(3453)
Advanced additive part-whole	16%	(1118)	35%	(2501)
Total	100%	(7204)	100%	(7204)

Table 3-3: Year 6: Addition and subtraction – students' initial and final assessment

In contrast to the previous set of tables, which looked at the percentage of students prior to and after instruction irrespective of their starting point, a different way of looking at patterns of achievement for specific year groups of students is to track achievement against the students' starting point. It can be observed that the patterns for each year group in addition and subtraction are similar to those in previous years, with the number of students remaining at the same stage at the end of the project increasing with each advancing stage.

Six percent of Year 4 students (see Table 3-4) who were emergent at the start of the project were still emergent at the end. This was an improvement from 19% remaining at emergent in

[1] Due to rounding, not all percentages add to 100.
[2] Any variations in the total are the result of minor errors in data entry.

2002. The 25% moving to advanced counting and 23% moving to early additive (a move of at least four stages) and a smaller number (4%) moving to advanced additive (five stages) were similar to 2002 figures. As the stages of sophistication in student thinking increased, so the percentage of students remaining at a particular stage increased. As for 2002, 23% of the 2003 students shifted from early additive part-whole to advanced additive part-whole.

Year 5 results followed a similar pattern to Year 4 (see Table 3-5). In addition and subtraction, 9% of Year 5 students who were emergent at the start of the project were still emergent at the end, compared to 12% in 2002. A higher percentage (33%) than 2002 (22%) moved to advanced counting, while those moving to early additive (a move of at least four stages) increased from 26% in 2002 to 45% in 2003. Fewer students (5%) moved from emergent to advanced additive (five stages) than in 2002, when 19% made this move. The important shift is from advanced counting to early additive part-whole, and 49% made this shift. Fewer students (27%) moved between early and advanced additive part-whole. These figures are similar to those in 2002.

A similar percentage of Year 6 students (14%; see Table 3-6) in 2002 and 2003 who were emergent at the start of the project were still emergent at the end. However, a higher percentage of students in 2003 (35%) moved to advanced counting than in 2002 (18%). The percentage moving from emergent to early additive (a move of at least four stages; 31%) and to advanced additive (five stages; 17%) was similar to 2002 figures. The percentage of students moving from advanced counting to early additive part-whole (53%) and between early and advanced additive part-whole (37%) was greater in Year 6 than in Years 4 and 5 (see above). This pattern confirmed that reported in 2002.

	Initial													
Final	**Emergent**		**One-to-one counting**		**Counting from one on materials**		**Counting from one by imaging**		**Advanced counting**		**Early additive part-whole**		**Advanced additive part-whole**	
Emergent	6%	(4)	1%	(1)	0%	(2)	1%	(4)	0%	(14)	0%	(3)	0%	(1)
One-to-one counting	16%	(11)	13%	(15)	0%	(2)	0%	(2)	0%	(0)	0%	(0)	0%	(0)
Counting from one on materials	20%	(14)	26%	(30)	18%	(111)	0%	(2)	0%	(0)	0%	(1)	0%	(0)
Counting from one by imaging	6%	(4)	19%	(21)	22%	(135)	16%	(65)	0%	(15)	0%	(0)	0%	(0)
Advanced counting	25%	(17)	33%	(38)	50%	(316)	62%	(257)	49%	(1929)	3%	(66)	0%	(0)
Early additive part-whole	23%	(16)	8%	(9)	10%	(63)	21%	(86)	48%	(1898)	74%	(1947)	3%	(9)
Advanced additive part-whole	4%	(3)	0%	(0)	0%	(1)	0%	(0)	3%	(102)	23%	(613)	97%	(288)
Total	100%	(69)	100%	(114)	100%	(630)	100%	(416)	100%	(3958)	100%	(2630)	100%	(298)

Table 3-4: Year 4: Patterns of improvement through the stages in addition and subtraction

Final	Initial						
	Emergent	**One-to-one counting**	**Counting from one on materials**	**Counting from one by imaging**	**Advanced counting**	**Early additive part-whole**	**Advanced additive part-whole**
Emergent	9% (6)	2% (1)	0% (0)	1% (1)	0% (8)	0% (10)	0% (1)
One-to-one counting	2% (1)	29% (18)	0% (0)	0% (0)	0% (0)	0% (0)	0% (0)
Counting from one on materials	2% (1)	21% (13)	20% (39)	1% (2)	0% (2)	0% (0)	0% (0)
Counting from one by imaging	5% (3)	9% (6)	16% (31)	19% (33)	0% (9)	0% (1)	0% (0)
Advanced counting	33% (21)	29% (18)	55% (104)	58% (100)	46% (1292)	2% (52)	0% (0)
Early additive part-whole	45% (29)	11% (7)	9% (17)	21% (37)	49% (1397)	71% (2220)	2% (9)
Advanced additive part-whole	5% (3)	0% (0)	0% (0)	1% (1)	5% (132)	27% (836)	98% (537)
Total	100% (64)	100% (63)	100% (191)	100% (174)	100% (2840)	100% (3119)	100% (547)

Table 3-5: Year 5: Patterns of improvement through the stages in addition and subtraction

Final	Initial						
	Emergent	**One-to-one counting**	**Counting from one on materials**	**Counting from one by imaging**	**Advanced counting**	**Early additive part-whole**	**Advanced additive part-whole**
Emergent	14% (8)	3% (1)	2% (2)	1% (1)	0% (7)	0% (7)	0% (1)
One-to-one counting	0% (0)	13% (5)	0% (0)	0% (0)	0% (0)	0% (0)	0% (0)
Counting from one on materials	3% (2)	24% (9)	34% (39)	0% (0)	0% (2)	0% (0)	0% (0)
Counting from one by imaging	0% (0)	18% (7)	16% (19)	20% (15)	0% (9)	0% (1)	0% (0)
Advanced counting	35% (20)	32% (12)	31% (36)	51% (38)	39% (953)	2% (53)	0% (3)
Early additive part-whole	31% (18)	11% (4)	17% (20)	24% (18)	53% (1306)	62% (2064)	2% (23)
Advanced additive part-whole	17% (10)	0% (0)	0% (0)	3% (2)	7% (167)	37% (1231)	98% (1091)
Total	100% (58)	100% (38)	100% (116)	100% (74)	100% (2444)	100% (3356)	100% (1118)

Table 3-6: Year 6: Patterns of improvement through the stages in addition and subtraction

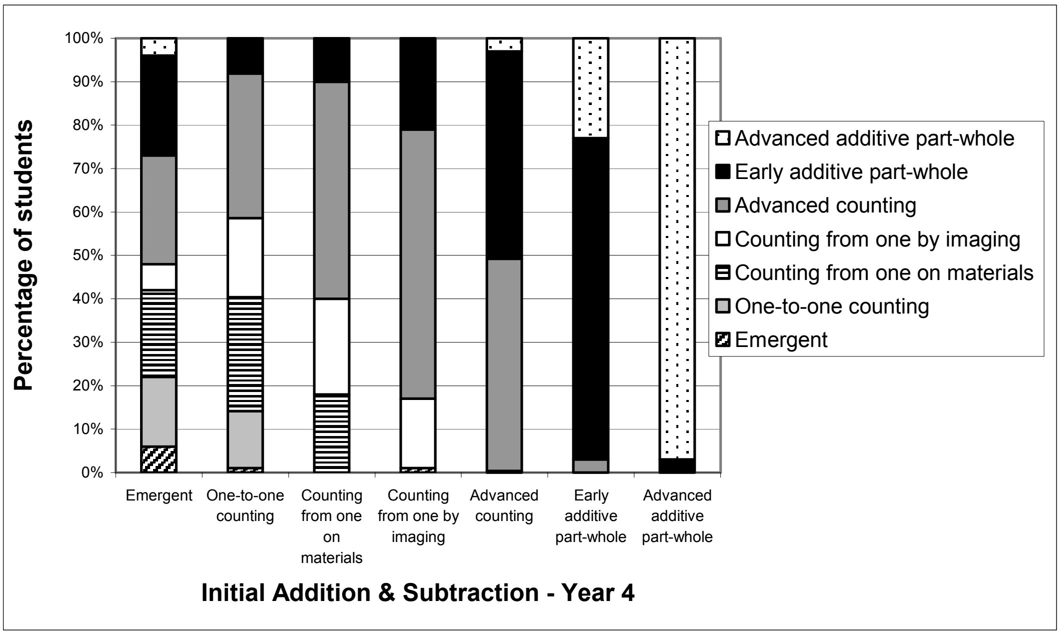

Figure 3-1: Year 4: Patterns of improvement through the stages in addition and subtraction

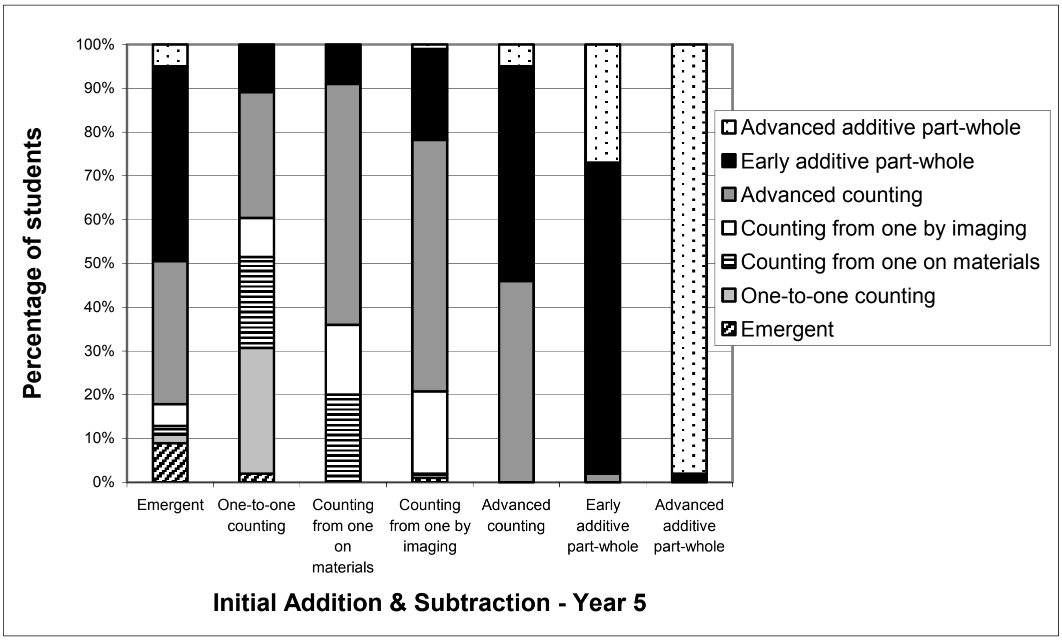

Figure 3-2: Year 5: Patterns of improvement through the stages in addition and subtraction

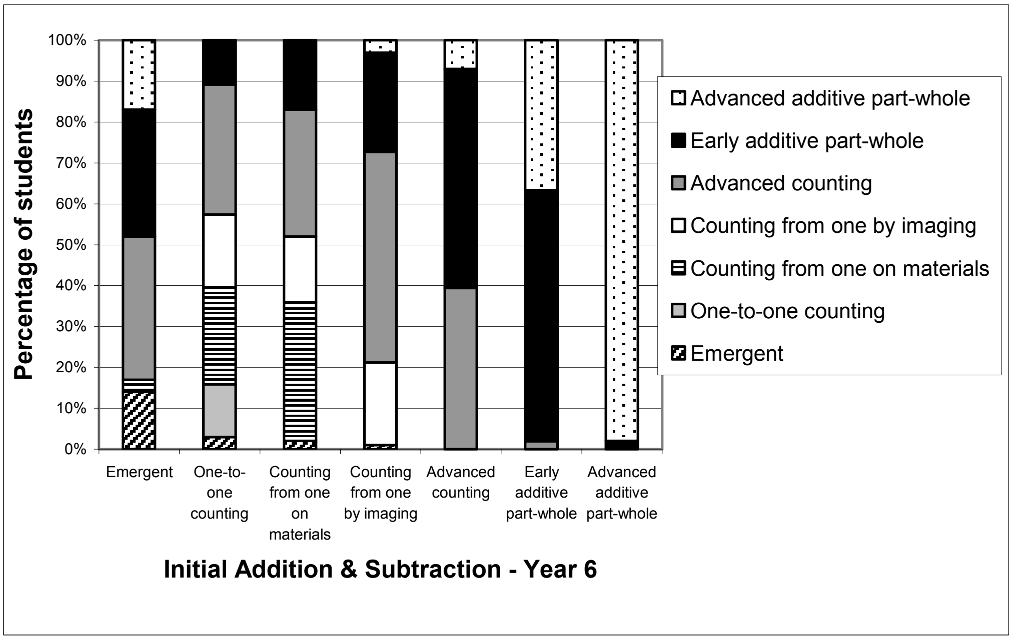

Figure 3-3: Year 6: Patterns of improvement through the stages in addition and subtraction

Another way of looking at patterns of improvement is to look across the age groups (see Table 3-7) using initial and final interview data. For instance, for Year 4 students the greatest shift was between counting-based stages and part-whole stages. The number of students at the counting-based stages decreased from 64% to 37% (a reduction of 27 percentage points). At Year 5 there was a 22% reduction of those who remained at the counting-based stages. The students in Year 6 made a similar decrease at the counting-based stages from 39% to 18% (a reduction of 21 percentage points).

At the part-whole stages, Year 4 students at early additive part-whole increased from 32% to 50% (an increase of 18 percentage points) and at advanced additive part-whole the number increased from 4% to 12% (an increase of 8 percentage points). In Year 5 the increase at early additive part-whole was 9 percentage points, with the larger increase being at advanced additive part-whole (14 percentage points). Compared to the younger students the gains for Year 6 students were less at the early additive part-whole stage (being 47% to 48%) but higher at the advanced additive part-whole stage (being a gain of 19 percentage points, from 16% to 35%). All the above figures were similar to those reported for 2002.

Stage	Year 4 Initial	Year 4 Final	Year 5 Initial	Year 5 Final	Year 6 Initial	Year 6 Final
Emergent	1%	0%	1%	0%	1%	0%
One-to-one counting	1%	0%	1%	0%	1%	0%
Counting from one on materials	8%	2%	3%	1%	2%	1%
Counting from one by imaging	5%	3%	2%	1%	1%	1%
Advanced counting	49%	32%	40%	23%	34%	16%
Early additive part-whole	32%	50%	44%	53%	47%	48%
Advanced additive part-whole	4%	12%	8%	22%	16%	35%
Total	100%	100%	100%	100%	100%	100%

Table 3-7: Addition and subtraction – patterns of improvement by year

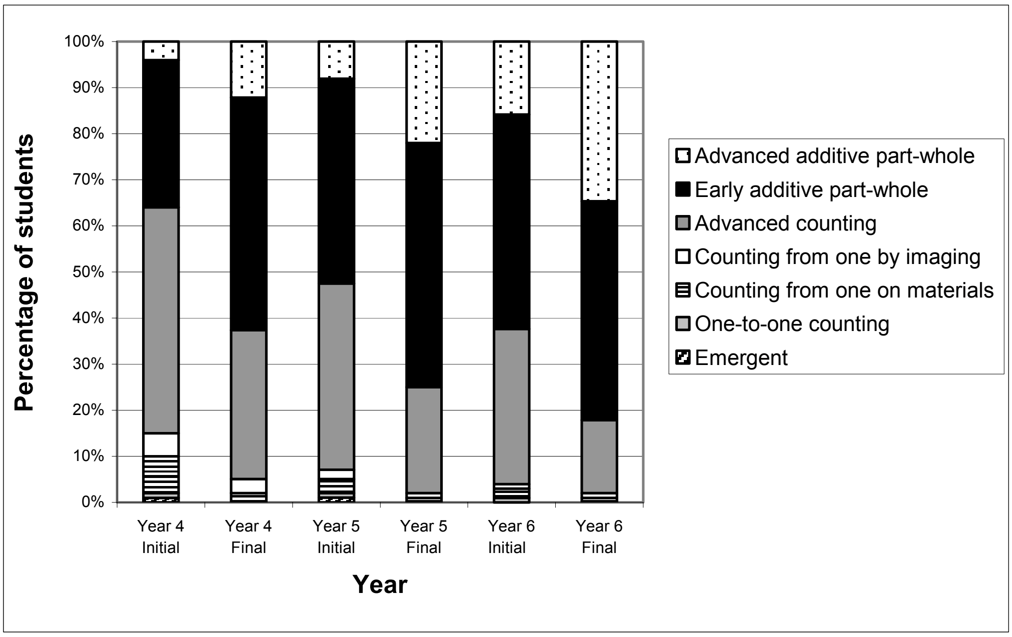

Figure 3-4: Addition and subtraction – patterns of improvement by year

Multiplication and Division

The patterns of results for multiplication and division were similar to those in 2002. The percentages of students moving to more advanced stages increased across the year levels and were greater than the figures for proportions and ratios but less than the figures for addition and subtraction.

In Year 4 the percentages for multiplication and division using counting-based strategies prior to instruction in 2003 were identical to those in 2002, with only 22% using part-whole strategies. As for 2002 the percentage using part-whole strategies had increased to 50% after instruction (see Table 3-8).

Table 3-9 shows that in multiplication and division, the majority of students in Year 5, as for Year 4, were at the advanced counting stage at the start of the project. Those 'not assessed' (8%) were about half as many as in Year 4, which possibly indicates a different pattern in teachers' choice of the form used in the diagnostic test. These patterns were the same in 2002. Initially only 39% were using part-whole strategies for multiplication and division. This percentage increased to 69% after instruction. The figures in 2002 were similar.

Table 3-10 (page 16) shows that just over half (59%) of the students in Year 6 in multiplication and division were using part-whole thinking prior to instruction. The percentage using part-whole strategies increased to 81%, which is similar to the 2002 figures.

Stage	Initial		Final	
Not assessed	18%	(1472)	7%	(562)
Counting from one by imaging	18%	(1489)	6%	(494)
Advanced counting	42%	(3368)	37%	(2993)
Early additive part-whole	15%	(1228)	27%	(2200)
Advanced additive part-whole	6%	(513)	20%	(1636)
Advanced multiplicative part-whole	1%	(45)	3%	(230)
Total	100%	(8115)	100%	(8115)

Table 3-8: Year 4: Multiplication and division – students' initial and final assessment

Stage	Initial		Final	
Not assessed	8%	(591)	4%	(263)
Counting from one by imaging	11%	(754)	3%	(245)
Advanced counting	39%	(2705)	24%	(1654)
Early additive part-whole	22%	(1783)	29%	(2054)
Advanced additive part-whole	15%	(1043)	31%	(2151)
Advanced multiplicative part-whole	2%	(122)	9%	(631)
Total	100%	(6998)	100%	(6998)

Table 3-9: Year 5: Multiplication and division – students' initial and final assessment

Stage	Initial		Final	
Not assessed	5%	(371)	2%	(163)
Counting from one by imaging	7%	(531)	3%	(147)
Advanced counting	29%	(2087)	15%	(1102)
Early additive part-whole	29%	(2079)	25%	(1763)
Advanced additive part-whole	25%	(1796)	38%	(2750)
Advanced multiplicative part-whole	5%	(340)	18%	(1279)
Total	100%	7204)	100%	(7204)

Table 3-10: Year 6: Multiplication and division – students' initial and final assessment

The shifts in students' thinking through the patterns of achievement confirm the shifts seen in the previous year (refer to Tables 3-11, 3-12 and 3-13). In multiplication and division, 18% of Year 4 students at counting from one by imaging remained at this stage after instruction. This compares with 22% for 2002. The larger shifts from counting from one by imaging to advanced counting by the end of the project (a move of one stage) showed an identical percentage of 58% for 2002 and 2003. The percentage that moved from advanced counting to early additive part-whole was 37%, while 17% moved to advanced additive part-whole.

An increased number of Year 5 students (21%) remained at the stage of counting from one by imaging after instruction compared to 11% in 2002. As for 2002 the larger shifts were 51% who moved from counting from one by imaging to advanced counting by the end of the project (a move of one stage) and 39% who moved from advanced counting to early additive part-whole. All these percentages are similar to those for Year 4 students (see above). The percentages for Year 5 students shifting from early additive part-whole to advanced additive part-whole (48%) and from advanced additive part-whole to advanced multiplicative part-whole (31%) are similar to the 2002 figures and greater than for Year 4 students (46% and 21% respectively) as discussed above.

In Year 6 the percentage remaining at the stage of counting from one by imaging after instruction (18%) was similar to previous years' results of 17% in 2001 and 21% in 2002. As with 2002 the larger shifts in student achievement were the 46% who moved from counting from one by imaging to advanced counting by the end of the project (a move of one stage) and the 39% who moved from advanced counting to early additive part-whole. These percentages are similar to those for younger students. Again as with Year 5 students, the percentages are higher for those shifting from early additive part-whole to advanced additive part-whole (53%) and from advanced additive part-whole to advanced multiplicative part-whole (38%), confirming the pattern of 2002.

Final	Initial											
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole	
Not assessed	36%	(528)	1%	(8)	1%	(20)	0%	(2)	1%	(4)	0%	(0)
Counting from one by imaging	14%	(201)	18%	(272)	1%	(19)	0%	(2)	0%	(0)	0%	(0)
Advanced counting	38%	(566)	58%	(867)	44%	(1495)	5%	(61)	1%	(4)	0%	(0)
Early additive part-whole	10%	(142)	17%	(254)	37%	(1244)	44%	(542)	4%	18)	0%	(0)
Advanced additive part-whole	2%	(33)	6%	(84)	17%	(572)	46%	(569)	73%	(377)	2%	(1)
Advanced multiplicative part-whole	0%	(2)	0%	(4)	0%	(18)	4%	(52)	21%	(110)	98%	(44)
Total	100%	(1472)	100%	(1489)	100%	(3368)	100%	(1228)	100%	(513)	100%	(45)

Table 3-11: Year 4: Patterns of improvement through the stages in multiplication and division

Final	Initial											
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole	
Not assessed	40%	(235)	0%	(4)	0%	(10)	0%	(6)	1%	(8)	0%	(0)
Counting from one by imaging	12%	(71)	21%	(161)	0%	(10)	0%	(3)	0%	(0)	0%	(0)
Advanced counting	29%	(169)	51%	(381)	40%	(1071)	2%	(32)	0%	(0)	0%	(0)
Early additive part-whole	13%	(75)	20%	(147)	39%	(1047)	42%	(755)	3%	(29)	1%	(1)
Advanced additive part-whole	6%	(38)	8%	(60)	19%	(522)	48%	(852)	65%	(678)	1%	(1)
Advanced multiplicative part-whole	0%	(3)	0%	(1)	2%	(45)	8%	(135)	31%	(327)	98%	(120)
Total	100%	(591)	100%	(754)	100%	(2705)	100%	(1783)	100%	(1043)	100%	(122)

Table 3-12: Year 5: Patterns of improvement through the stages in multiplication and division

Final	Initial											
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole	
Not assessed	39%	(145)	0%	(1)	0%	(8)	0%	(7)	0%	(2)	0%	(0)
Counting from one by imaging	9%	(34)	18%	(97)	1%	(15)	0%	(1)	0%	(0)	0%	(0)
Advanced counting	29%	(107)	46%	(246)	35%	(728)	1%	(19)	0%	(2)	0%	(0)
Early additive part-whole	12%	(44)	24%	(125)	39%	(817)	36%	(745)	2%	(32)	0%	(0)
Advanced additive part-whole	10%	(37)	2%	(62)	23%	(469)	53%	(1094)	60%	(1078)	3%	(10)
Advanced multiplicative part-whole	1%	(4)	0%	(0)	2%	(50)	10%	(213)	38%	(682)	97%	(330)
Total	100%	(371)	100%	(531)	100%	(2087)	100%	(2079)	100%	(1796)	100%	(340)

Table 3-13: Year 6: Patterns of improvement through the stages in multiplication and division

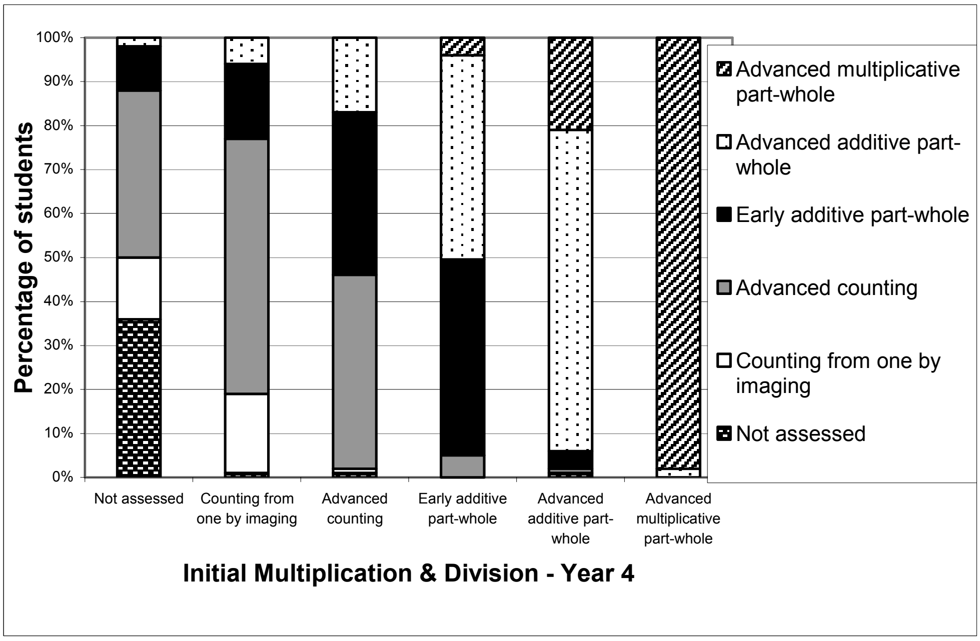

Figure 3-5: Year 4: Patterns of improvement through the stages in multiplication and division

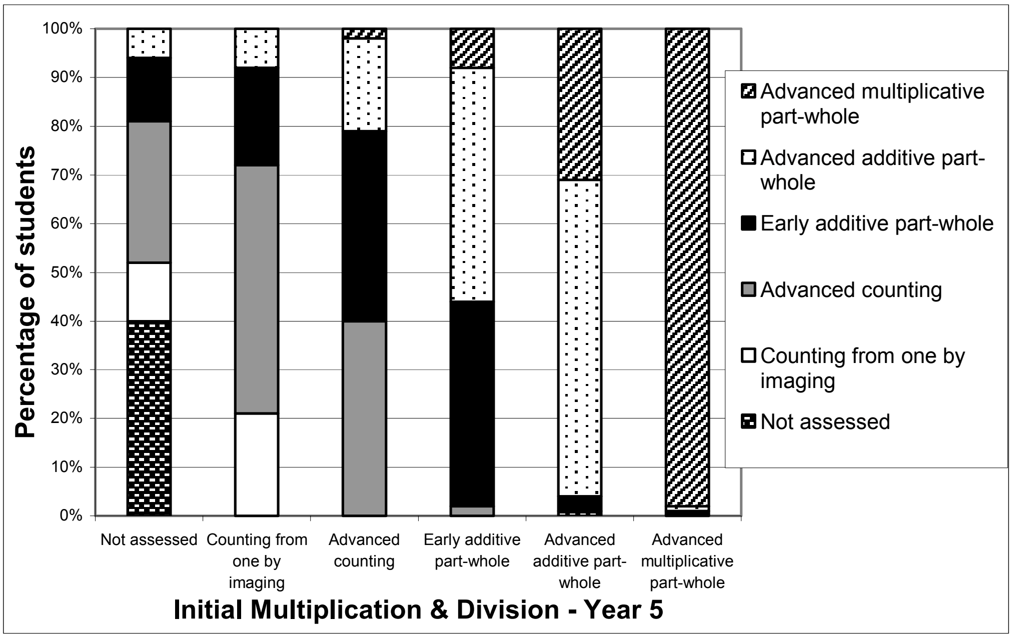

Figure 3-6: Year 5: Patterns of improvement through the stages in multiplication and division

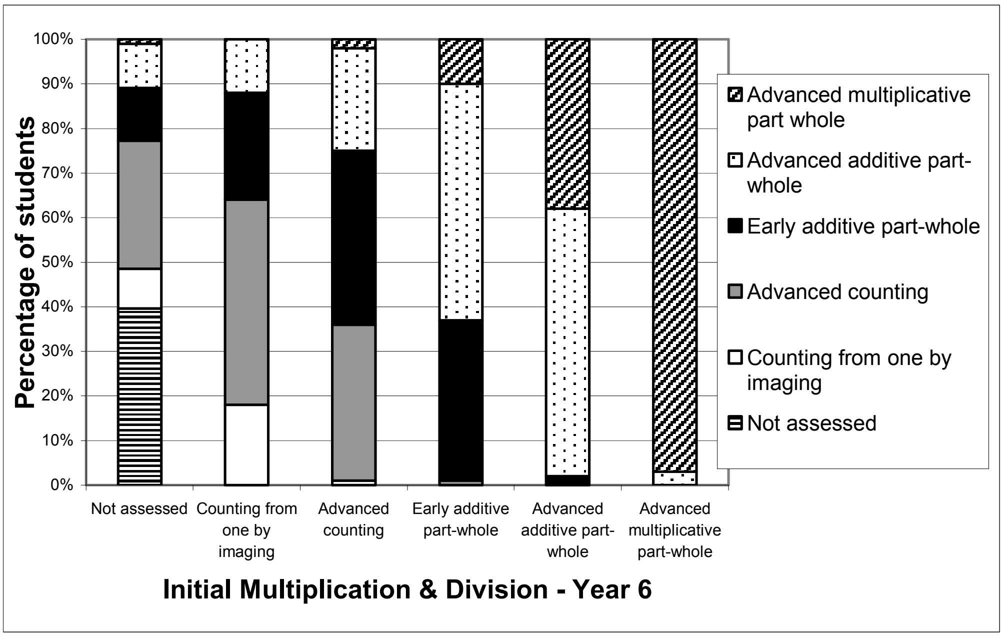

Figure 3-7: Year 6: Patterns of improvement through the stages in multiplication and division

Table 3-14 shows the patterns of improvement across the age groups, using initial and final interview data. The greatest shift for Year 4 students was between counting-based stages and part-whole stages. The number of students at the counting-based stages decreased from 78% to 50% (a reduction of 28 percentage points). At Year 5 there was a 27% reduction of those who remained at the counting-based stages. Those students in Year 6 made a similar decrease at the counting-based stages from 41% to 20% (a reduction of 21 percentage points).

At the part-whole stages, Year 4 students at early additive part-whole increased from 15% to 27% (an increase of 12 percentage points). At advanced additive part-whole the number increased from 6% to 20% (an increase of 14 percentage points) and at advanced multiplicative part-whole from 1% to 3%. In Year 5 the increase at early additive part-whole was 7%, with the larger increase being at advanced additive part-whole (16%). At advanced multiplicative part-whole the increase was smaller (from 2% to 9%). Compared to the younger students there was a decreased number of Year 6 students at the early additive part-whole stage (29% to 25%), but larger increases at the more advanced stages of advanced additive part-whole stage (13 percentage points) and at the advanced multiplicative part-whole stage (13 percentage points).

Stage	Year 4 Initial	Year 4 Final	Year 5 Initial	Year 5 Final	Year 6 Initial	Year 6 Final
Not assessed	18%	7%	8%	4%	5%	2%
Counting from one by imaging	18%	6%	11%	3%	7%	3%
Advanced counting	42%	37%	39%	24%	29%	15%
Early additive part-whole	15%	27%	22%	29%	29%	25%
Advanced additive part-whole	6%	20%	15%	31%	25	38%
Advanced multiplicative part-whole	1%	3%	2%	9%	5%	18%
Total	100%	100%	100%	100%	100%	100%

Table 3-14: Multiplication and division - patterns of improvement by year

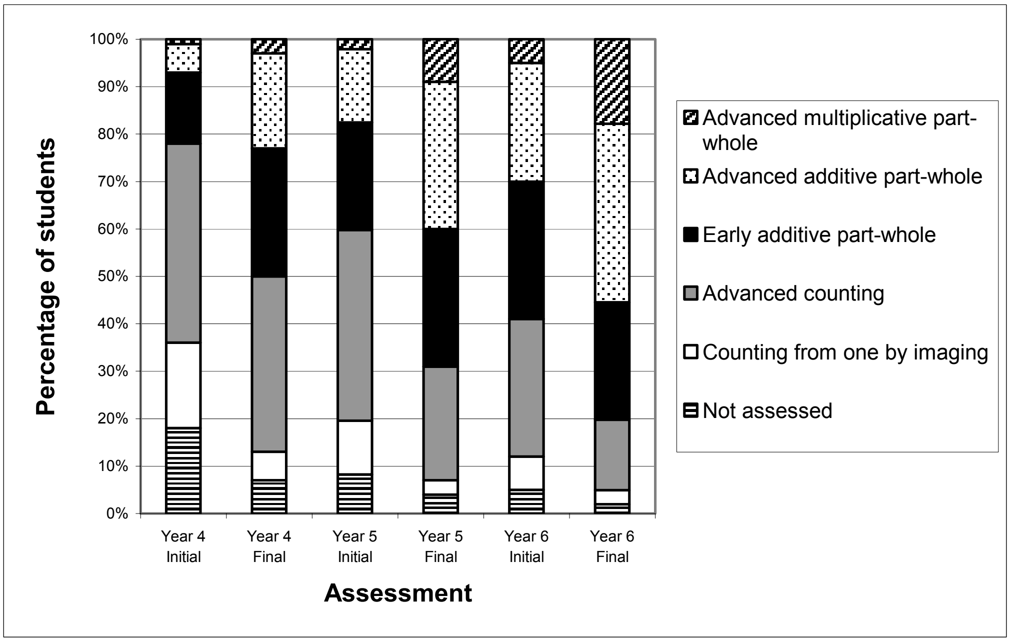

Figure 3-8: Multiplication and division - patterns of improvement by year

Proportions and Ratios

The pattern of achievement for proportions and ratios followed the same pattern as 2002, with students starting at a lower stage than for the other domains. In Year 4 most students were either using counting-based strategies (65%) or were not assessed (19%). The figures for 2002 were 65% and 21% respectively. The proportion of students with part-whole strategies prior to instruction was 15% compared to 14% in 2002. After instruction the proportion with part-whole strategies increased to 41% compared to 39% in 2002.

In Year 5, the majority of students were either using counting-based strategies (60%) or were not assessed (10%) prior to instruction. This was slightly better than for the Year 4 students. A smaller proportion (30%) had part-whole strategies. This compares to 15% for Year 4 students. By the end of the project the number of students with part-whole strategies had increased to 59%, which is slightly lower than for the other operational domains. In Year 6 just under half (45%) of the students were using part-whole thinking prior to instruction. This increased after instruction to 71%, which is almost the same as in 2002.

Stage	Initial		Final	
Not assessed	19%	(1565)	8%	(626)
Counting from one by imaging	17%	(1411)	4%	(308)
Advanced counting	48%	(3923)	47%	(3853)
Early additive part-whole	12%	(954)	28%	(2232)
Advanced additive part-whole	3%	(227)	11%	(897)
Advanced multiplicative part-whole	0%	(34)	2%	(185)
Advanced proportional part-whole	0%	(1)	0%	(14)
Total	100%	(8115)	100%	(8115)

Table 3-15: Year 4: Proportions and ratios – students' initial and final assessment

Stage	Initial		Final	
Not assessed	10%	(712)	6%	(394)
Counting from one by imaging	12%	(835)	2%	(162)
Advanced counting	48%	(3348)	33%	(2297)
Early additive part-whole	21%	(1465)	31%	(2157)
Advanced additive part-whole	7%	(528)	20%	(1421)
Advanced multiplicative part-whole	2%	(110)	7%	(506)
Advanced proportional part-whole	0%	(9)	1%	(61)
Total	100%	(6998)	100%	(6998)

Table 3-16: Year 5: Proportions and ratios – students' initial and final assessment

Stage	Initial		Final	
Not assessed	7%	(511)	4%	(312)
Counting from one by imaging	8%	(576)	1%	(104)
Advanced counting	40%	(2865)	23%	(1645)
Early additive part-whole	26%	(1864)	27%	(1970)
Advanced additive part-whole	14%	(1038)	27%	(1951)
Advanced multiplicative part-whole	4%	(317)	15%	(1056)
Advanced proportional part-whole	1%	(33)	2%	(166)
Total	100%	(7204)	100%	(7204)

Table 3-17: Year 6: Proportions and ratios – students' initial and final assessment

When students' starting place is taken into account, the pattern of improvement is consistent with that found for addition and subtraction and multiplication and division, in that fewer students shifted with the increasing sophistication of stages. This confirms the 2002 figures. The percentage of Year 4 students remaining at the counting from one by imaging stage at the end of the project was 12% (down from 18% in 2002). Again the larger shifts were from counting from one by imaging to advanced counting (65%), from advanced counting to early additive part-whole (32%) and from early additive part-whole to advanced additive part-whole (30%).

In Year 5, 14% of students remained at the counting from one by imaging stage at the end of the project. Again the larger shifts were from counting from one by imaging to advanced counting (55%), from advanced counting to early additive part-whole (35%) and from early additive part-whole to advanced additive part-whole (37%). The percentage moving from advanced additive part-whole to advanced multiplicative part-whole (34%) and from advanced multiplicative part-whole to advanced proportional part-whole (26%) was similar to

2002. Similarly a small proportion of students (11%) moved back two levels to advanced additive part-whole.

In Year 6, 12% of students remained at the counting from one by imaging stage at the end of the project. Again the larger shifts were from counting from one to advanced counting (51%), from advanced counting to early additive part-whole (35%) and from early additive part-whole to advanced additive part-whole (41%). Compared to younger students a greater percentage moved from advanced additive part-whole to advanced multiplicative part-whole (41%) and from advanced multiplicative part-whole to advanced proportional part-whole (25%). All these percentages are very similar to the 2002 figures.

Final	Initial						
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole	Advanced proportional part-whole
Not assessed	38% (593)	1% (13)	0% (19)	0% (1)	0% (0)	0% (0)	0% (0)
Counting from one by imaging	7% (100)	12% (178)	1% (28)	0% (2)	0% (0)	0% (0)	0% (0)
Advanced counting	45% (708)	65% (915)	56% (2188)	4% (40)	1% (2)	0% (0)	0% (0)
Early additive part-whole	9% (141)	17% (237)	32% (1276)	60% (569)	4% (9)	0% (0)	0% (0)
Advanced additive part-whole	1% (21)	5% (67)	10% (378)	30% (283)	65% (148)	0% (0)	0% (0)
Advanced multiplicative part-whole	0% (2)	0% (1)	1% (34)	6% (58)	28% (64)	77% (26)	0% (0)
Advanced proportional part-whole	0% (0)	0% (0)	0% (0)	0% (1)	2% (4)	23% (8)	100% (1)
Total	100% (1565)	100% (1411)	100% (3923)	100% (954)	100% (227)	100% (34)	100% (1)

Table 3-18: Year 4: Patterns of improvement through the stages in proportions and ratios

Final	Initial						
	Not assessed	Counting from one by imaging	Advanced counting	Early additive part-whole	Advanced additive part-whole	Advanced multiplicative part-whole	Advanced proportional part-whole
Not assessed	51% (363)	0% (4)	0% (13)	1% (8)	1% (6)	0% (0)	0% (0)
Counting from one by imaging	5% (34)	14% (116)	0% (12)	0% (0)	0% (0)	0% (0)	0% (0)
Advanced counting	31% (223)	55% (456)	47% (1588)	2% (28)	0% (2)	0% (0)	0% (0)
Early additive part-whole	9% (64)	22% (187)	35% (1158)	50% (729)	4% (19)	0% (0)	0% (0)
Advanced additive part-whole	4% (28)	8% (65)	5% (484)	37% (539)	57% (302)	2% (2)	11% (1)
Advanced multiplicative part-whole	0% (0)	1% (7)	3% (92)	10% (146)	34% (182)	72% (79)	0% (0)
Advanced proportional part-whole	0% (0)	0% (0)	0% (1)	0% (6)	3% (17)	26% (29)	89% (8)
Total	100% (712)	100% (835)	100% (3348)	100%(1456)	100% (528)	100% (110)	100% (9)

Table 3-19: Year 5: Patterns of improvement through the stages in proportions and ratios

Final	Initial													
	Not assessed		Counting from one by imaging		Advanced counting		Early additive part-whole		Advanced additive part-whole		Advanced multiplicative part-whole		Advanced proportional part-whole	
Not assessed	57%	(291)	1%	(43)	0%	(11)	0%	(7)	0%	(0)	0%	(0)	0%	(0)
Counting from one by imaging	4%	(18)	12%	(71)	1%	(13)	0%	(2)	0%	(0)	0%	(0)	0%	(0)
Advanced counting	27%	(137)	51%	(295)	41%	(1175)	2%	(36)	0%	(2)	0%	(0)	0%	(0)
Early additive part-whole	8%	(41)	23%	(131)	35%	(1007)	42%	(775)	2%	(16)	0%	(0)	0%	(0)
Advanced additive part-whole	3%	(15)	12%	(68)	19%	(546)	41%	(761)	53%	(554)	2%	(7)	0%	(0)
Advanced multiplicative part-whole	2%	(8)	1%	(8)	4%	(108)	15%	(271)	41%	(429)	73%	(230)	6%	(2)
Advanced proportional part-whole	0%	(1)	0%	(0)	0%	(5)	1%	(12)	4%	(37)	25%	(80)	94%	(31)
Total	100%	(511)	100%	(576)	100%	(2865)	100%	(1864)	100%	(1038)	100%	(317)	100%	(33)

Table 3-20: Year 6: Patterns of improvement through the stages in proportions and ratios

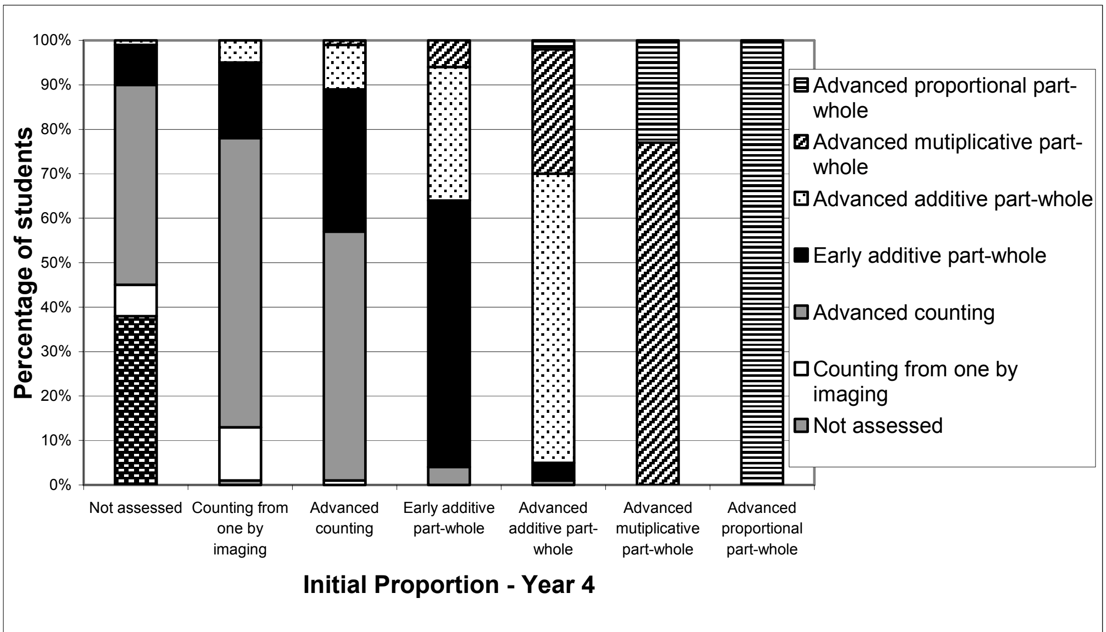

Figure 3-9: Year 4: Patterns of improvement through the stages in proportions and ratios

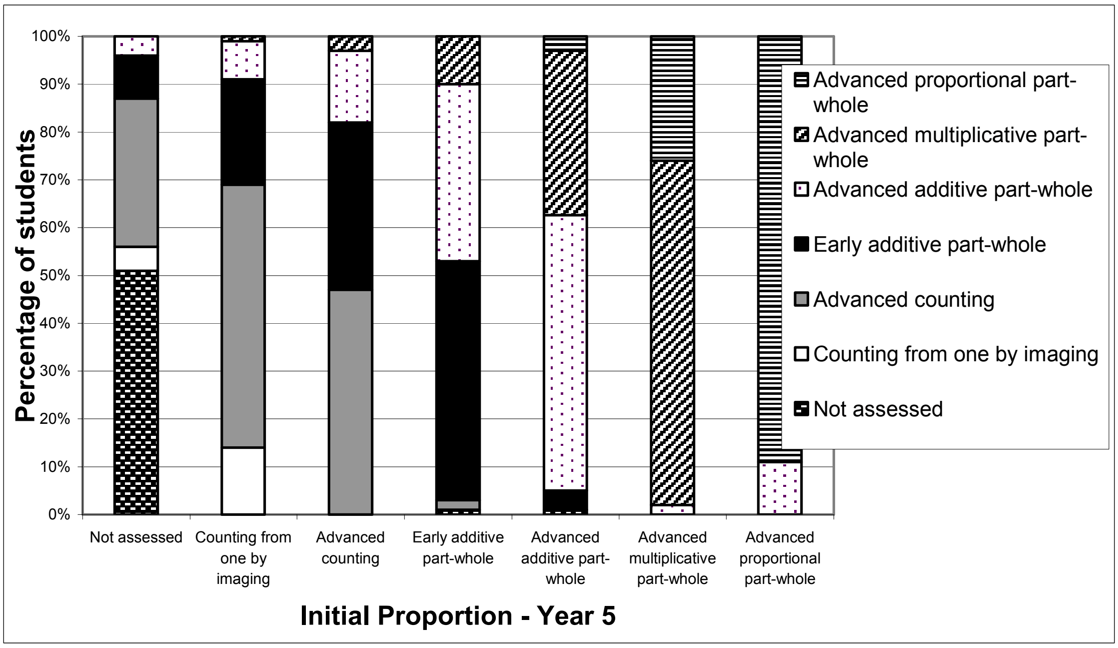

Figure 3-10: Year 5: Patterns of improvement through the stages in proportions and ratios

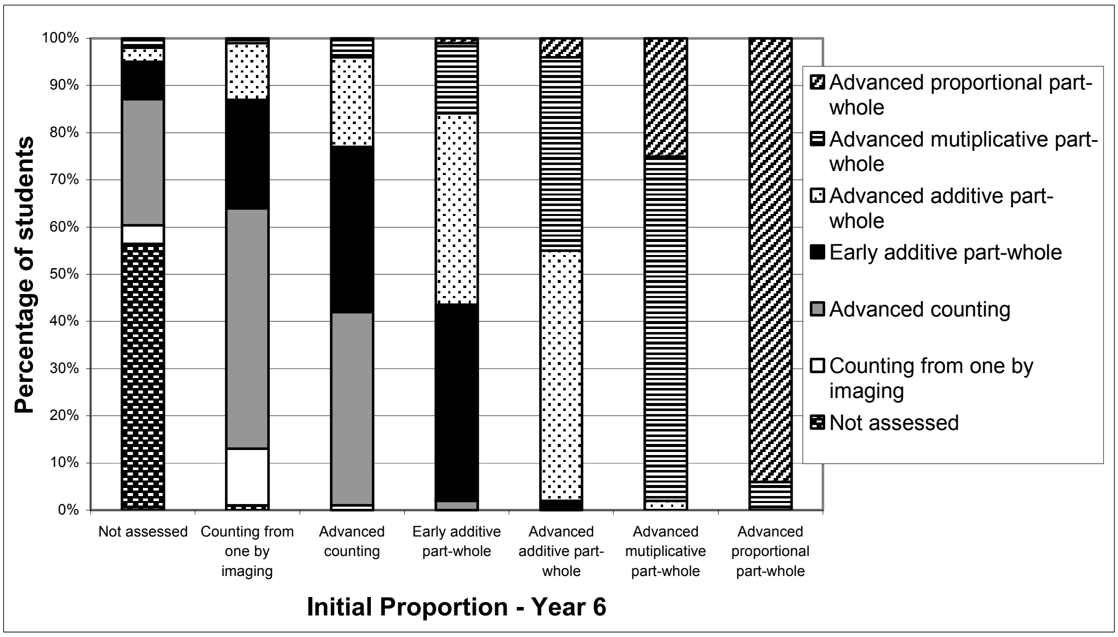

Figure 3-11: Year 6: Patterns of improvement through the stages in proportions and ratios

Table 3-21 shows the patterns of improvement across the age groups, using initial and final interview data. As for the other operational domains the greatest shift for Year 4 students was between counting-based stages and part-whole stages. The number of students at the counting-based stages decreased from 84% to 59% (a reduction of 25 percentage points). At Year 5 there was a 29 percentage-point reduction of those who remained at the counting-based stages. The percentage of students in Year 6 at the counting-based stages similarly decreased from 55% to 28% (a reduction of 27 percentage points).

At the part-whole stages, Year 4 students at early additive part-whole increased from 12% to 28% (an increase of 16 percentage points). The gains (7%) for the more advanced part-whole stages were slight. In Year 5 the increase at early additive part-whole was 10% and at advanced additive part-whole 13%. At advanced multiplicative part-whole the increase was smaller (from 2% to 7%). Compared to the younger students a similar number of Year 6 students were at the early additive part-whole stage, but there were larger increases at the more advanced stages of advanced additive part-whole (13%) and at advanced multiplicative part-whole (11%).

Stage	Year 4 Initial	Year 4 Final	Year 5 Initial	Year 5 Final	Year 6 Initial	Year 6 Final
Not assessed	19%	8%	10%	6%	7%	4%
Counting from one by imaging	17%	4%	12%	2%	8%	1%
Advanced counting	48%	47%	48%	33%	40%	23%
Early additive part-whole	12%	28%	21%	31%	26%	27%
Advanced additive part-whole	3%	11%	7%	20%	14%	27%
Advanced multiplicative part-whole	0%	2%	2%	7%	4%	15%
Advanced proportional part-whole	0%	0%	0%	1%	1%	2%
Total	100%	100%	100%	100%	100%	100%

Table 3-21: Proportions and ratios - patterns of improvement by year

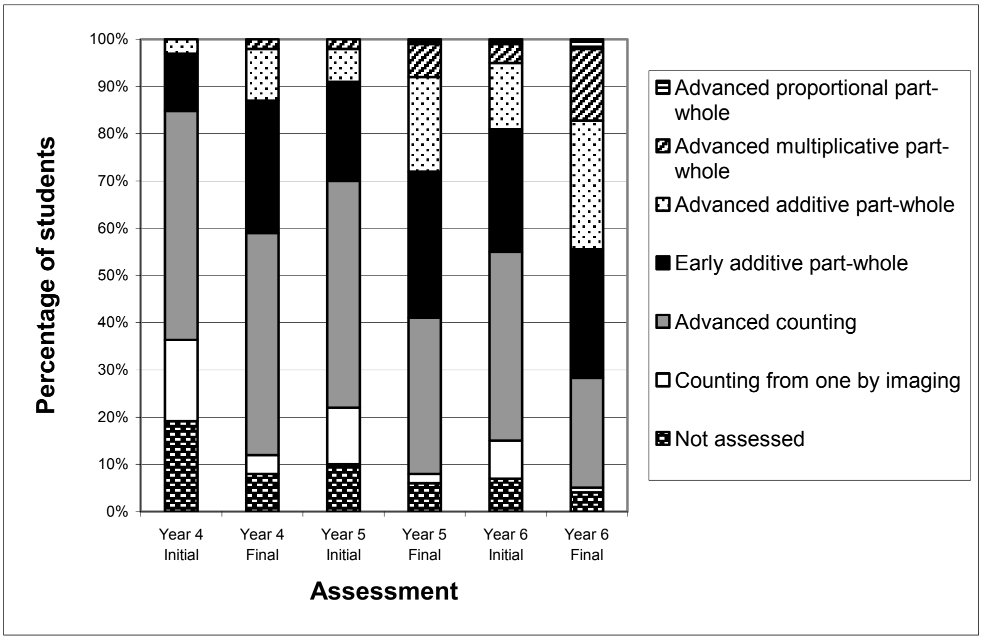

Figure 3-12: Proportions and ratios - patterns of improvement by year

Patterns of Development in the Knowledge Aspects

The pattern of achievement in the knowledge aspects confirmed the 2002 results and was similar in terms of stages gained over the period of instruction. Because Form A of the diagnostic interview was not used with the majority of students, number identification and decimals were not assessed.

Stage	Initial		Final	
Not assessed	80%	(6485)	89%	(7209)
Emergent number identification	0%	(18)	0%	(10)
Numerals to 10	1%	(61)	0%	(20)
Numerals to 20	1%	(82)	0%	(27)
Numerals to 100	7%	(573)	2%	(176)
Numerals to 1,000	11%	(896)	8%	(673)
Total	100%	(8115)	100%	(8115)

Table 3-22: Year 4: Number identification - initial and final assessment

Stage	Initial		Final	
Not assessed	90%	(6321)	93%	(6533)
Emergent number identification	0%	(26)	1%	(44)
Numerals to 10	0%	(11)	0%	(4)
Numerals to 20	0%	(25)	0%	(5)
Numerals to 100	2%	(166)	1%	(63)
Numerals to 1,000	6%	(449)	5%	(349)
Total	100%	(6998)	100%	(6998)

Table 3-23: Year 5: Number identification - initial and final assessment

Stage	Initial		Final	
Not assessed	93%	(6716)	95%	(6847)
Emergent number identification	0%	(28)	0%	(29)
Numerals to 10	0%	(6)	0%	(3)
Numerals to 20	0%	(26)	0%	(10)
Numerals to 100	1%	(64)	1%	(34)
Numerals to 1,000	5%	(364)	4%	(281)
Total	100%	(7204)	100%	(7204)

Table 3-24: Year 6: Number identification – initial and final assessment

Stage	Initial		Final	
Not assessed	96%	(7818)	90%	(7314)
Emergent decimal identification	3%	(223)	5%	(397)
Decimal identification	1%	(58)	4%	(297)
Ordered decimals	0%	(12)	1%	(74)
Rounded decimals	0%	(3)	0%	(26)
Decimal conversions	0%	(1)	0%	(7)
Total	100%	(8115)	100%	(8115)

Table 3-25: Year 4: Decimals – initial and final assessment

Stage	Initial		Final	
Not assessed	92%	(6426)	78%	(5483)
Emergent decimal identification	5%	(312)	5%	(361)
Decimal identification	3%	(180)	10%	(716)
Ordered decimals	1%	(54)	4%	(300)
Rounded decimals	0%	(22)	2%	(103)
Decimal conversions	0%	(4)	1%	(35)
Total	100%	(6998)	100%	(6998)

Table 3-26: Year 5: Decimals – initial and final assessment

Stage	Initial		Final	
Not assessed	83%	(5962)	65%	(4665)
Emergent decimal identification	5%	(339)	4%	(304)
Decimal identification	8%	(601)	16%	(1124)
Ordered decimals	3%	(223)	8%	(594)
Rounded decimals	1%	(65)	5%	(392)
Decimal conversions	0%	(14)	2%	(125)
Total	100%	(7204)	100%	(7204)

Table 3-27 Year 6: Decimals – initial and final assessment

The largest shift in FNWS for all students in all year groups was the move from FNWS up to 1,000 and up to 1,000,000. The percentages increased with each year group and followed the same pattern as in 2002.

Stage	Initial		Final	
Emergent FNWS	2%	(118)	1%	(92)
Initial FNWS up to 10	0%	(40)	0%	(7)
FNWS up to 10	2%	(143)	0%	(29)
FNWS up to 20	6%	(500)	2%	(118)
FNWS up to 100	38%	(3107)	18%	(1457)
FNWS up to 1000	48%	(3893)	64%	(5172)
FNWS up to 1,000,000	4%	(314)	15%	(1240)
Total	100%	(8115)	100%	(8115)

Table 3-28: Year 4: FNWS - initial and final assessment

Stage	Initial		Final	
Emergent FNWS	2%	(159)	3%	(175)
Initial FNWS up to 10	0%	(18)	0%	(6)
FNWS up to 10	1%	(38)	0%	(8)
FNWS up to 20	2%	(165)	1%	(45)
FNWS up to 100	22%	(1537)	9%	(608)
FNWS up to 1000	63%	(4406)	59%	(4108)
FNWS up to 1,000,000	10%	(675)	29%	(2048)
Total	100%	(6998)	100%	(6998)

Table 3-29: Year 5: FNWS - initial and final assessment

Stage	Initial		Final	
Emergent FNWS	2%	(179)	3%	(189)
Initial FNWS up to 10	0%	(10)	0%	(5)
FNWS up to 10	0%	(21)	0%	(10)
FNWS up to 20	1%	(81)	0%	(24)
FNWS up to 100	14%	(970)	5%	(323)
FNWS up to 1000	64%	(4611)	50%	(3564)
FNWS up to 1,000,000	19%	(1332)	43%	(3089)
Total	100%	(7204)	100%	(7204)

Table 3-30: Year 6: FNWS – initial and final assessment

For BNWS the pattern was similar to other knowledge aspects as well as to the 2002 results, with the largest shift being from BNWS up to 1000 and above. For Year 4 students this moved from 47% prior to instruction to 74% after instruction, for Year 5, 66% prior to instruction to 83% after instruction and for Year 6, 75% prior to instruction to 89% after instruction. Again all the percentages confirmed the 2002 figures.

Stage	Initial		Final	
Emergent BNWS	2%	(175)	2%	(125)
Initial BNWS up to 10	1%	(110)	0%	(34)
BNWS up to 10	5%	(432)	1%	(78)
BNWS up to 20	7%	(558)	2%	(172)
BNWS up to 100	37%	(2993)	22%	(1750)
BNWS up to 1000	44%	(3572)	59%	(4774)
BNWS up to 1,000,000	3%	(275)	15%	(1182)
Total	100%	(8115)	100%	(8115)

Table 3-31: Year 4: BNWS - initial and final assessment

Stage	Initial		Final	
Emergent BNWS	3%	(186)	3%	(210)
Initial BNWS up to 10	1%	(33)	0%	(7)
BNWS up to 10	2%	(117)	0%	(26)
BNWS up to 20	3%	(224)	1%	(61)
BNWS up to 100	27%	(1863)	13%	(888)
BNWS up to 1000	57%	(3958)	56%	(3912)
BNWS up to 1,000,000	9%	(617)	27%	(1894)
Total	100%	(6998)	100%	(6998)

Table 3-32: Year 5: BNWS - initial and final assessment

Stage	Initial		Final	
Emergent BNWS	3%	(213)	3%	(214)
Initial BNWS up to 10	0%	(10)	0%	(9)
BNWS up to 10	1%	(65)	0%	(16)
BNWS up to 20	2%	(113)	1%	(40)
BNWS up to 100	19%	(1346)	7%	(504)
BNWS up to 1000	58%	(4201)	47%	(3404)
BNWS up to 1,000,000	17%	(1256)	42%	(3017)
Total	100%	(7204)	100%	(7204)

Table 3-33: Year 6: BNWS - initial and final assessment

For grouping, across all year groups, the movement of students was between the stages of grouping with tens and tens in 100. In 2003, at Year 4 the percentage of students prior to instruction was down on the 2002 figure of 59% to 53%. By the end of instruction the percentage had increased to 71%, down from 80% in 2002. Prior to instruction 66% of Year 5 and Year 6 students were at these stages but by the end of instruction this had remained at 66% for Year 5 and decreased to 54% for Year 6. Twenty-one percent of Year 5 students and 37% of Year 6 students reached the more advanced levels of knowledge.

Stage	Initial		Final	
Not assessed	1%	(116)	1%	(91)
Non-grouping with 5s & within 10	21%	(1663)	4%	(356)
With 5s & within 10	23%	(1895)	14%	(1094)
With 10s	40%	(3223)	41%	(3345)
10s in 100	13%	(1037)	30%	(2428)
10s & 100s in whole numbers	2%	(137)	7%	(540)
10s, 100s & 1000s in whole numbers	1%	(38)	3%	(223)
10ths, 100ths & 1000ths in decimals	0%	(6)	1%	(38)
Total	100%	(8115)	100%	(8115)

Table 3-34: Year 4: Grouping - initial and final assessment

Stage	Initial		Final	
Not assessed	2%	(171)	3%	(187)
Non-grouping with 5s & within 10	9%	(645)	2%	(149)
With 5s & within 10	16%	(1145)	8%	(549)
With 10s	44%	(3056)	32%	(2230)
10s in 100	22%	(1565)	34%	(2393)
10s & 100s in whole numbers	4%	(285)	12%	(834)
10s, 100s & 1000s in whole numbers	2%	(3104)	7%	(496)
10ths, 100ths & 1000ths in decimals	0%	(27)	2%	(160)
Total	100%	(6998)	100%	(6998)

Table 3-35: Year 5: Grouping - initial and final assessment

Stage	Initial		Final	
Not assessed	3%	(182)	2%	(180)
Non-grouping with 5s & within 10	5%	(385)	2%	(113)
With 5s & within 10	13%	(908)	5%	(367)
With 10s	39%	(2828)	24%	(1710)
10s in 100	27%	(1917)	30%	(2159)
10s & 100s in whole numbers	8%	(590)	17%	(1232)
10s, 100s &1000s in whole numbers	4%	(307)	14%	(1007)
10ths, 100ths & 1000ths in decimals	1%	(87)	6%	(436)
Total	100%	(7204)	100%	(7204)

Table 3-36: Year 6: Grouping - initial and final assessment

For fractions the movement of students across all year groups was greatest for ordered unit fractions. At Year 4 this increased from 10% prior to instruction to 41% after instruction (up from 33% in 2002), at Year 5, 20% prior to instruction to 45% after instruction and in Year 6, 28% prior to instruction to 41% after instruction. All these figures reflect a slight increase from those in 2002.

Stage	Initial		Final	
Not assessed	19%	(1534)	8%	(629)
Non-fractions of regions	52%	(4248)	16%	(1316)
Assigned unit fractions	17%	(1416)	28%	(2241)
Ordered unit fractions	10%	(844)	41%	(3360)
Coordinated numerators & denominators	1%	(62)	6%	(462)
Equivalent fractions	0%	(6)	1%	(81)
Ordered fractions	0%	(5)	0%	(26)
Total	100%	(8115)	100%	(8115)

Table 3-37: Year 4: Fractions - initial and final assessment

Stage	Initial		Final	
Not assessed	10%	(672)	6%	(404)
Non-fractions of regions	40%	(2766)	10%	(670)
Assigned unit fractions	28%	(1942)	23%	(1615)
Ordered unit fractions	20%	(1427)	45%	(3129)
Coordinated numerators & denominators	2%	(145)	12%	(818)
Equivalent fractions	1%	(35)	3%	(243)
Ordered fractions	0%	(11)	2%	(119)
Total	100%	(6998)	100%	(6998)

Table 3-38: Year 5: Fractions - initial and final assessment

Stage	Initial		Final	
Not assessed	6%	(455)	4%	(293)
Non-fractions of regions	29%	(2061)	7%	(477)
Assigned unit fractions	30%	(2168)	18%	(1326)
Ordered unit fractions	28%	(1984)	41%	(2965)
Coordinated numerators & denominators	5%	(341)	17%	(1241)
Equivalent fractions	2%	(157)	8%	(603)
Ordered fractions	1%	(38)	4%	(299)
Total	100%	(7204)	100%	(7204)

Table 3-39: Year 6: Fractions - initial and final assessment

Chapter 4: The Shift from Counting-based to Part-whole Strategies

Overview

The shift from counting-based to part-whole strategies is an important marker in judging the progress of students participating in the Numeracy Project. Part-whole thinking enables students to think about numbers in more complex ways and to develop more sophisticated strategies when solving problems. Many commentators have noted that this is critical to students' later success in mathematics (Thomas & Ward, 2001; Thomas, Tagg & Ward, 2003; Wright, 1998; Young-Loveridge, 2001). More traditional approaches to teaching number through an early introduction to algorithms in the middle primary years have typically not provided students with opportunities to manipulate numbers in increasingly sophisticated ways, which underpin the development of part-whole thinking.

This chapter focuses only on those who initially had counting-based strategies, in contrast to the previous chapter, which examined the achievement patterns of the entire sample. It traces the development of part-whole thinking of this set of students for each year level across the operational domains of addition and subtraction, multiplication and division, and proportions and ratios. The reason that this chapter omits those who already have part-whole strategies is that it is the shift to part-whole strategies that is the key to becoming proficient in numeracy. Through this analysis it may be possible to shed more light on the characteristics of those who successfully make this shift during the course of instruction and to provide extra assistance to those groups of students for whom this shift is more elusive. To this end it examines the effect of gender, ethnicity, decile and regions on the shift to part-whole thinking, as well as comparing the knowledge held by those making a shift to part-whole thinking with those who retain counting-based strategies at the conclusion of the project.

For the purposes of examining the shift in thinking, the results presented below treat all stages prior to and including advanced counting as counting-based and hence not making the shift to part-whole. The term "no shift" has been used to indicate this group of students. The critical cross-over for students is between the stages of advanced counting and early additive part-whole. It is important to note here that the stages of understanding on the Framework are not intended to represent even-sized steps.

To put these results in context, it is important to note the proportion of students in the total sample who started with counting-based strategies for addition and subtraction. In 2003 this was 53% compared to 52% in 2002 and 41% in 2001. These differences between 2001, 2002 and 2003 could be caused by the effect of the project being extended (8,095 students in 2001, 33,209 in 2002 and 22,317 in 2003). As noted in the 2002 report, schools involved in 2001 were more likely to have participated in professional development as part of the Numeracy Development Project pilot. They may have also benefited from working with a very experienced group of facilitators who were developing the project in the first year of its implementation. An important characteristic of the project has been the iterative process of evidence-based policy formulation marked by concurrent phases of policy formulation, implementation and evaluation (see Higgins with Parsons and Hyland, 2003).

The following tables show the shift between counting-based strategies and part-whole strategies for the three operational domains. The percentage decreased across the three operational domains, with addition and subtraction presenting the most improved results and proportions and ratios the smallest gains. For addition and subtraction the percentage of students across year groups shifting to part-whole strategies was the same as for 2002, being 50% of students. These percentages are a drop from the 2001 figure of 63%. A possible reason for this is that changes were made to the diagnostic interview for 2002. For multiplication and division the percentage of students across all year groups who moved to part-whole strategies was 44% and for proportions and ratios 39%. These figures were similar to those for 2002. The percentage of students, who initially used counting-based strategies and shifted to part-whole strategies increased at each year level for each operational domain. The proportion of those who shifted to more advanced stages increased at each year level. The percentages for each year group were similar to those from 2002. About half of the Year 4 (48%) and Year 5 students (49%) in addition and subtraction shifted to part-whole strategies. Slightly more Year 6 students (57%) made the shift. A similar pattern emerged for the operational domain of multiplication and division. Slightly fewer Year 4 (37%) than Year 5 students (48%) and Year 6 students (54%) made the shift to part-whole. For proportions and ratios about half the Year 6 students (49%) shifted to part-whole, with those at Year 5 (43%) and Year 4 (31%) showing lower percentages than the older students.

Compared to 2002 there was a big increase in addition and subtraction in the number of Year 4 students who in 2003 became advanced additive (from 2% in 2002 to 12% in 2003). The percentage of students who became advanced additive in multiplication and division across all year groups increased from 11% in 2002 to 14% in 2003. As for 2002 the greatest shift across all year groups and operational domains was to the early additive stage. This stage is important compared to advanced counting as it allows students to solve more complex problems through splitting and joining numbers. Fewer students moved to the stages of advanced additive and advanced multiplicative part-whole, representing more sophisticated part-whole thinking. Virtually no students shifted to advanced proportional part-whole.

Final	No shift	Became early additive	Became advanced additive	Totals
Year 4	52% (3009)	36% (2072)	12% (719)	100% (5800)
Year 5	51% (1709)	45% (1487)	4% (136)	100% (3332)
Year 6	43% (1185)	50% (1366)	7% (179)	100% (2730)
Totals	50% (5903)	41% (4927)	9% (1034)	100% (11862)

Table 4-1: Years 4, 5 & 6: Addition and subtraction - shift of those initially using counting-based strategies to part-whole strategies

Final	No shift	Became early additive	Became advanced additive	Became advanced multiplicative	Totals
Year 4	63% (3976)	26% (1640)	11% (689)	0% (24)	100% (6329)
Year 5	52% (2112)	32% (1269)	15% (620)	1% (49)	100% (4050)
Year 6	46% (1381)	33% (986)	19% (568)	2% (54)	100% (2989)
Totals	56% (7469)	29% (3895)	14% (1877)	1% (127)	100% (13368)

Table 4-2: Years 4, 5 & 6: Multiplication and division - shift of those initially using counting-based strategies to part-whole strategies

Final	No shift	Became early additive	Became advanced additive	Became advanced multiplicative	Became advanced proportional	Totals
Year 4	69% (4742)	24% (1654)	7% (466)	0% (37)	0% (0)	100% (6899)
Year 5	57% (2809)	29% (1409)	12% (577)	2% (99)	0% (1)	100% (4895)
Year 6	51% (2014)	30% (1179)	16% (629)	3% (124)	0% (6)	100% (3952)
Totals	61% (9565)	27% (4242)	10% (1672)	2% (260)	0% (7)	100% (15746)

Table 4-3: Years 4, 5 & 6: Proportions and ratios - shift of those initially using counting-based strategies to part-whole strategies

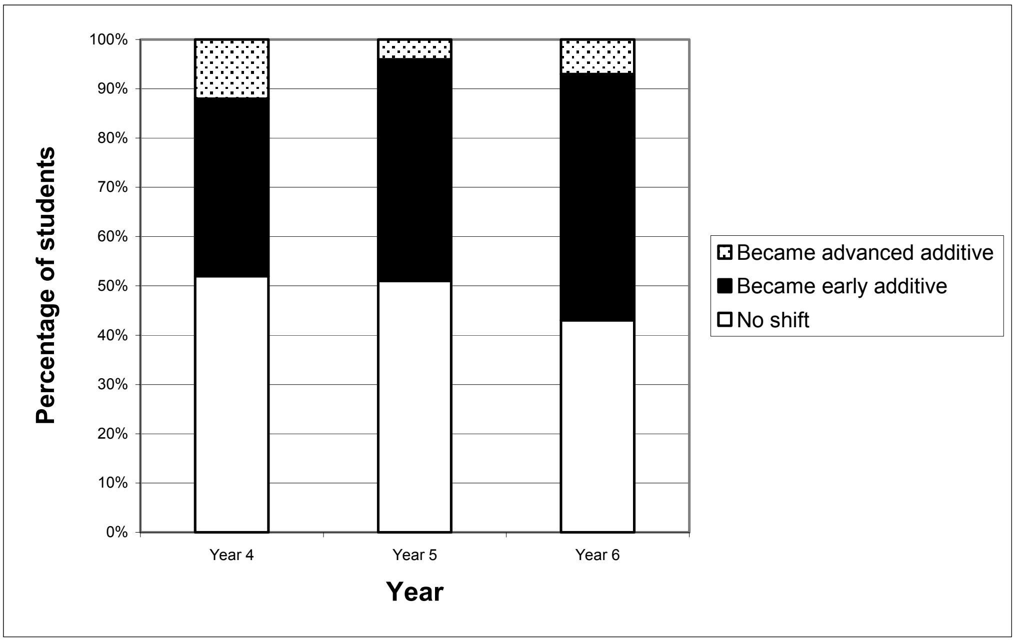

Figure 4-1: Years 4, 5 & 6: Addition and subtraction - shift of those initially using counting-based strategies to part-whole strategies

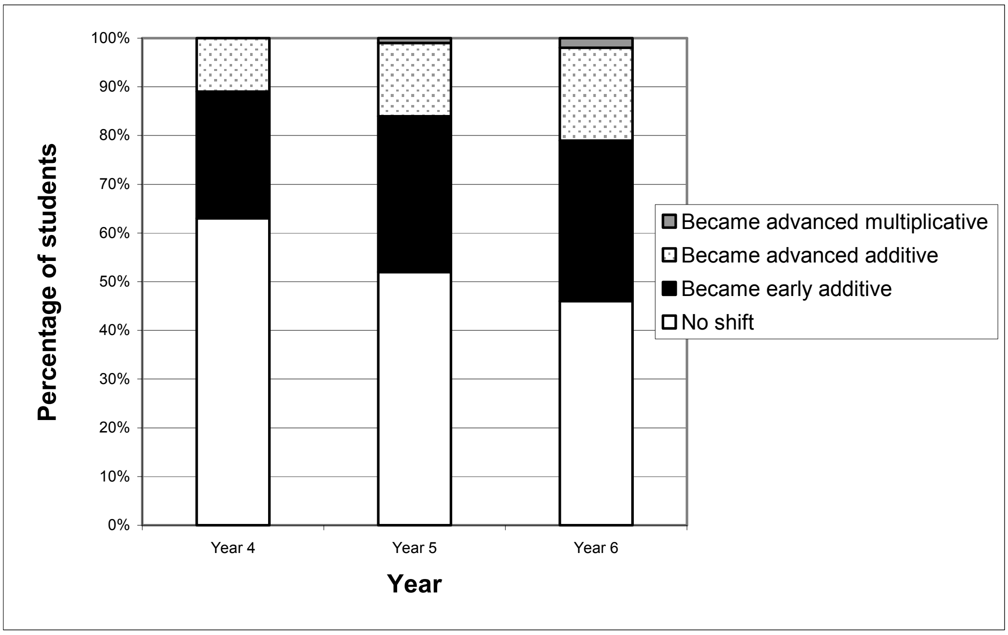

Figure 4-2: Years 4, 5 & 6: Multiplication and division - shift of those initially using counting-based strategies to part-whole strategies

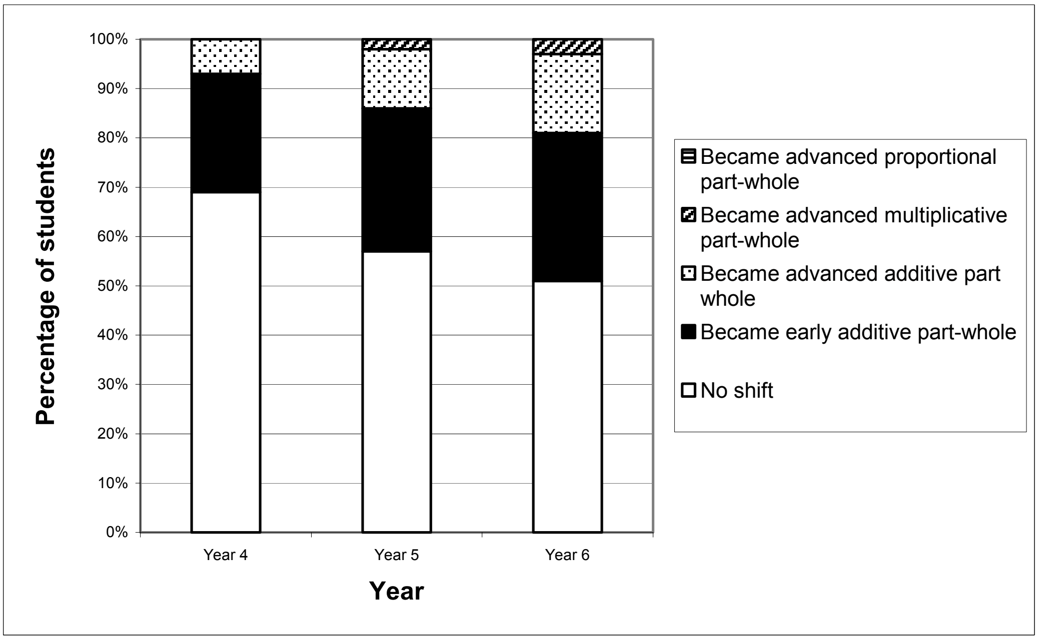

Figure 4-3: Years 4, 5 & 6: Proportions and ratios - shift of those initially using counting-based strategies to part-whole strategies

	2001	2002	2003
No shift	37% (1230)	50% (8519)	50% (5903)
Became early additive	52% (1707)	46% (7904)	41% (4927)
Became advanced additive	11% (373)	4% (683)	9% (1034)

Table 4-4: Years 4, 5 & 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – 2001, 2002 & 2003 comparison

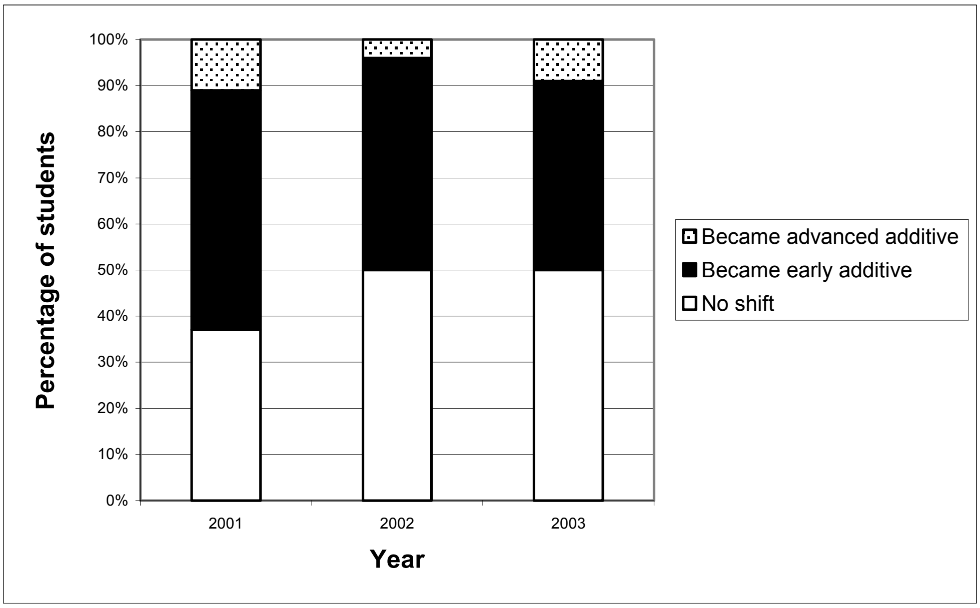

Figure 4-4: Years 4, 5 & 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – 2001, 2002 & 2003 comparison

The next section considers the effect of gender, ethnicity, decile, region, and knowledge on student achievement in addition and subtraction for all year groups.

Gender appears to have little impact on those students shifting to part-whole thinking. This is consistent with the 2001 and 2002 results for ANP (Higgins, 2002 and 2003). For Year 4 students, the boys did slightly better than the girls, with more boys (44%) shifting to part-whole strategies than girls (40%). The gap in the scores decreased to a difference of 2 percentage points for the older age groups. The results for students in Years 5 and 6 can be found in the appendix.

Gender	No shift	Became early additive	Became advanced additive	Totals
Female	60% (1639)	39% (1063)	1% (43)	100% (2745)
Male	56% (1370)	41% (1009)	3% (63)	100% (2442)
Total	58% (3009)	40% (2072)	2% (106)	100% (5187)

Table 4-5: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by gender

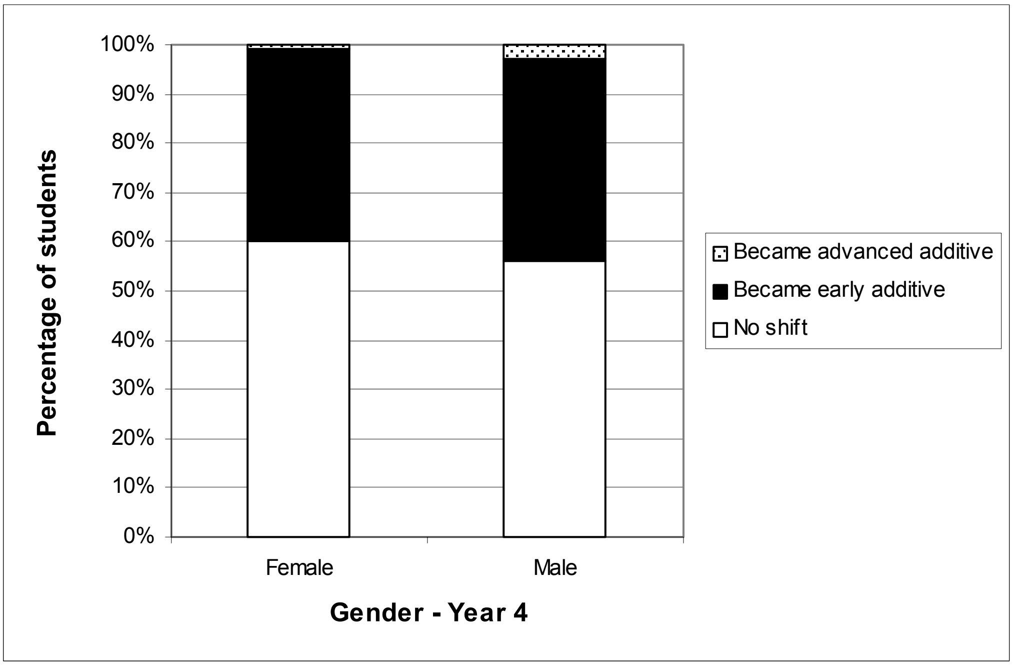

Figure 4-5: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by gender

The pattern of results in addition and subtraction for particular ethnic groups did differ from those of 2002. It needs to be noted from the outset that for this analysis (of those beginning with counting-based strategies) there is only a small proportion of Asian, Pasifika and Other ethnic groups in the sample. Also, the number of Asian students who began with counting-based strategies was only half as many as in 2002. The figures need to be read with these factors in mind.

The effect of ethnicity on student progress in addition and subtraction appears to lessen as students move up the year levels. At Year 4 (see Table 4-6) the range across the different ethnic groups for those remaining with counting-based strategies was notable, being 26 percentage points, compared to 22 at Year 5 and 15 at Year 6. This compares to 24 for Year 4, 21 for Year 5 and 19 for Year 6 in 2002. The results for students in Years 5 and 6 can be found in the appendix.

Ethnicity	No shift		Became early additive		Became advanced additive		Totals	
New Zealand European	53%	(1488)	45%	(1273)	2%	(63)	100%	(2824)
Māori	65%	(958)	34%	(488)	1%	(21)	100%	(1467)
Pasifika	69%	(367)	30%	(159)	1%	(6)	100%	(532)
Asian	43%	(59)	53%	(72)	4%	(5)	100%	(136)
Other	60%	(137)	35%	(80)	5%	(11)	100%	(228)
Total	58%	(3009)	40%	(2072)	2%	(106)	100%	(5187)

Table 4-6: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by ethnicity

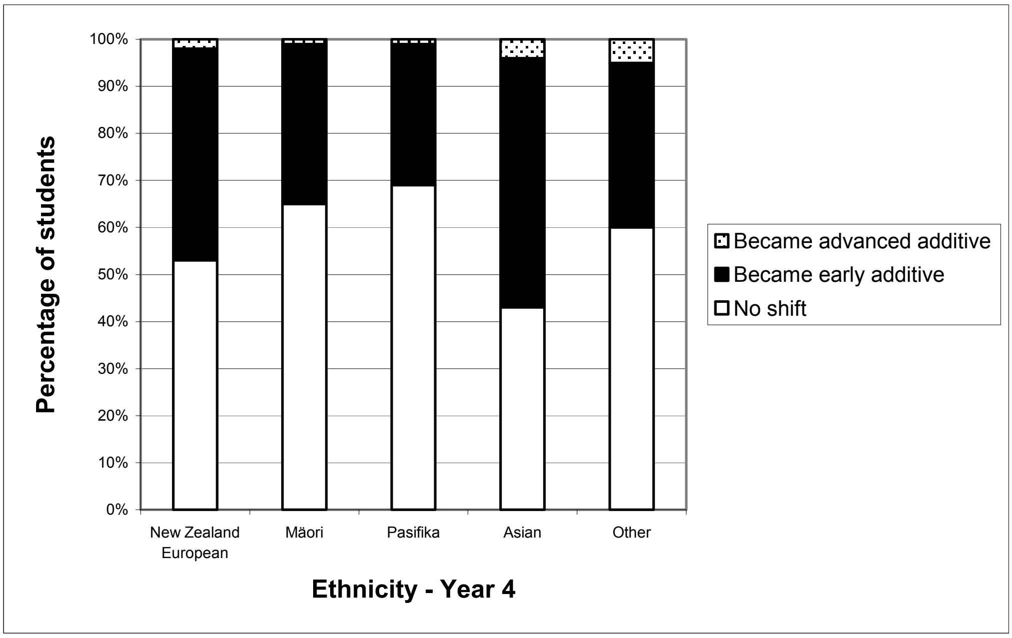

Figure 4-6: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by ethnicity

When compared to the 2002 results there appears to be a decline in the percentage of students adopting part-whole strategies in addition and subtraction for most ethnic groups at Years 5 and 6 (see Table 4-7). In Year 6 the percentages of those becoming part-whole declined for all ethnic groups except Pasifika students, with those students categorised as Other and Māori declining the most.

More Asian students than those of any other ethnic group shifted to part-whole strategies at Year 4 (57% Asian compared to 47% New Zealand European) and at Year 5 (59% Asian compared to 53% New Zealand European). This, however, was not the case for Year 6, with 62% New Zealand European compared to 58% Asian. Compared to 2002 more Pasifika students shifted to part-whole at Year 4 (31% in 2003 compared to 27% in 2002). For Years 5 and 6 the results for Pasifika were similar to the 2002 percentages. When compared to the 2002 figures, Māori students declined for each successive year group. At Year 4, 35% of Māori students compared to 37% in 2002 adopted part-whole strategies, for Year 5 this was 45% in 2003 compared to 48% in 2002, and in Year 6, 51% in 2003 compared to 57% in 2002.

Ethnicity	Year	No shift		Became early additive		Became advanced additive	
NZ European	2001	34%	(699)	53%	(1038)	13%	(244)
	2002	44%	(3842)	51%	(4397)	5%	(429)
	2003	48%	(2885)	48%	(2915)	4%	(243)
Māori	2001	39%	(285)	50%	(366)	11%	(79)
	2002	54%	(2636)	43%	(2100)	3%	(140)
	2003	58%	(1944)	39%	(1312)	3%	(108)
Pasifika	2001	49%	(205)	46%	(190)	5%	(21)
	2002	64%	(1433)	34%	(777)	2%	(45)
	2003	63%	(705)	34%	(374)	3%	(31)
Asian	2001	33%	(44)	52%	(70)	15%	(20)
	2002	46%	(276)	47%	(284)	8%	(46)
	2003	42%	(111)	51%	(133)	7%	(18)
Other	2001	34%	(27)	54%	(43)	11%	(9)
	2002	49%	(350)	48%	(346)	3%	(23)
	2003	55%	(258)	41%	(191)	4%	(21)

Table 4-7: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by ethnicity

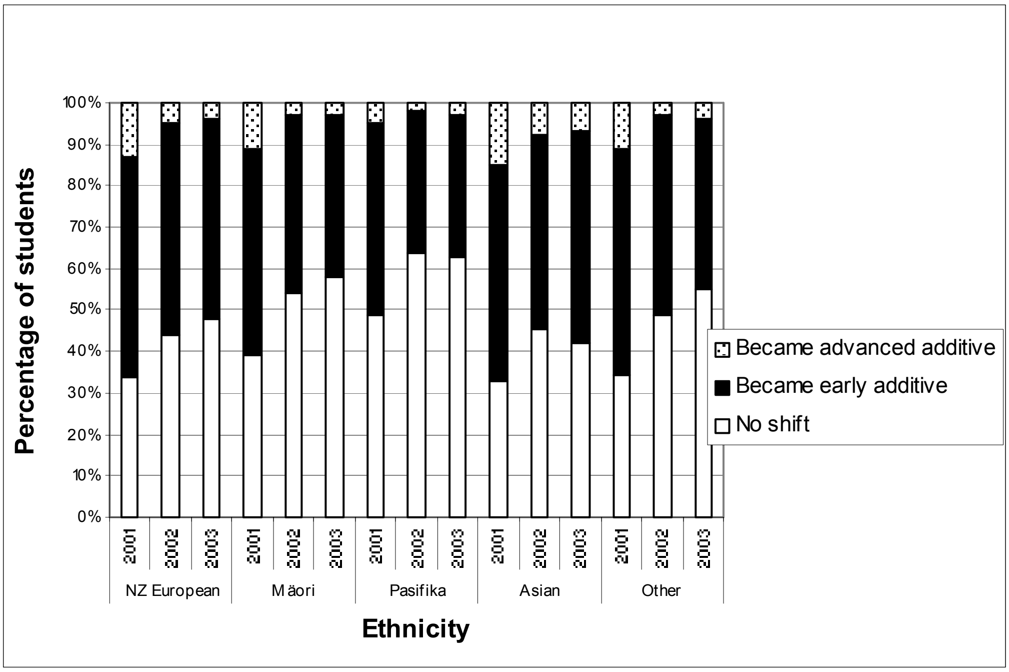

Figure 4-7: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by ethnicity

The effect of decile on the patterns of achievement was similar to that in previous years, with increasing numbers of students shifting to part-whole strategies across the composite groups of low (deciles 1 to 3), middle (deciles 4 to 7) and high (deciles 8 to 10) decile rankings (see Table 4-8). More students adopted part-whole strategies in 2003 than in 2002 for each decile group. The middle deciles improved the most, from 45% in 2002 to 50% of students shifting to part-whole strategies in 2003.

The pattern across individual deciles is more variable than in 2002 (see Table 4-9). However, deciles 1 and 2 for each year group have fewer students adopting part-whole strategies. At Year 4 the spread from a low of 36% for decile 9 to a high of 72% in decile 2 for those remaining with counting-based strategies was much greater than for the other year groups and for those in 2002. For Year 4 the spread was 36 percentage points compared to 23 at Year 5 and 24 at Year 6. This compares to 24 in Year 4, 23 in Year 5 and 28 in Year 6 in 2002. The results for students in Years 5 and 6 can be found in the appendix, along with the results for the regions.

Decile	Year	No shift	Became early additive	Became advanced additive
Low (1-3)	2001	40% (592)	50% (731)	10% (146)
	2002	64% (1714)	35% (925)	2% (34)
	2003	60% (2791)	37% (1701)	3% (163)
Middle (4-7)	2001	36% (466)	52% (675)	12% (149)
	2002	55% (1347)	43% (1032)	2% (49)
	2003	50% (2056)	46% (1921)	4% (151)
High (8-10)	2001	31% (172)	55% (301)	14% (78)
	2002	47% (626)	51% (681)	3% (36)
	2003	44% (1056)	51% (1230)	5% (119)

Table 4-8: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by aggregated decile

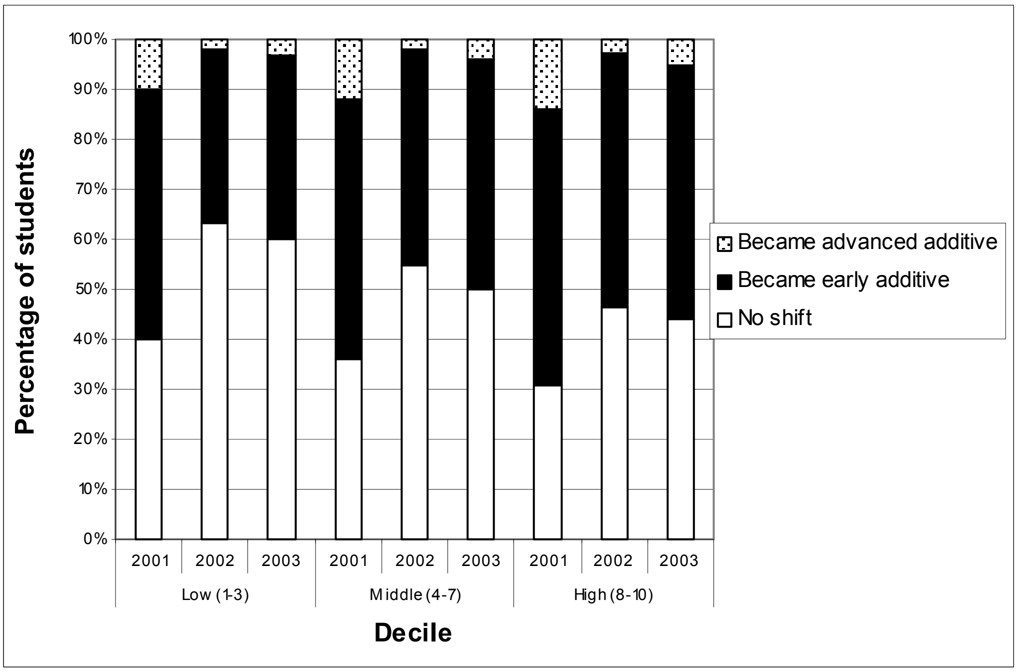

Figure 4-8: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by aggregated decile

Decile	No shift	Became early additive	Became advanced additive	Totals
1	68% (422)	30% (185)	2% (11)	100% (618)
2	72% (546)	27% (209)	1% (10)	100% (765)
3	60% (407)	39% (264)	1% (10)	100% (681)
4	49% (199)	50% (203)	1% (4)	100% (406)
5	59% (438)	38% (278)	3% (21)	100% (737)
6	52% (234)	45% (206)	3% (15)	100% (455)
7	56% (190)	43% (145)	1% (5)	100% (340)
8	58% (283)	40% (196)	2% (9)	100% (488)
9	36% (97)	61% (161)	3% (8)	100% (266)
10	45% (193)	52% (225)	3% (13)	100% (431)
Total	58% (3009)	40% (2072)	2% (106)	100% (5187)

Table 4-9: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by decile

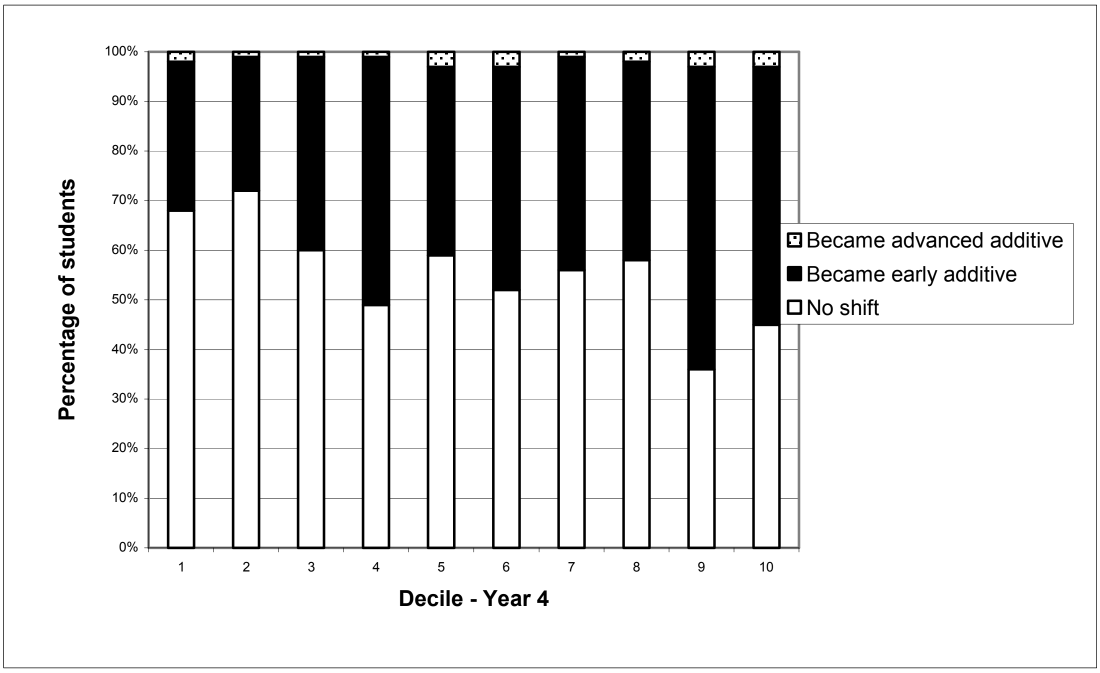

Figure 4-9: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - final status by decile

Decile	Year	No shift		Became early additive		Became advanced additive	
1	2001	62%	(136)	34%	(74)	4%	(8)
	2002	60%	(1734)	38%	(1111)	2%	(66)
	2003	61%	(1026)	36%	(617)	3%	(56)
2	2001	31%	(183)	54%	(321)	15%	(86)
	2002	54%	(976)	43%	(753)	3%	(54)
	2003	62%	(991)	34%	(535)	4%	(61)
3	2001	41%	(273)	51%	(336)	8%	(52)
	2002	53%	(1180)	44%	(968)	3%	(70)
	2003	54%	(774)	44%	(622)	2%	(34)
4	2001	36%	(242)	52%	(355)	12%	(82)
	2002	51%	(1179)	45%	(1029)	4%	(98)
	2003	47%	(460)	48%	(462)	5%	(46)
5	2001	35%	(35)	47%	(47)	17%	(17)
	2002	45%	(898)	50%	(1000)	4%	(83)
	2003	53%	(822)	44%	(687)	3%	(46)
6	2001	32%	(87)	57%	(154)	11%	(29)
	2002	47%	(382)	50%	(404)	4%	(30)
	2003	46%	(428)	49%	(450)	5%	(43)
7	2001	42%	(102)	49%	(119)	9%	(21)
	2002	46%	(451)	51%	(506)	3%	(34)
	2003	51%	(346)	47%	(322)	2%	(16)
8	2001	28%	(67)	52%	(124)	19%	(46)
	2002	41%	(541)	54%	(716)	6%	(74)
	2003	52%	(501)	45%	(442)	3%	(31)
9	2001	33%	(12)	58%	(21)	8%	(3)
	2002	37%	(362)	55%	(541)	9%	(88)
	2003	38%	(193)	57%	(289)	5%	(26)
10	2001	33%	(93)	56%	(156)	10%	(29)
	2002	37%	(370)	57%	(573)	6%	(65)
	2003	39%	(362)	54%	(499)	7%	(62)

Table 4-10: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by decile

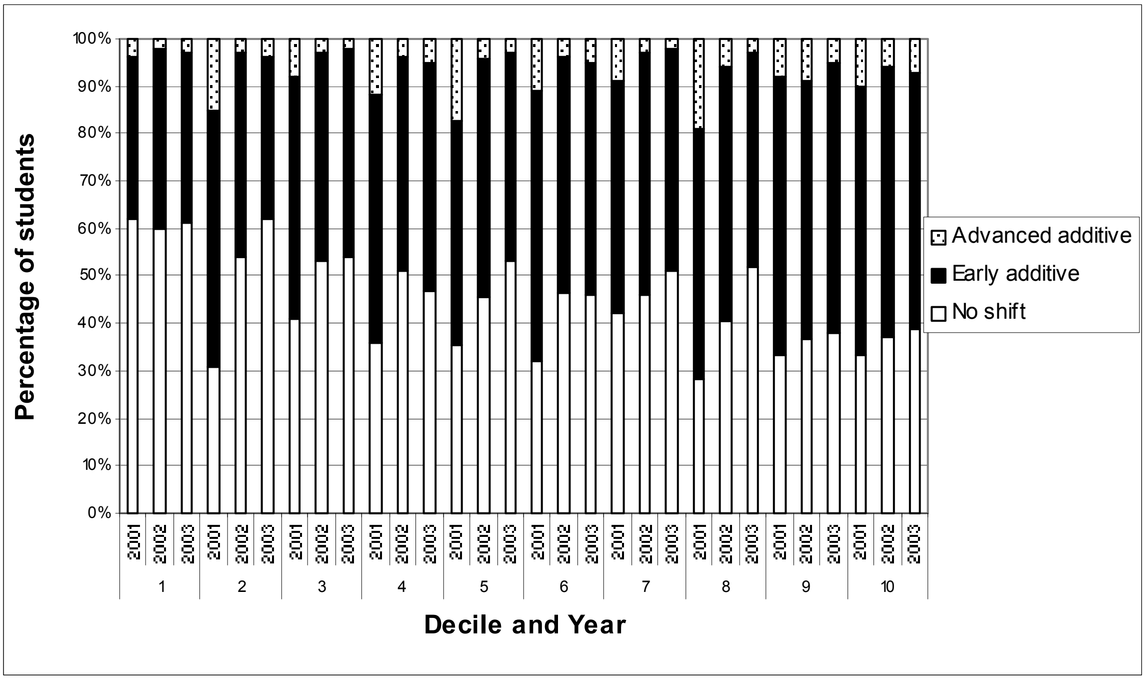

Figure 4-10: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies - 2001, 2002 & 2003 comparison by decile

The next set of tables compares the knowledge base of those retaining counting-based strategies with that of those who adopt part-whole strategies. As for 2002 the comparison was not possible for the knowledge domains of number identification and decimals and percentages as the majority of students were not assessed. For the remaining knowledge domains the levels of knowledge for those with part-whole based strategies were consistently higher than for those with counting-based strategies. The results for 2003 in general were better than those for 2002. The knowledge of those adopting part-whole strategies increased across all knowledge domains for all year groups (the results for students in Years 5 and 6 can be found in the appendix). For those retaining counting-based strategies, the percentages for 2003 increased for all knowledge domains assessed except for grouping, where the results were similar to those in 2002.

Strategies	Emergent	Initial up to 10	Up to 10	Up to 20	Up to 100	Up to 1,000	Up to 1,000,000
FNWS							
Counting based	2% (64)	0% (7)	1% (28)	4% (113)	36% (1114)	53% (1635)	4% (119)
Part-whole	1% (28)	0% (0)	0% (1)	0% (5)	7% (343)	70% (3537)	22% (1121)
BNWS							
Counting based	2% (76)	1% (32)	3% (77)	5% (162)	40 (1225)	46% (1414)	3% (94)
Part-whole	1% (49)	0% (2)	0% (1)	0% (10)	10% (525)	67% (3360)	22% (1088)

Table 4-11: Year 4: FNWS and BNWS - knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s & within 10s	With 5s & within 10	With 10s	10s in 100	10s & 100s in whole numbers	10s, 100s & 1000s in whole numbers	10ths, 100ths & 1000ths in decimals
Counting based	2% (63)	11% (329)	27% (839)	46% (1431)	13% (399)	1% (18)	0% (1)	0% (0)
Part-whole	1% (28)	1% (27)	5% (255)	38% (1914)	40% (2029)	10% (522)	4% (222)	1% (38)

Table 4-12: Year 4: Grouping - knowledge for counting-based and part-whole

	Not assessed	Non-fractions of regions	Assigned unit fractions	Ordered unit fractions	Coordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting based	19% (580)	25% (781)	32% (998)	23% (707)	0% (12)	0% (1)	0% (1)
Part-whole	1% (49)	11% (535)	25% 1243	53% (2653)	9% (450)	2% (80)	0% (25)

Table 4-13: Year 4: Fractions - knowledge for counting-based and part-whole

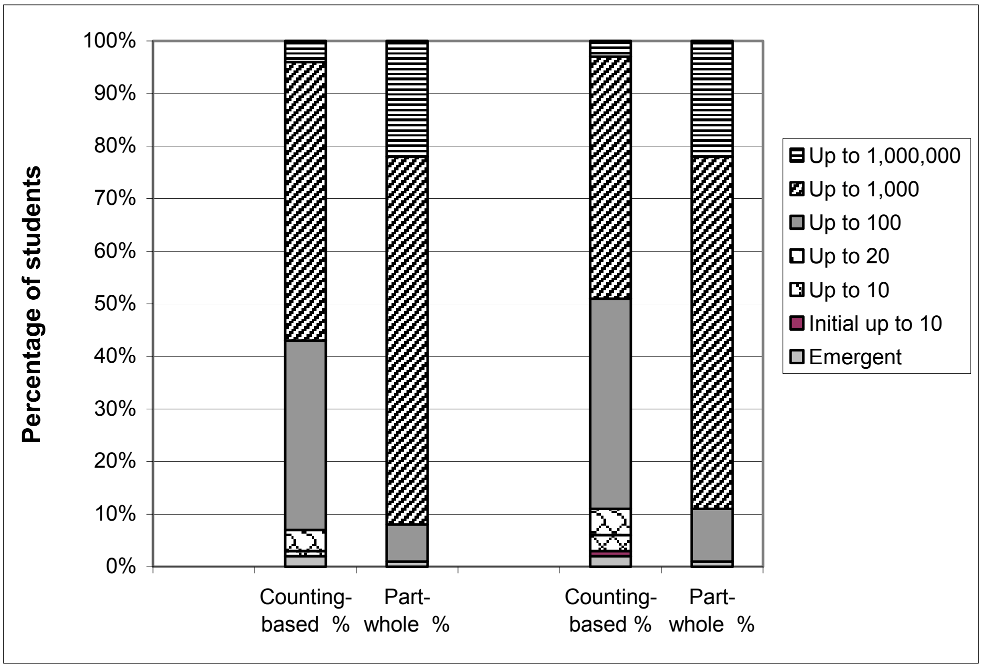

Figure 4-11: Year 4: FNWS and BNWS - knowledge for counting-based and part-whole

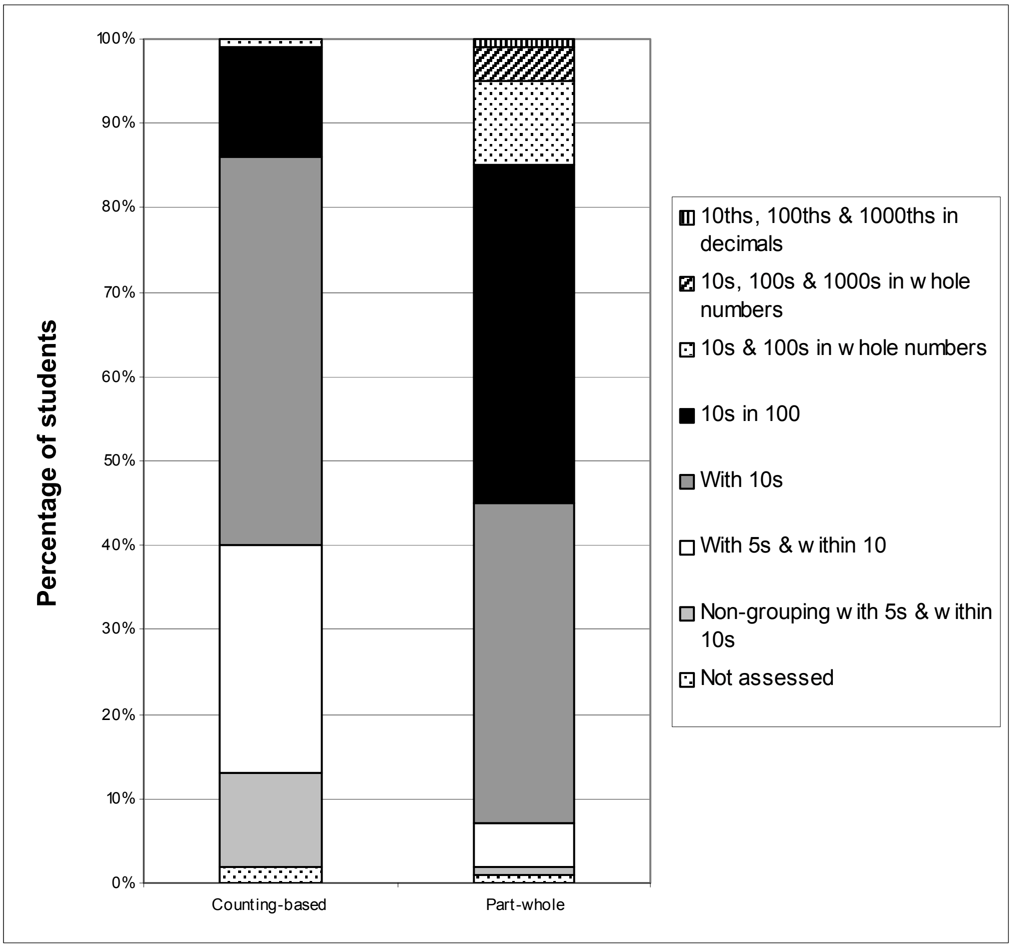

Figure 4-12: Year 4: Grouping - knowledge for counting-based and part-whole

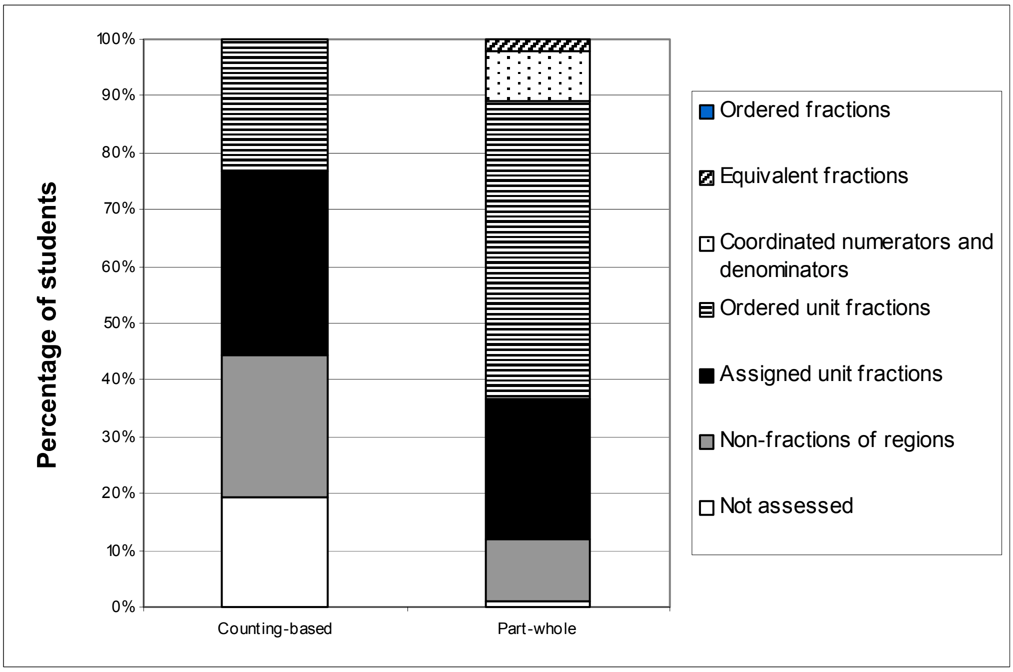

Figure 4-13: Year 4: Fractions - knowledge for counting-based and part-whole

Chapter 5: Pedagogy of Facilitation

Introduction

This case study builds on the 2001 model (Higgins, 2002), which identified context as central to effective facilitation. The importance of a facilitator being able to quickly assess a teacher's working context is emerging as a significant theme. The lens that a facilitator uses to assess a teacher's working context (as schematically represented in Figure 5-1) is framed by their beliefs about what constitutes effective professional development; their knowledge of mathematics; their pedagogical knowledge; and their knowledge of learners' cognitions in mathematics.

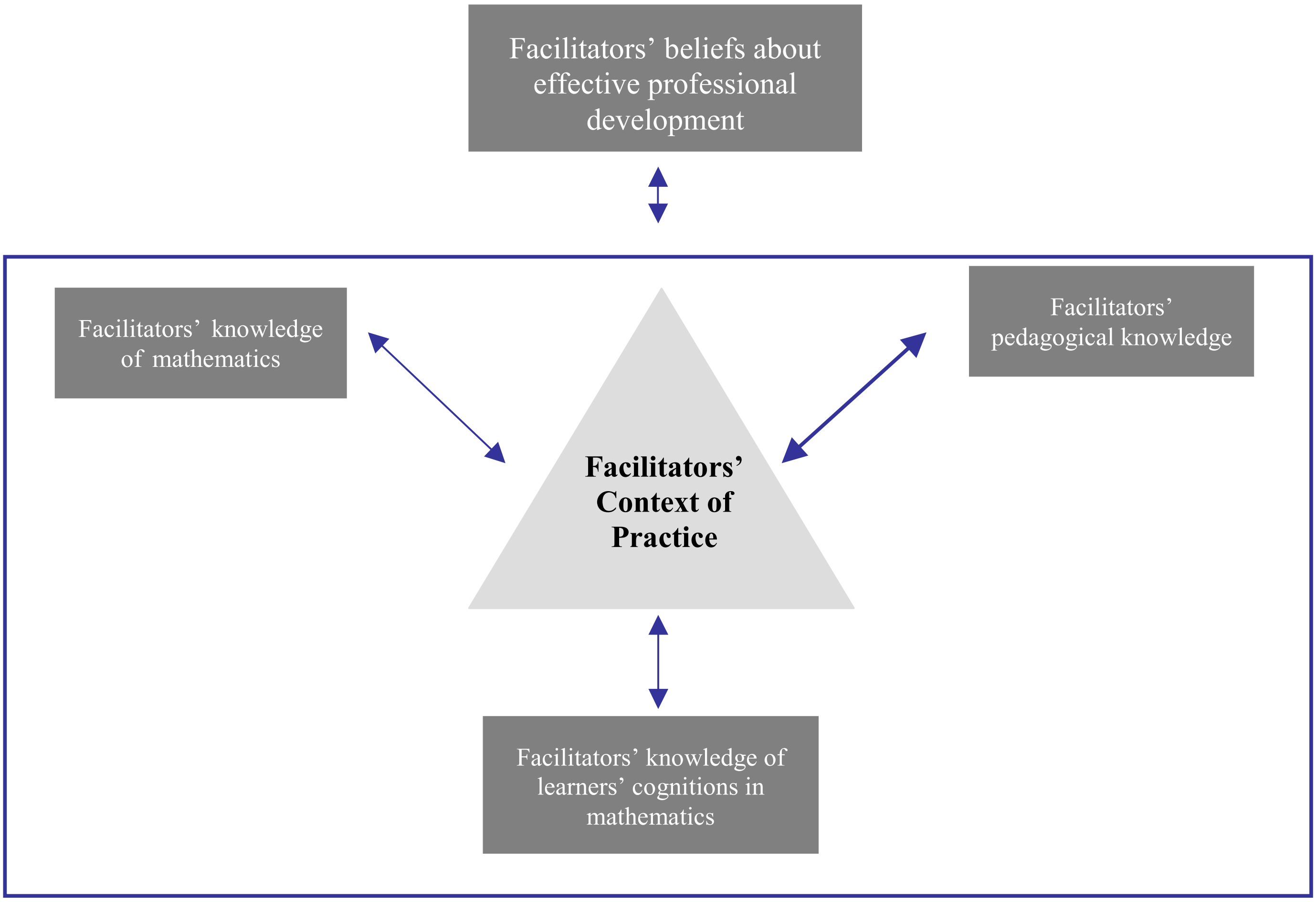

Figure 5-1: Facilitators' context of practice
(Adapted from Fennema & Franke, 1992, p.162, Higgins, 2002, p.9)

The complexity of the facilitator's context of practice arises from the nesting of the facilitator's practice within the context of the teacher's practice. The *inside nesting* of the facilitator's role within the everyday practice of teaching and learning lays the foundation for effective implementation and starts the conditions needed for sustainability.

Using a frame of structure and agency and using knowledge of effective teaching (Hattie, 2002; Thomas, Tagg & Ward, 2003) the case study aims to identify dimensions of effective facilitation and its consequences for effective implementation, internalisation and sustainability. The process by which this happens is marked by a facilitator's ability to create a situation in which participants can interpret and internalise the ideas of the project in their own context and thus provide a path to autonomy. Quality facilitation (as with quality teaching; Alton-Lee, 2003), while having some generic characteristics, needs to be defined in terms of the discipline/subject area. Expert facilitation is based on the context within which one facilitates rather than based on the skills of facilitation; in other words, an expert facilitator is an expert teacher of teachers in a specific content area.

Key dimensions of facilitation include a "seamlessness" between modelling and observation; the use of structure in the form of frameworks in the observation and associated feedback on teacher practice; and the ability to quickly assess the teachers' context of practice. Effective accomplishment of these processes is a necessary pre-condition for effective implementation and sustainability.

The overall analytical frame is from a socio-cultural interpretativist theoretical perspective, drawing mainly on the work of Remillard (1999), Cobb & McClain (2002), Cobb et al. (2003) and Sfard & Kieran (2001). A facilitator's role is to mediate the level of abstraction between tangible, concrete actions and guiding concepts.

Structure

The notion of structure appears important in underpinning the work of facilitators in the Numeracy Project. This analysis of structure draws on Sewell's (1992) definition in which he argued for a dual character, in that structure is "composed simultaneously of schemas, which are virtual, and of resources[3], which are actual" (p. 13). The two schema underpinning the Numeracy Project are levels of abstraction in children's understanding of number and levels of abstraction in representations of mathematical ideas. These schemas can be inferred from the physical manifestation in the form of the resources. These resources are, respectively, the Number Framework (Ministry of Education, 2004a) and the associated diagnostic interview, and the Teaching Model (Ministry of Education, 2004b). Sewell suggested:

> If resources are effects of schemas, it is also true that schemas are the effects of resources. If schemas are to be sustained or reproduced over time – and without sustained reproduction they can hardly be counted as structural – they must be validated by the accumulation of resources that their enactment engenders. Schemas not empowered or regenerated by resources would eventually be abandoned and forgotten, just as resources without cultural schemas to direct their use would eventually dissipate and decay. Sets of schemas and resources may properly be said to constitute *structures* only when they mutually imply and sustain each other over time. (p. 13)

What Sewell is highlighting here is the dialectal relationship between the virtual and actual components of the structure. Each component gives meaning to the other component and is essential to the overall meaning of the structure. Any change in one component creates a change in the other. An important part of the process is the validation of the structural components through the implementation of the key ideas embodied in the resources of the project. In this way the ideas are sustained through being reproduced over a long period of time.

Applied to the structures of the Numeracy Project, validation of the stages of children's mathematical thinking is embodied in the written form of the Framework and associated

[3] Sewell's use of the term "resources" in specific contrast to schema should not be confused with the more general meaning commonly applied to classroom materials.

diagnostic interview. The validation of the schema of the representation of mathematical ideas for teaching purposes occurs through the resource of the Teaching Model and the associated practices. The practices of the school community endorse both the resources of the Number Framework and the Teaching Model. The dialectic relationship between the mental schema and the resources in the case of the Numeracy Project can be seen to, as Sewell suggested above, "mutually imply and sustain each other over time".

It is also important to consider assumptions underlying the structure and its dynamic effects. These assumptions are that teaching is mediated learning through interaction and that the pedagogy of mathematics and mathematics education is discipline-specific and contextually-bound.

One of the particular issues that Sewell addressed in his theory of structure was the possibility of change in structure. Sewell's notion of a dynamic structure shows how core ideas form a dialectic relationship with the related resources to account for change from within the project. He argued for "a conceptual vocabulary that makes it possible to show how the ordinary operations of structures can generate transformations" (p. 16). It is this characteristic that accounts for the capacity of the project to incorporate modifications during the implementation phase and yet retain the essence of the project. It is also useful for examining the ways in which facilitators implement the project in a range of settings.

In addition Sewell (1992) suggested five key axioms (underlying principles) as a means of accounting for *internally instigated change,* that is, initiated conditions that allow a facilitator to work with teachers *inside* their everyday teaching practices. This is the locus of authentic implementation and the starting point for sustainability.

The five key axioms are related to the structure as a whole, and to the schema and the resources as dual components that form the structure. These were "the multiplicity of structures, the transposability of schemas, the unpredictability of resource accumulation, the polysemy of resources, and the intersection of structures" (p. 16). The multiplicity of structures accounts for variation in deep and surface features within different school settings; the transposability of schema refers to the way in which a facilitator helps teachers to apply and extend the schema or underlying ideas in new contexts; the unpredictability of resource accumulation arises through modification and differential validation arising from changes to the schema; the polysemy or multiple meanings of resources comes through ambiguity and reinterpretation; and the intersection of structures through the transposition by facilitators from schemas that conflict with those underlying the resources.

The combination of these five key axioms helps explain how structures remain intact yet at the same time change when enacted in dynamic, diverse contexts[4]. This apparent contradiction in terms is important to an analysis of the implementation of the Numeracy Project in which key structures represent the integral heart of the project yet at the same time are open to transformation by all those who play any part in enacting it in the classroom.

Of relevance to the Numeracy Project are the changes from within the project that form a dynamic through which the project continues to develop as it is implemented. This process stands in contrast to many other teacher development projects where there is a separation between development and implementation. As argued elsewhere the Numeracy Project comprises a dynamic approach to the policy process, with interdependent and interrelated aspects of policy formulation, implementation and evaluation (Higgins et al., 2003). In

[4] Context is used here as a term that encompasses not only the physical context of the classroom, but also the attributes of the teachers and children *within* that class.

essence, the *inside* dynamic is genuinely where the intended improvement of practice occurs in a way that is efficacious for teachers and lays the authentic foundation for sustainability.

The rest of the chapter will examine the evidence in terms of Sewell's theory of structure by firstly discussing participants' comments about the structural elements of the Numeracy Project and also by examining evidence of these elements in the delivery – specifically through class modelling and the associated feedback sessions.

Views of the Structural Components

Effective facilitation appeared to initially centre on the structural components (i.e. the Number Framework and the Teaching Model) of the Numeracy Project rather than on classroom activities. Both teachers and facilitators appeared to see the structures as a frame that could be adjusted to fit the *inside* context[5]. The Framework is seen as central to the project, as attested to in the following comment.

> I think it's really important that they get to know the Framework … to me it's one of the key things. … I am constantly bringing them back to the Framework. (Nancy, Facilitator, 2003)

The key point about the Framework is its use by teachers in classrooms *inside* their own practice. It is this *inside* use that gives it a dynamic quality and by which sustained use can be achieved.

> It's about giving simple, understandable, credible, reasonable structures for teachers to use. (Roger, Facilitator, 2003)

The explanatory power of the Number Framework is realised as teachers use it in their classrooms. Through seeing that it reflects the types of thinking that they have observed in the children they teach, the new ideas begin to be internalised.

> But starting from the Framework and using that to develop the pedagogy. … It's the foundation of what we do and I think teachers, initially they don't realise that … [Later] they start to say "Oh this makes so much more sense to us now, we can see how children are learning, we can see the progression, and we can understand it and we've got something to handle." (Fiona, Facilitator, 2003)

The structure provides direction for teachers.

> When we present them with the Framework is without doubt the most powerful [time]. They get this enormous sense of knowing that they are going to know where the students are, they are going to know where they have been and where to take them next … they have never had that – knowing where from and where to. (Erica, Facilitator, 2003)

This confirms some facilitators' comments in the 2001 project (Higgins, 2002):

5 This reference to the Number Framework includes both the underlying ideas as explained through the stages of the framework.

> If you can keep your sights on the Framework [you have] a clear sense of those progressions embedded in your head. It doesn't matter what resources you particularly have in front of you or don't [have]. You can make do with a few things and you have got then the sense of where your children are and where you will be too, and so that's really what I have been trying to get teachers to see as the aims of this project. (Kay, Facilitator, ANP 2001)

The structure enables the dynamic to build momentum, to avoid prescription. It enables teachers to become autonomous in implementing the key ideas of the project.

> That's what we're doing, we're giving teachers a structure without giving them "You will do page this or that". … I'm in favour of resources … whatever you might be using, [but] it's which bit are you going to choose to meet the needs of a framework and the kids within that framework now, not just because you have got that resource to use. … And that's the bit that's missing. Teachers need to know the framework. (Emma, Facilitator, ANP 2001)

Multiple Interpretations and Transposition of Structure

Multiple interpretations of the structural components of the project enabled all participants to contextualise the professional development through transposing and extending the essence of the structure *inside* the settings in which they operate.

Interpretation of structure and the way facilitators and teachers use it governs their respective agencies (or capacity to act). Facilitator and teacher agency arises from the capacity to transpose and extend the schema to new contexts (Sewell, 1992). In order for facilitators to be able to extend and transpose the schema of the levels of complexity in understanding and representing number ideas successfully in unfamiliar cases or settings, they need a sound understanding of the schema.

> How do you bring the Number Framework to life? You bring [it] to life through real kids and teachers interpreting it in terms of what their own students are doing. (William, Facilitator, 2003)

> The framework has clearly been developed from what students are showing us rather than from where mathematicians have decided students should get to. … I think it has been very easy for teachers to see that this is what the students can do. (Erica, Facilitator, 2003)

> I believe the way to proceed in terms of professional development, whether it's teachers or facilitators, is to work from the specific, with examples of real kids doing the stuff, to a generalised sort of classification system which gives you the lead-in to interpret that stuff and then reflect back onto some more specifics to thrash it out really. (William, Facilitator, 2003)

Transformation of the structure over time appears to be a critical precondition in the sustainability of the project.

> You can argue about the detail and whether we need more stages or not, I think that that argument is not now going to take place because I think we are getting to the point where the structure's there, danger ossification. But

> for teachers to see, look this is only giving you a way of looking roughly at the cognition of the kids you've got. (Roger, Facilitator, 2003)

> We always emphasise that it's not a programme … we say it's an evolving project … there will continue to be subtle changes as it gets refined. (Erica, Facilitator, 2003)

The structure is important as it clarifies that the project is about concepts rather than activities. It sharpens the focus on the key aim of generating students' understanding of increasingly complex mathematical ideas and allows the facilitator and the teacher to interpret the structure to fit *within* their context and *inside* their everyday practice.

In-class Modelling and Feedback

In-class modelling has been identified as an important aspect contributing to the success of the Numeracy Project (Higgins, 2002; Thomas and Ward., 2002). The pedagogy of the project is centred on in-class modelling of an interactive approach, structured by the Number Framework and the Teaching Model. By situating the professional development in the teachers' classrooms, teachers were positioned as enactors and interpreters of the fundamental ideas of the project in terms of their classroom context. The professional development was generated *internally* rather than being imposed from the outside as in a more traditional approach to teacher education. Both teachers and facilitators put their own interpretations on the structure through using it *inside* their own teaching. This is a precondition for teachers' internalisation, efficacy, and authentication of the new teaching practices.

The ability of a facilitator to quickly assess the teacher's context of practice is important in tailoring the professional development programme to specific teachers' needs.

> We've got to be able to set up this climate of trust so that the teachers will reveal more of that context than what we are actually seeing. (Erica, Facilitator, 2003)

> What you've got to try and get them to do as facilitators is act as filters in the sense that they observe that complexity that's going on in classrooms with teaching and then are able to filter out of that the key aspect that they're going to concentrate on with a particular teacher. (William, Facilitator, 2003)

> They've picked up on the model … they've then gone and tried it out and now … they are actually growing their own teaching [methodology] from the basic data that I gave them. (Steve, Facilitator, 2003)

The pivotal role of in-class modelling allowed the facilitator to contextualise the professional development for individual teachers *inside* their own teaching practice. This orients the facilitators to a starting point of the teacher rather than the programme itself and enables the facilitator to create the pre-condition necessary to the end goal of teachers autonomously developing their practice for themselves *inside* the school.

Conclusion

> People say that you have to empower people but in actual fact the person has to empower themselves … you can't go around empowering people, it has to come from them … but as facilitators we need to get them so they want to ask more … they want to challenge more. (Nancy, Facilitator, 2003)

The challenge in professional development programmes is to foster a dynamic that can lead teachers and school to sustaining a project from *within* the school and classroom.

Foundational conditions of teachers' everyday practice fuel the dynamic of implementing, internalising and sustaining improvement to practice. The role of the facilitator *needs to be modified* as a teacher internalises changes to their classroom practice and a school community becomes capable of sustaining the project.

Chapter 6: Sustainability

Issues of sustainability are important to any externally generated initiative with the ceasing of the external support. The degree to which the essence of the project is reflected in sustained classroom practices is governed by the extent to which teachers within a school internalise the classroom implementation.

The role of the facilitator evolves during the formative stages of implementation and the internalisation process. As this progression proceeds, the facilitator then shifts from an emphasis on individual teachers working collegially to that of working with the lead teacher and principal on fostering a school-wide dynamic that supports sustainability of the changes.

Implementing and sustaining the project can be defined by the continuing enactment of the structural elements of the project. The analysis of structure draws on Sewell's (1992) notion of structure described in the previous chapter. Facilitators and lead teachers can be seen as playing the part of mediator or interpreter of "meaning" between the structure of the project and the structure of the school. An important consideration is the degree to which the positions of lead teacher and facilitator in particular school settings are effective in endorsing and accommodating the structural elements of the project within the school structure.

Sustainability arises from the "inside" dynamic, especially the efficacy of a school staff interpreting a professional development project and transforming their practices. In other words, what happens inside the school is of overall importance to the extent to which the ideas of the project are established, internalised, and authenticated.

Multiple interpretations of the structural components of the project (and their shared common ground) enable all participants to interrelate the development through transposing and extending the essence of the structure (the Framework and the Teaching Model) and the settings in which they operate.

Interpretation of structure, and the way facilitators and teachers use it, governs their respective action in implementing the ideas of the project within the frameworks in which they are embedded, namely, the school structure and the project structure. This chapter examines the progression that fuels the school-wide dynamic essential to sustaining the project beyond its initial implementation phase.

Individual Teachers: Starting Point

For individual teachers the internalisation process begins with their sense of new ideas being viable through their pedagogy. The imperative for teachers to make changes to their practice, as has been reported previously (Higgins, 2001 and 2002), arises from evidence of shifts in children's thinking as articulated in the Number Framework. This, for teachers, is evidence of efficacy and collectively it serves to authenticate long-term commitment.

> I think there's been a change in attitude from a lot of our teachers and that they've seen success, they've seen the results. … Children [are] making progress and they've seen that the children are really enjoying maths now and so I think a lot of them are more committed to the actual teaching of maths. (Carol, Lead Teacher, 2003)

> Getting staff through that realisation that their professional judgement, their listening to the children, their looking at the child … to say, "OK, right, I'm quite confident that they've got that understanding, they're using that skill, they're using that strategy, they're ready to move on." (Maureen, Lead Teacher, 2003)

> When you see the change in your teaching and when you feel … for me … it was something like … and a few people have said this to me … hey I actually feel like I know what I'm teaching now, because sometimes I felt like you could stick me in a dark room and I might as well as just throw a dart and if I got it OK that was fine … but it's made all those links and a lot of people are like that … oh yeah, I can see what is happening now. (Diane, Lead Teacher, 2003)

The in-class modelling by facilitators was powerful in providing teachers with a sense of efficacy because it took place inside individual teachers' classrooms.

> The in-class modelling I thought was a real key. It wasn't removed from your classroom. It was actually real. (Beryl, Lead Teacher, 2003)

> You listen and the best way to learn of course is to see it in action and so … OK the maths people came in and they demonstrated and so therefore teachers actually saw it with their kids and thought, "Oh yeah, that's how it goes." (Owen, Principal, 2003)

The powerful impact on teachers of seeing the Framework enacted in their classrooms, both in the sense of providing clear goals for children's learning and as an emphasis on children explaining their mathematical thinking, provides an incentive for teachers to transform their practice.

> People are just so excited by the children actually having to explain how they do it and how they know it before actually ticking off to say, OK, well this child is at this particular stage or level. … Children need to ask questions but they also need to be able to substantiate and find and discover and prove what they did and why it worked or why it didn't work, so that whole pedagogy of enquiry. (Maureen, Lead Teacher, 2003)

Peer Support Structures: Internalising the Initiative

Peer support is an important component of the internalisation process for individual teachers. In broad terms this is about teachers talking together about the changes they are making to their classroom practice and their developing knowledge of the Framework and how best to develop children's understanding of number.

> I think it's been the most successful externally stimulated initiative that we've been involved in and I think the model should be interpreted by the Ministry as being really successful, therefore … if this school's part of any further initiatives it should at least be based on the same criteria as the Numeracy Project … I think the cooperative notion of teachers working together too and sharing a completely new methodology of staging children … coming to grips with how children learn mathematics. (Neil, Principal, 2003)

Some schools encouraged teachers not only to talk together, but also to observe each other's practice as a means of developing it. One lead teacher described the characteristics of those likely to sustain the changes as being "open minded, willing to learn and to change their ways of maths teaching where applicable, keen to share and to be observed". She explained that:

> We very much encourage teachers not to lock themselves in their own classrooms. Here we have an open door policy where with good teaching practice we get teachers, as many of the teachers as possible, especially the new teachers, to go into those rooms and actually observe. That's where they are going to pick up their good teaching practice. (Linda, Lead Teacher, 2003)

More formal structures, such as the "critical friend" model or buddy teachers, provided effective processes for peer support.

> We have continued [the model of professional development] with a critical-friend-type model. This involves buddying them up with somebody working with children at the same stage. The critical friend has made it ongoing. (Steve, Principal, 2003)

> We have used the critical friend model to go in and observe each other. (Beryl, Lead Teacher, 2003)

> We've implemented buddy observations. We've got lots of teachers that go in and out of other classes and watch people taking maths in action. (Linda, Lead Teacher, 2003)

Consideration of effective feedback on classroom practice led some schools to consider the use of video.

> While our teachers talk the talk, that's one thing, we insist that we have a common vocabulary and all teachers can talk. The second thing is through that talk we've got the intellectual knowledge and the next step is then them observing a good model and then the next step is being observed, which then we can focus more specifically on what coaching and feedback has to happen. … Next year one of our major focuses is the use of the video, teachers seeing themselves and being very much part of the mentoring and coaching and feedback model. … Adopting the methodology of increased observation. That was the part that was the weakest part. … It really woke us up with a big jolt because the school's quite strong in change mentality and getting better and better at it. (Neil, Principal, 2003)

Through talking about their practice, teachers have the opportunity to consider multiple interpretations of the stages of the Framework as well as the Teaching Model. The ideas under discussion are referenced to the everyday practice of their classrooms.

School-wide Dynamic: Conditions for Sustaining the Project

The dynamic arising from a collective focus on raising student achievement through developing teacher knowledge and refining pedagogy creates a necessary but not sufficient condition for sustaining the project. The lead teachers and facilitators are important as interpreters of "meaning" between the structures of the project and those of the school.

> I still have always believed that it has to be whole-school professional development. … [The Numeracy Project] was good because everyone was together and we're all learning the same thing at the same time. (Carolyn, Principal, 2003)

> You need whole-school development and teachers need to be working together with a common goal. … I don't think that my belief has changed but I think that it's been enhanced through the Numeracy Project development because there was a real expectation for our teachers … that they were going to have to make the grade … through this development. (Carol, Lead Teacher, 2003)

> I think that because we've embraced it as a whole staff, it's far more powerful than one team or just a few teachers doing it. (Nigel, Principal, 2003)

The sustainability of the project is influenced by the fit between the aims of the project and the aims of school. The more closely these align, the better the chances of longer-term sustainability. The possibilities for such alignment are improved when a school develops a school-wide focus led by the principal, with the assistance and guidance of both the facilitator and lead teacher.

> In terms of sustained change through professional development, I think it's going to happen with teachers if it's part of a bigger picture, so if they're doing numeracy for example, they're doing it because it's a school-wide focus to change the numeracy delivery in classrooms and it needs to be supported. (Nigel, Principal, 2003)

It is also important to fit the project into the school structure. In some cases this involves a reinterpretation of the key ideas guided by the facilitator and lead teacher.

> …. We needed to do the pedagogy and where the school was going as a whole within curriculum programmes and then fit the numeracy in … within that framework. (Maureen, Lead Teacher, 2003)

> I feel that this numeracy programme has come alongside us, it hasn't directed us from the top and I think that's been accepted and people basically enjoyed it. … It's such an excellent model in its own way of helping teachers change. I think it's been well thought out. (Carolyn, Principal, 2003)

> Each initiative is interpreted in terms of the school culture and where we joined because we're aware that we need to get better and the Ministry supplying some of impetus through financing and making sure they've got good people facilitating it. … We take what we want from it. (Neil, Principal, 2003)

The reinterpretation process may also result in adjustments to school structures. Flexible structures provided a school with options.

> … it's making the school organisation extremely flexible. … The structure of ours is to do whatever we need to do. (Neil, Principal, 2003)

> It's far more effective to look overall at your curriculum delivery and that's what we did. We started with our pedagogy, what were we doing and why were we doing it, and then we looked at our total curriculum package. So we've reviewed the whole shooting box basically, and I think in terms of numeracy because it was part of our bigger picture stuff, that's what makes it real. (Nigel, Principal, 2003)

The quality of guidance from the facilitator is important to the process.

> I think the whole thing to do with any in-service depends on the credibility of the facilitator and if you haven't got a good facilitator or you … doesn't relate as well to your staff … forget it … the projects not going to go and that's in any subject … any area. (Owen, Principal, 2003)

> [The staff's] enthusiasm and dedication really, they've really come on board with it and without that it would be a pretty hard ship to sail in terms of something new like this. They've grasped it fully. … It went back to the enthusiasm and the quality of our facilitator getting people on board for a start, but also the professionalism of my staff to be willing to come on board and make changes. (Tony, Principal, 2003)

The shared nature of the initiative contributes to the effectiveness of a lead teacher's position.

> That you all talk the same talk. (Beryl, Lead Teacher, 2003)

> [As everyone knows what I am talking about] I feel … that has helped me to sustain it as a lead teacher. (Diane, Lead Teacher, 2003)

Sustaining School-wide Support: Conditions for Authentication and Learning

Creating a school-wide dynamic described by one principal as "a common thread of interest" was contrasted to developments outside the school by several of those interviewed.

> This leaving the school and going to one-day courses is probably a waste of time as far as I'm concerned. I believe it must be a whole school shared with consistent language across the school and that opportunity to watch someone else work in your classroom is just so powerful, it was a brilliant approach and … it really is a good way of learning. So this whole-school thing is what I believe in now. (Beryl, Lead Teacher, 2003)

> I guess part of the shift however has been in recent times a focus more on whole-school development as opposed to going outside of school and so that's a significant shift because you're all working on the same project and you can all manage or be involved in the same project and be on the same wavelength. Sometimes … it's very hard for the facilitator to accommodate all people [when] you have people coming from all sorts of schools to an outside environment. (Edward, Principal, 2003)

The focus on achievement data was the single most important factor in a common school focus as it provided a measure of success.

> The focus on achievement has really kept us going with it because it's seen as so important. (Trina, Lead Teacher, 2003)
>
> Student data is [about] the next step learning, reporting to parents, reporting to the board. So to look at school-wide improvement overall but if you aren't looking for the next step then you're not going to be showing much school-wide improvement overall because you've kind of like missed the boat. The big step is being able to see where the next step is for the children. … So again it's always looking at this is where you're at … what do you expect next from these children and there's a lot more I suppose dialogue, shall we say, in the staff room a lot more mathematical dialogue now than what there has been for a while, which is really great … of talking of what children are doing and how they're doing it and things like this and a lot more discussion on strategies that work, materials that are working well and sharing those activities that are doing well. (Maureen, Lead Teacher, 2003)
>
> There's a real shared knowledge and there's a shared language in the staff room too and teachers have often observed one another and they really focus on their achievement data. … I feel the teachers are certainly working together. (Carolyn, Principal, 2003)

One school found that it was useful to share the results of the diagnostic interview with the children as this provided motivation and a sense of ownership to the children. Some schools also found that they "changed and tweaked and adapted" the assessment as they became more familiar with it. This school used the "I can" sheets[6] as part of their formative assessment process. Some schools also noted that the improvement in teaching resulted in longer periods of consolidation in which changes in children's understanding appeared less obvious.

> Now if everybody is actually teaching good programmes it means that they personally don't get that jump start. … We need to consolidate each of those stages and make sure that we're not just moving children on because it's politically nice to do so and we need to make sure that we're actually teaching through all those stages. Realistically, the more you see, the more in depth you look at each stage of the amount of knowledge and the amount of different ways the children are to be seen to be using all those strategies. … If there's a wee bit of a plateau here it could be because of very good programmes coming right through. (Maureen, Lead Teacher, 2003)

Part of sustainability is interpreting and adapting the structures for a specific situation. One school described how they developed additional benchmarks to close the gap between stage four and stage five.

> What we did is we developed our own benchmarks and we have a stage four developing, a stage four consolidating, a stage five developing and a stage five consolidating and what that does is just closes those gaps and shifts the goalposts and the expectations a little bit. … They are huge steps as you go

[6] The "I can" sheets are part of the assessment component of the Numeracy Project.

> further up the framework but as I say numeracy is a philosophy, it's not a programme and we've just tried to adapt it to meet the needs of our students and our staff and to meet our goals. (Diane, Lead Teacher, 2003)

As the project becomes embedded in the school culture, the focus of the role of facilitator may change from one focused on organisation to that of mentor and coach. This is yet another example of transformation that emerges from the school-wide dynamic that fuels the sustainability.

Conclusion

There appear to be three important components that fuel the dynamic of sustainability: innovation which leads to internalisation, which leads to sustainability. Innovation in teaching practices amongst a small group of staff provides a foundation for individuals to internalise changes to their practice. A collective, internalised effort creates a dynamic from which changes can be sustained at the school level. The role of the facilitator and lead teacher as interpreters of "meaning" of structural elements shifts as the dynamic of sustainability evolves.

> I think we were stopping before at the talking the talk and the teachers' intellectual knowledge and we were hoping it translated into practice. So because of that we are spending a lot more time observing and a lot more time giving feedback and a lot more time modelling, so the model, observing, feedback, coaching parts have been strengthened hugely because of [the Numeracy Project]. (Neil, Principal, 2003)

> There are probably three key characteristics in terms of sustaining the change. The first one would be support; a teacher who is supported will obviously make the change and sustain it. Another one would be challenge; a teacher is challenged about what they are delivering in their programme and that whole pedagogy thing of why you're doing what you're doing. And the third thing I think would be success; it's seeing the success of the children within the programme and seeing the development that they've made and as a teacher feeling successful around what you're teaching the children in terms of numeracy. (Nigel, Principal, 2003)

The progression from initiation to sustaining practices unfolds through cooperative relationships between the facilitator, lead teacher, principal and teachers. These relationships undergo reconfiguration as the progression occurs. Ultimately the teachers and the school gain a sense of autonomy that is bounded by the project's network of dynamic schools where teachers feel a sense of efficacy evidenced by student learning.

Conclusion

For the third year running, the majority of students improved during their participation in the Advanced Numeracy Project. As in 2001 and 2002, this growth was irrespective of students' age, ethnicity, gender, and decile. Across all year levels and as for previous years, the results for addition and subtraction were slightly better than for the other operational domains, with the number of students remaining at the same stage at the end of the project increasing with each advancing stage. The patterns of results for the operational domain of multiplication and division followed a similar pattern to those in 2002. The percentages of students moving to the more advanced stages of this domain increased across the year levels and were greater than the figures for the domain of proportions and ratios but less than the figures for addition and subtraction.

When students' starting place is taken into account, their pattern of achievement for proportions and ratios was consistent with that found for the addition and subtraction and multiplication and division operational domains in that fewer students shifted as the stages increased in sophistication. This confirmed the 2002 figures. The pattern for proportions and ratios is the same as for 2002, with students starting at a lower stage than for the other domains. The pattern for the knowledge aspects also confirmed the 2002 results and was similar in terms of stages gained over the period of instruction.

The shift from counting-based to part-whole strategies is an important marker in judging the progress of students. For all strategy domains, the percentage of students across year groups shifting to part-whole strategies was the same as for 2002. The percentage of students who initially used counting-based strategies and shifted to part-whole strategies increased at each year level for each operational domain. The proportion of those who shifted to more advanced stages increased at each year level. The percentages for each year group were similar to those from 2002. The levels of knowledge for those with part-whole strategies were consistently higher than for those with counting-based strategies. The results for 2003 in general were better than those for 2002.

Compared to 2002 there was a big increase in addition and subtraction in the number of Year 4 students who in 2003 became advanced additive. As for 2002 the greatest shift across all year groups and operational domains was to the early additive stage. Fewer students moved to the stages of advanced additive and advanced multiplicative part-whole, representing more sophisticated part-whole thinking. Virtually no students shifted to advanced proportional part-whole.

The effect of ethnicity on student progress in addition and subtraction appears to lessen as students move up the year levels. When compared to the 2002 results there appears to be a decline in scores for most ethnic groups at Years 5 and 6. In Year 6 the percentages of those becoming part-whole declined for all ethnic groups except Pasifika students with those students categorised as Other and Māori declining the most. Comparatively more Asian students than any other ethnic group shifted to part-whole strategies at Year 4 and at Year 5. The effect of decile on the patterns of achievement was similar to that in previous years, with increasing numbers of students shifting to part-whole strategies across the composite groups of low (deciles 1 to 3), middle (deciles 4 to 7) and high (deciles 8 to 10) decile rankings. More students adopted part-whole strategies in 2003 than in 2002 for each decile group.

Effective facilitation appeared to centre on the structural components of the Numeracy Project rather than on classroom activities. Facilitators who appeared effective centred their work with teachers on the structural components of the Numeracy Project rather than on classroom activities. Multiple interpretations of the structural components of the project enabled all participants to contextualise the professional development through transposing and extending the essence of the structure (the Framework and the Teaching Model) to the settings in which they operate. Transformation of structures over time appears to be a critical component in ensuring the sustainability of the project.

Three components that fuel the dynamic of sustainability at the school level are the initial innovation of classroom practice by individual teachers, the internalisation of these practices, firstly at the individual teacher level and then through the cooperative efforts of a group of staff. The role of the facilitator evolves during the formative stages of implementation and the internalisation process. As this progression proceeds, the facilitator then shifts from an emphasis on individual teachers working collegially to that of working with the lead teacher and principal on fostering a school-wide dynamic that supports the sustainability of the changes.

References

Alton-Lee, A. (2003). *Quality teaching for diverse students: A best evidence synthesis.* Wellington: Ministry of Education.

Cobb, P., & McClain, K. (2002). Supporting students' learning of significant mathematical ideas. In G. Wells & G. Claxton (Eds.), *Learning for life in the 21st century* (pp. 154-166). London: Blackwell.

Cobb, P., McClain, K., de Silva Lamberg, T., & Dean, C. (2003). Situating teachers' instructional practices in the institutional setting of the school and district. *Educational Researcher*, *32* (6), 13-24.

Fennema, E., & Franke, M. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday classrooms. *Journal for Research in Mathematics Education*, *30* (2), 148-170.

Hattie, J. (2002). What are the attributes of excellent teachers? In *Teachers make all the difference: What is the best evidence?* (pp. 3-26). Conference Proceedings. Wellington: NZCER.

Higgins, J. (2001). *Evaluation of the Year 4-6 Numeracy Exploratory Study. Exploring Issues in Mathematics Education*, July 2001. Wellington: Ministry of Education.

Higgins, J. (2002). *An evaluation of the Advanced Numeracy Project 2001. Exploring Issues in Mathematics Education*, July 2002. Wellington: Ministry of Education.

Higgins, J. (2003). *An evaluation of the Advanced Numeracy Project 2002. Exploring Issues in Mathematics Education*. Wellington: Ministry of Education.

Higgins, J. with Parsons, R. & Hyland, M. (2003). The Numeracy Development Project: Policy to practice. *New Zealand Annual Review of Education,* 12: 2002, 157-175.

Hughes, P. (2003). *Stages of teaching*. Auckland: Auckland College of Education.

Ministry of Education. (2004a). *Book 1: The number framework.* Wellington: Ministry of Education.

Ministry of Education (2004b). *Book 3: Getting started.* Wellington: Ministry of Education.

Newman, D., Griffin, P., & Cole, M. (1989). *The construction zone: Working for cognitive change in school*. Cambridge: Cambridge University Press.

Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics*, 9, 7-11.

Remillard, J. (1999). Prerequisites for learning to teach mathematics for all students. In W. Secada (Ed.), *Changing the faces of mathematics: Perspectives on multiculturalism and gender equity*. Reston, VA: National Council of Teachers of Mathematics.

Sewell, W. (1992). A theory of structure: Duality, agency, and transformation. *American Journal of Sociology, 98* (1).

Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, *8* (1), 42-76.

Thomas, G., Tagg, A., & Ward, J. (2003). *An evaluation of the Early Numeracy Project 2002. Exploring issues in mathematics education*. Wellington: Ministry of Education.

Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me In Too pilot project: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Thomas, G., & Ward, J. (2002). *An evaluation of the Early Numeracy Project 2001: Exploring issues in mathematics education*. Wellington: Ministry of Education.

Wright, R. (1998). An overview of a research-based framework for assessing and teaching early number. In C. Kanes, M. Goos & E. Warren (Eds.), *Proceedings of the 21st annual conference of the Mathematics Education Group of Australasia,* Vol. 2 (pp. 701-708). Brisbane: Griffiths University.

Young-Loveridge, J. (2001). Helping children move beyond counting to part-whole strategies. *Teachers and Curriculum,* 5, 72-78.

Appendix

Gender	No shift		Became early additive		Became advanced additive		Totals	
Female	52%	(906)	45%	(773)	3%	(48)	100%	(1727)
Male	50%	(803)	45%	(714)	5%	(88)	100%	(1605)
Total	51%	(1709)	45%	(1487)	4%	(136)	100%	(3332)

Table A-1: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

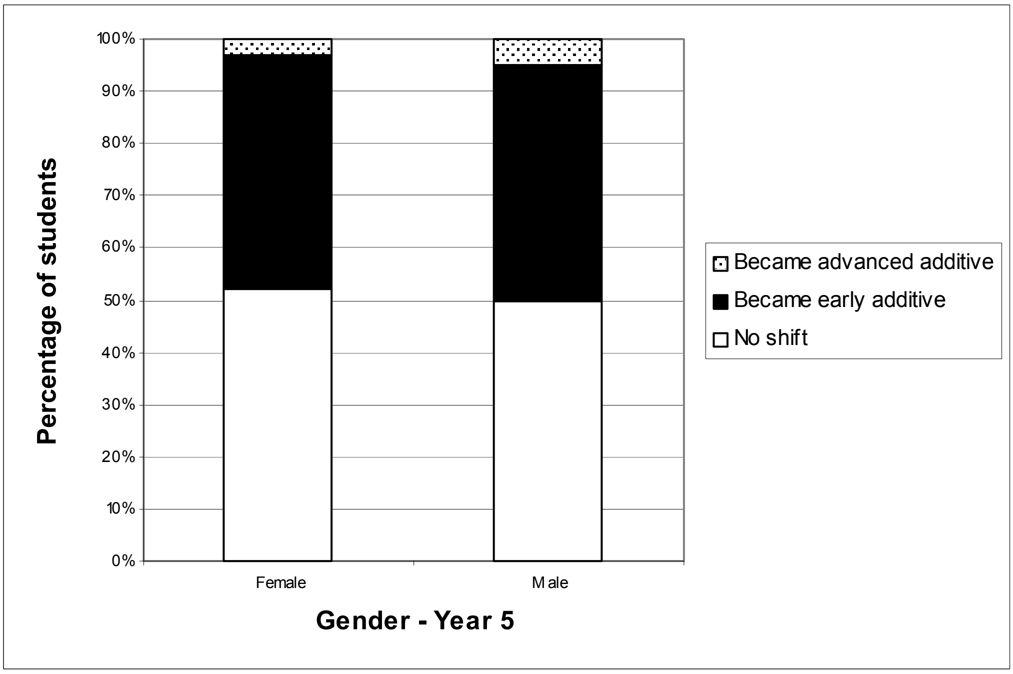

Figure A-1: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

Gender	No shift		Became early additive		Became advanced additive		Totals	
Female	44%	(659)	49%	(729)	7%	(98)	100%	(1486)
Male	42%	(526)	51%	(637)	7%	(81)	100%	(1244)
Total	43%	(1185)	50%	(1366)	7%	(179)	100%	(2730)

Table A-2: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

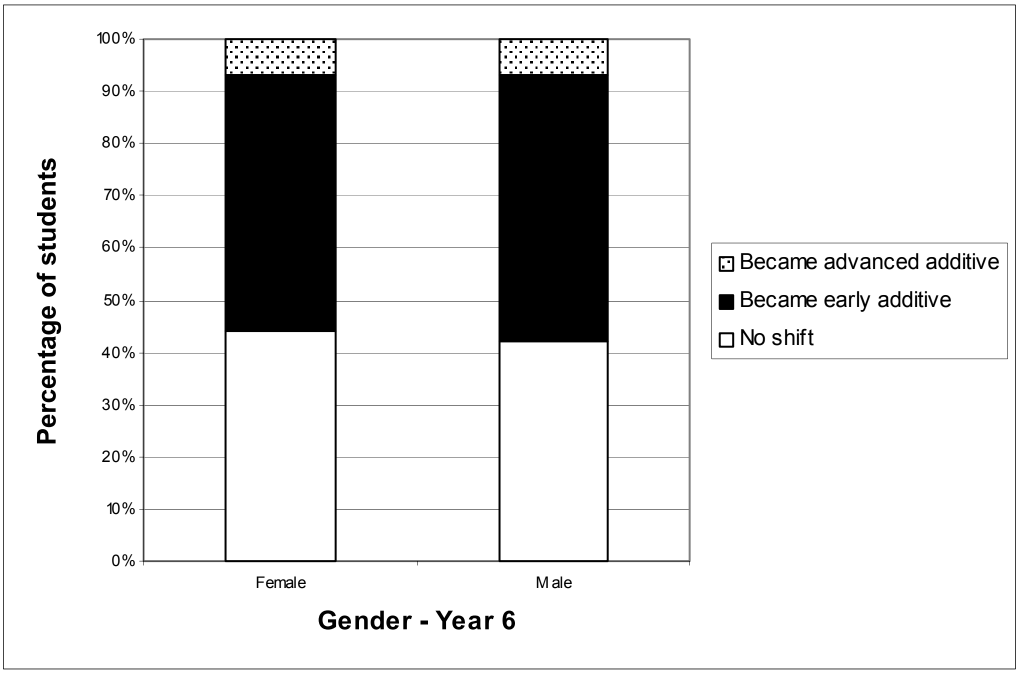

Figure A-2: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

Gender	No shift		Became early additive		Became advanced additive		Totals	
Female	54%	(3204)	43%	(2565)	3%	(189)	100%	(5958)
Male	51%	(2699)	45%	(2360)	4%	(232)	100%	(5291)
Total	52%	(5903)	44%	(4925)	4%	(421)	100%	(11249)

Table A-3: All years: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

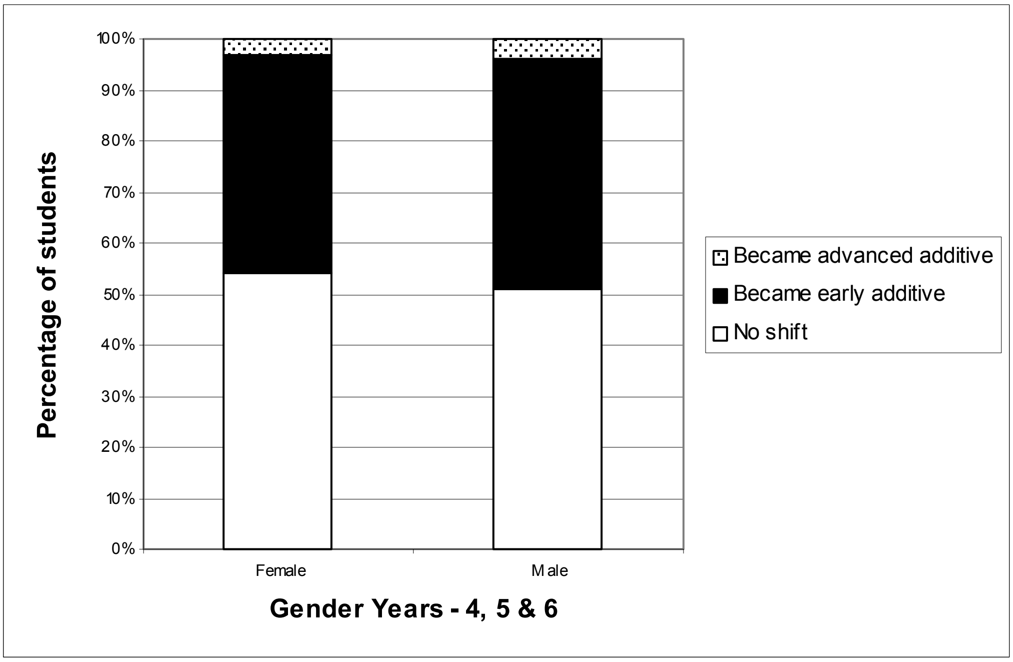

Figure A-3: All years: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by gender

Ethnicity	No shift	Became early additive	Became advanced additive	Totals
NZ European	47% (870)	48% (885)	5% (80)	100% (1835)
Māori	55% (548)	41% (405)	4% (40)	100% (993)
Pasifika	63% (191)	34% (102)	3% (9)	100% (302)
Asian	41% (27)	53% (35)	6% (4)	100% (66)
Other	54% (73)	44% (60)	2% (3)	100% (136)
Total	51% (1709)	45% (1487)	4% (136)	100% (3332)

Table A-4: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by ethnicity

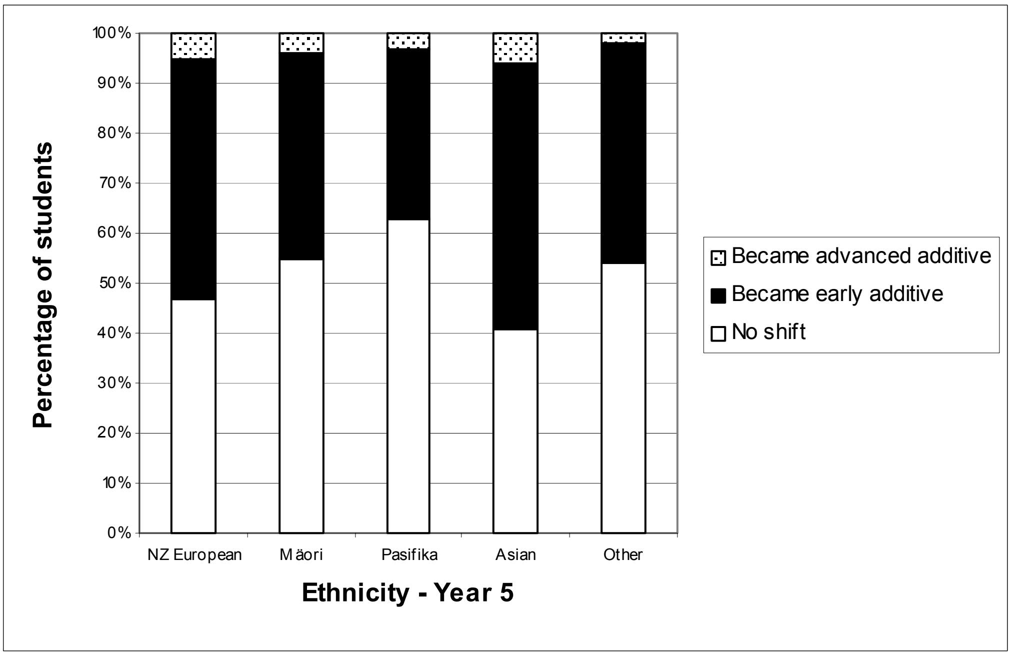

Figure A-4: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by ethnicity

Ethnicity	No shift	Became early additive	Became advanced additive	Totals
NZ European	38% (527)	55% (757)	7% (100)	100% (1384)
Māori	49% (438)	46% (419)	5% (47)	100% (904)
Pasifika	53% (147)	41% (113)	6% (16)	100% (276)
Asian	42% (25)	43% (26)	15% (9)	100% (60)
Other	45% (48)	48% (51)	7% (7)	100% (106)
Total	43% (1185)	50% (1366)	7% (179)	100% (2730)

Table A-5: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by ethnicity

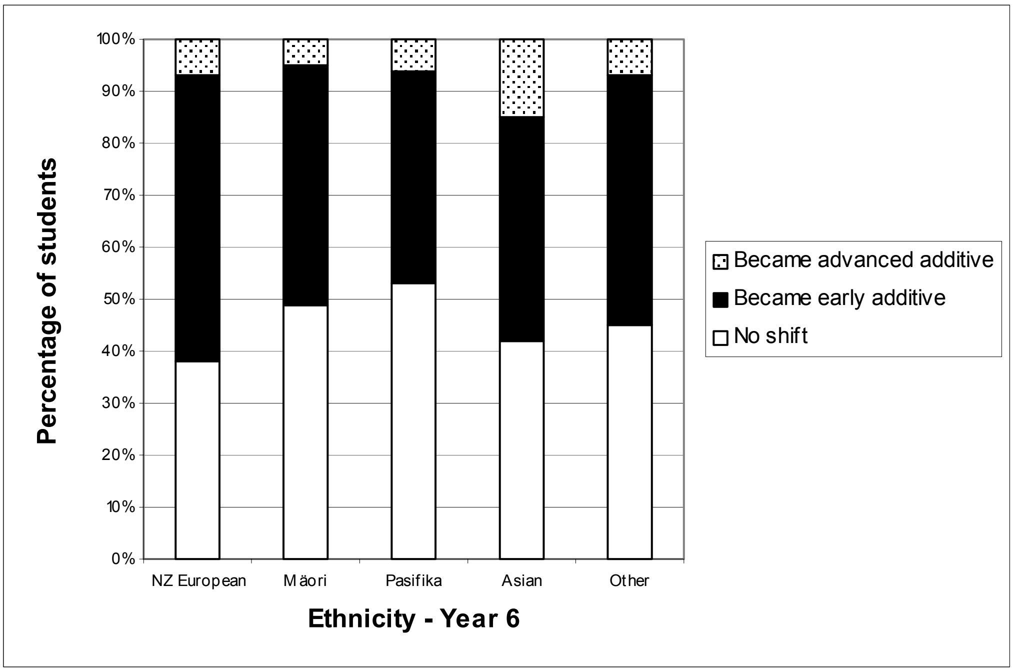

Figure A-5: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by ethnicity

Decile	No shift	Became early additive	Became advanced additive	Totals
1	62% (343)	34% (191)	4% (21)	100% (555)
2	56% (241)	38% (164)	6% (23)	100% (428)
3	51% (222)	46% (199)	3% (14)	100% (435)
4	52% (155)	44% (131)	4% (12)	100% (298)
5	54% (228)	44% (187)	2% (8)	100% (423)
6	45% (119)	51% (136)	4% (12)	100% (267)
7	46% (94)	52% (107)	2% (5)	100% (206)
8	47% (143)	48% (147)	5% (14)	100% (304)
9	40% (61)	53% (79)	7% (10)	100% (150)
10	39% (103)	55% (146)	6% (17)	100% (266)
Total	51% (1709)	45% (1487)	4% (136)	100% (3332)

Table A-6: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by decile

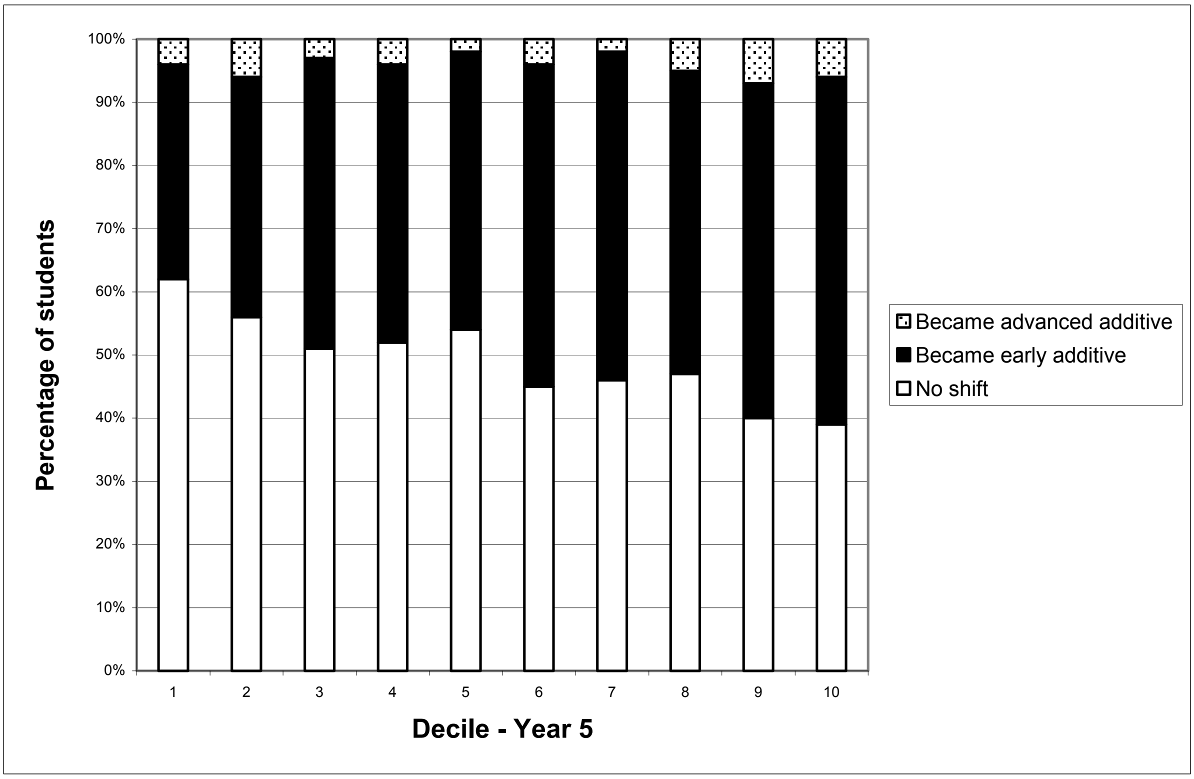

Figure A-6: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by decile

Decile	No change		Became early additive		Became advanced additive		Totals	
1	50%	(261)	46%	(241)	4%	(24)	100%	(526)
2	52%	(204)	41%	(162)	7%	(28)	100%	(394)
3	46%	(145)	51%	(159)	3%	(10)	100%	(314)
4	40%	(106)	49%	(128)	11%	(30)	100%	(264)
5	40%	(156)	56%	(222)	4%	(17)	100%	(395)
6	38%	(75)	54%	(108)	8%	(16)	100%	(199)
7	45%	(62)	51%	(70)	4%	(6)	100%	(138)
8	41%	(75)	55%	(99)	4%	(8)	100%	(182)
9	38%	(35)	53%	(49)	9%	(8)	100%	(92)
10	28%	(63)	57%	(128)	15%	(32)	100%	(223)
Total	43%	(1182)	50%	(1366)	7%	(179)	100%	(2727)

Table A-7: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by decile

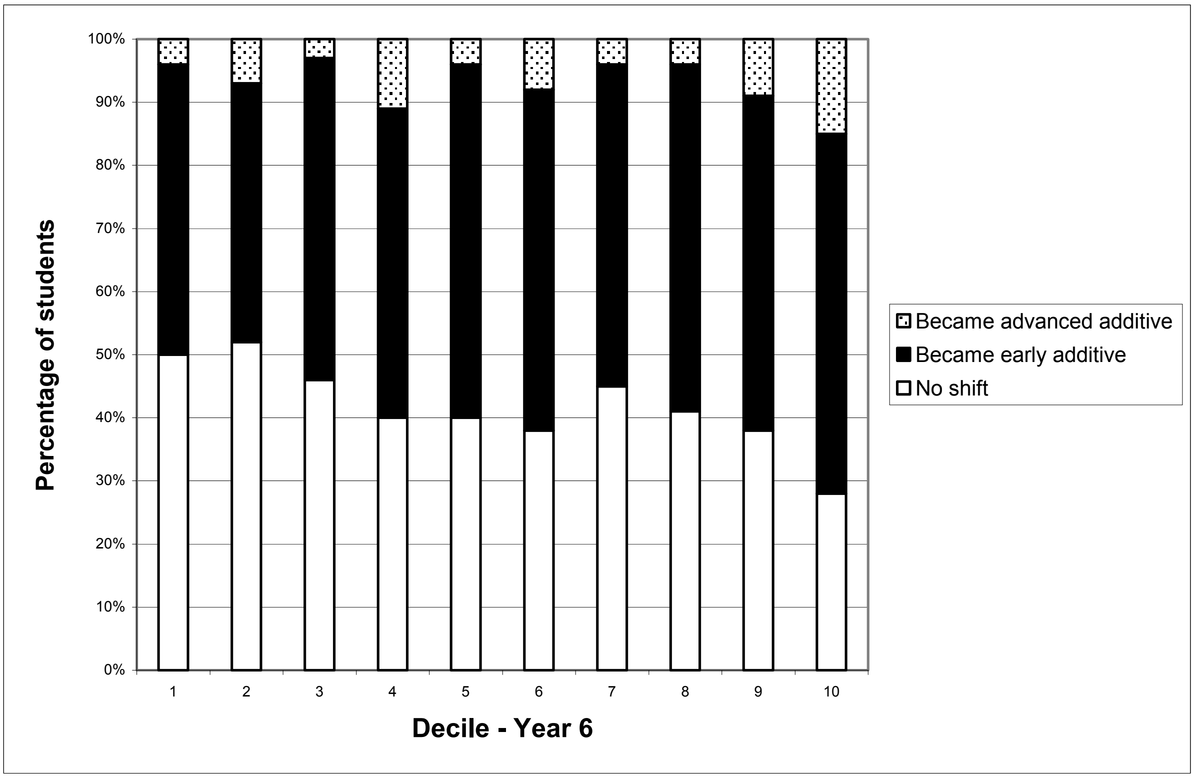

Figure A-7: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by decile

Strategies	Emergent	Initial up to 10	Up to 10	Up to 20	Up to 100	Up to 1,000	Up to 1,000,000
FNWS							
Counting-based	4% (64)	0% (6)	0% (8)	2% (44)	25% (443)	62% (1096)	6% (12)
Part-whole	2% (111)	0% (0)	0% (0)	0% (1)	3% (165)	58% (3012)	37% (1936)
BNWS							
Counting - based	4% (64)	0% (7)	1% (25)	3% (57)	32% (572)	53% (931)	7% (117)
Part-whole	3% (146)	0% (0)	0% (1)	0% (4)	6% (316)	57% (2981)	34% (1777)

Table A-8: Year 5: FNWS and BNWS – knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s & within 10s	With 5s & within 10	With 10s	10s in 100	10s & 100s in whole numbers	10s, 100s & 1000s in whole numbers	10ths, 100ths & 1000ths in decimals
Counting - based	4% (63)	8% (138)	23% (402)	47% (838)	16% (291)	2% (39)	0% (1)	0% (1)
Part-whole	2% (124)	0% (11)	3% (147)	27% (1392)	40% (2102)	15% (795)	9% (495)	3% (159)

Table A-9: Year 5: Grouping – knowledge for counting-based and part-whole

	Not assessed	Non-fractions of regions	Assigned unit fractions	Ordered unit fractions	Coordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting - based	16% (284)	19% (339)	35% (623)	28% (490)	2% (36)	0% (0)	0% (1)
Part-whole	2% (120)	6% (331)	19% (992)	51% (2639)	15% (782)	5% (243)	2% (118)

Table A-10: Year 5: Fractions - knowledge for counting-based and part-whole

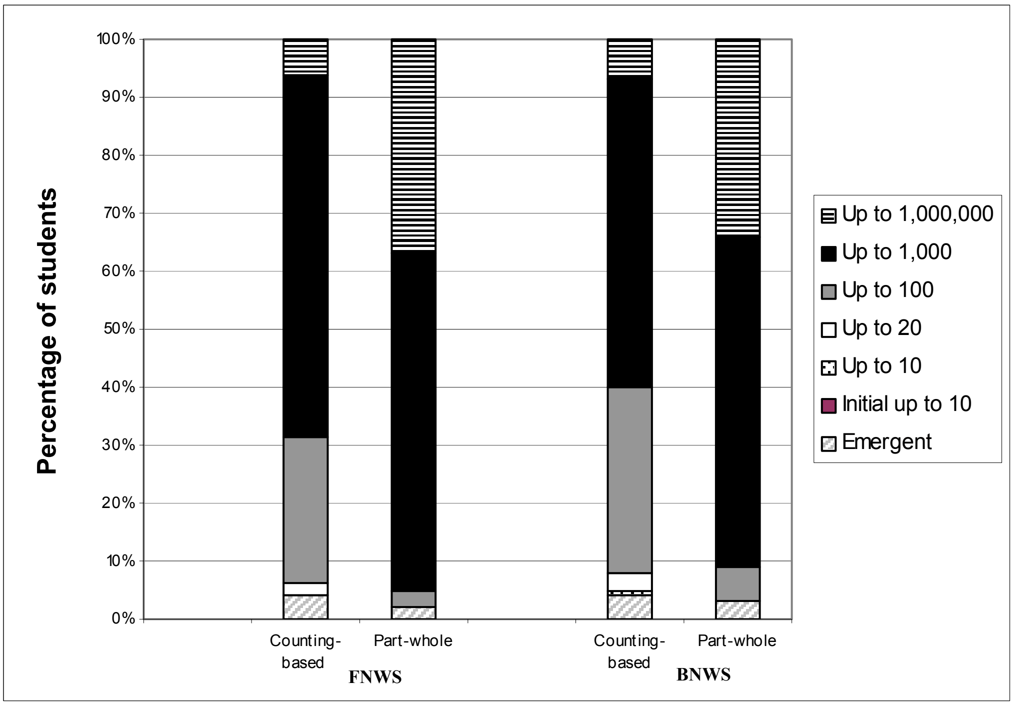

Figure A-8: Year 5: FNWS and BNWS - knowledge for counting-based and part-whole

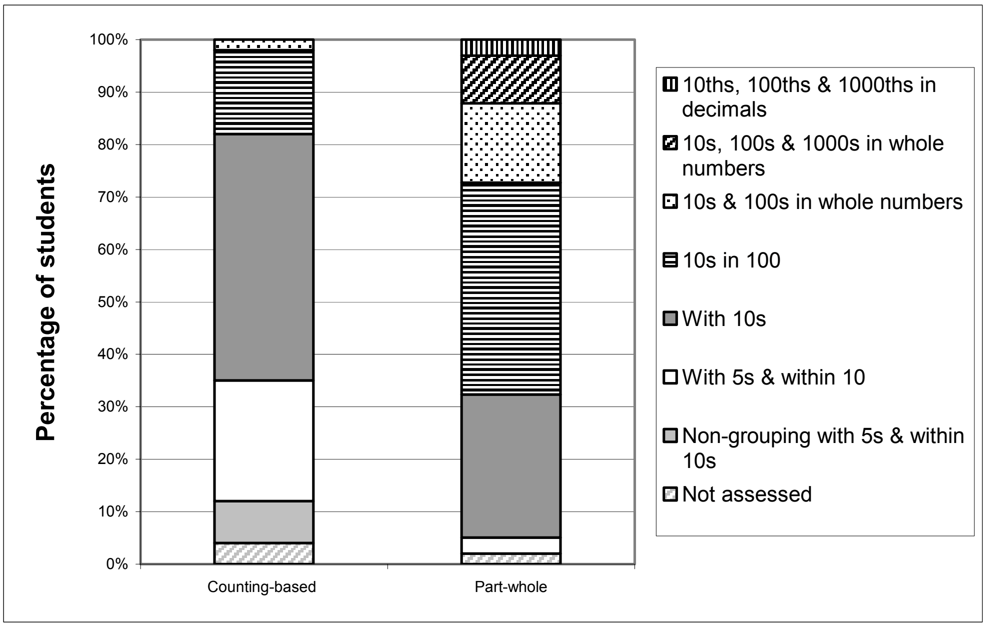

Figure A-9: Year 5: Grouping - knowledge for counting-based and part-whole

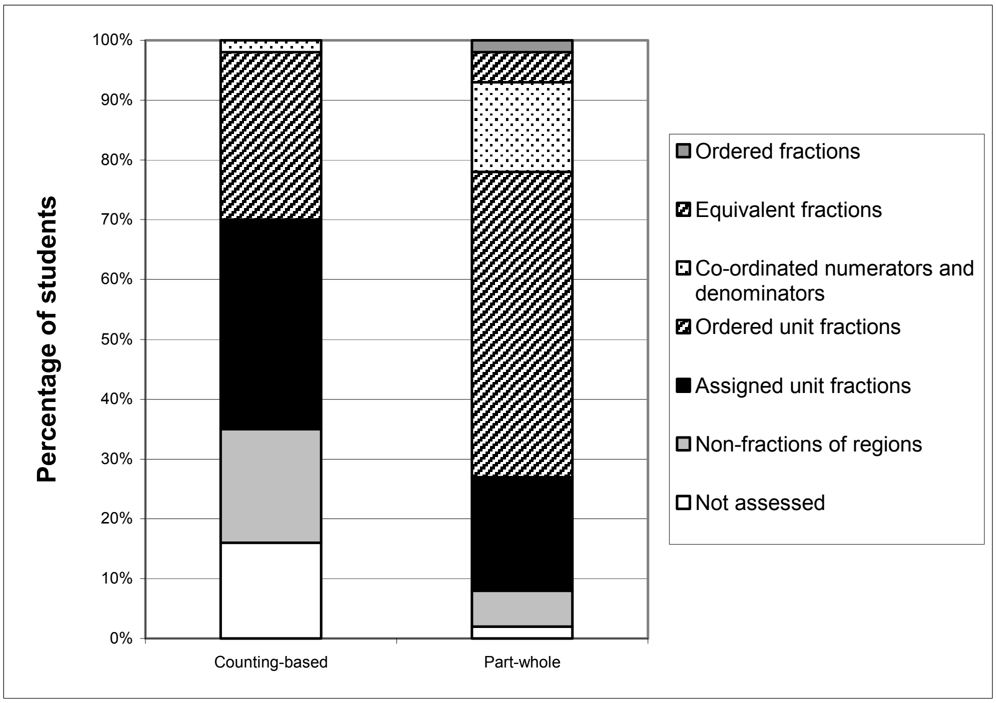

Figure A-10: Year 5: Fractions - knowledge for counting-based and part-whole

Strategies	Emergent	Initial up to 10	Up to 10	Up to 20	Up to 100	Up to 1,000	Up to 1,000,000
FNWS							
Counting - based	4% (52)	0% (4)	1% (10)	2% (24)	18% (229)	64% (803)	10% (128)
Part-whole	2% (137)	0% (1)	0% (0)	0% (0)	2% (94)	46% (2761)	50% (2961)
BNWS							
Counting - based	4% (56)	1% (9)	1% (15)	3% (39)	25% (314)	56% (700)	9% (117)
Part-whole	3% (158)	0% (0)	0% (1)	0% (1)	3% (190)	45% (2704)	49% (2900)

Table A-11: Year 6: FNWS and BNWS – knowledge for counting-based and part-whole

	Not assessed	Non-grouping with 5s & within 10s	With 5s & within 10	With 10s	10s in 100	10s & 100s in whole numbers	10s, 100s & 1000s in whole numbers	10ths, 100ths & 1000ths in decimals
Counting-based	4% (50)	8% (100)	18 % (224)	45% (568)	20% (245)	5% (59)	0% (4)	0% (0)
Part-whole	2% (130)	0% (13)	2% (143)	19%(1142)	32%(1914)	20%(1173)	17%(1003)	7% (436)

Table A-12: Year 6: Grouping – knowledge for counting-based and part-whole

	Not assessed	Non-fractions of regions	Assigned unit fractions	Ordered unit fractions	Coordinated numerators and denominators	Equivalent fractions	Ordered fractions
Counting-based	13% (164)	19% (240)	30% (378)	34% (428)	3% (38)	0% (2)	0% (0)
Part-whole	2% (129)	4% (237)	16% (948)	43% (2537)	20% (1203)	10% (601)	5% (299)

Table A-13: Year 6: Fractions – knowledge for counting-based and part-whole

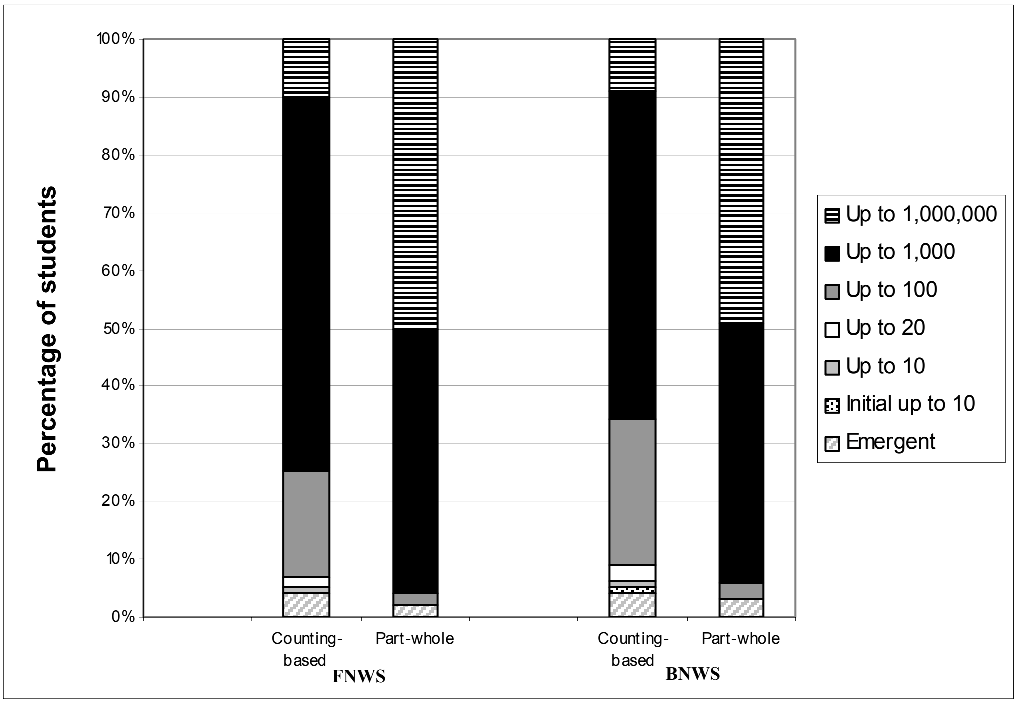

Figure A-11: Year 6: FNWS and BNWS – knowledge for counting-based and part-whole

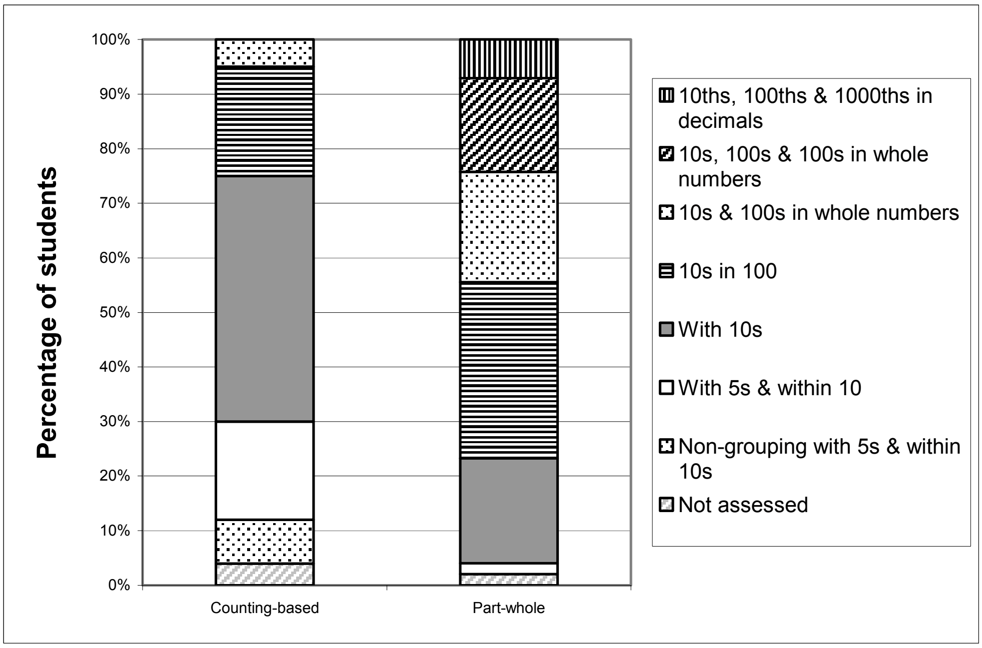

Figure A-12: Year 6: Grouping – knowledge for counting-based and part-whole

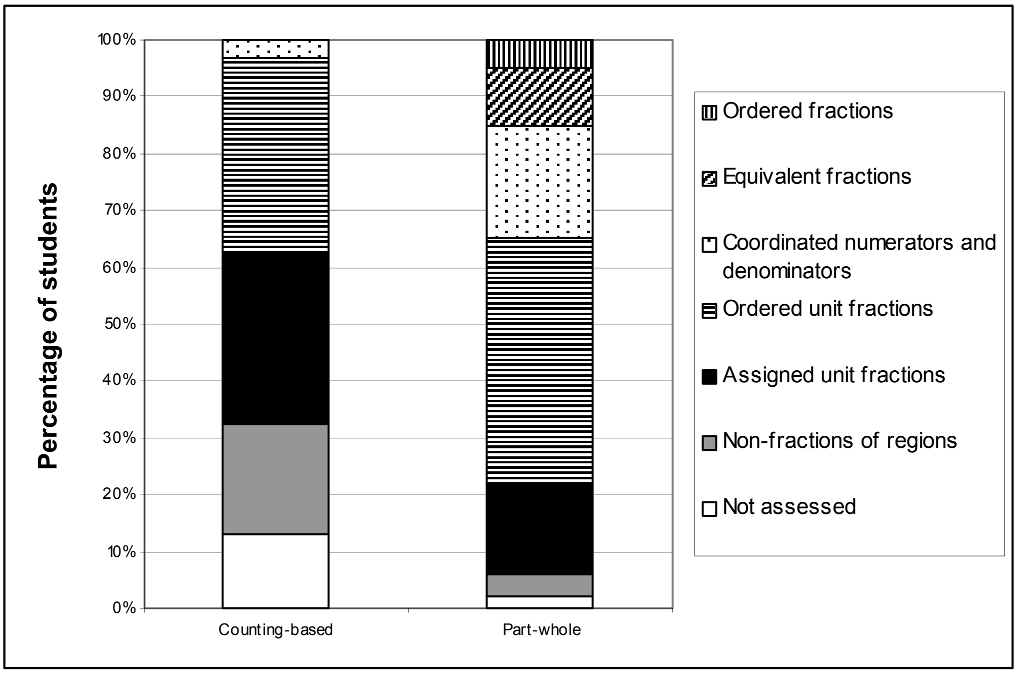

Figure A-13: Year 6: Fractions - knowledge for counting-based and part-whole

Region	No shift		Early additive		Advanced additive		Totals	
Auckland	61%	(773)	37%	(470)	2%	(29)	100%	(1272)
Canterbury	60%	(534)	38%	(336)	2%	(14)	100%	(884)
Massey	58%	(477)	40%	(330)	2%	(12)	100%	(819)
Northland	61%	(217)	36%	(127)	3%	(9)	100%	(353)
Otago	40%	(100)	59%	(149)	1%	(3)	100%	(252)
Southland	50%	(77)	47%	(73)	3%	(5)	100%	(155)
Waikato	59%	(589)	39%	(384)	2%	(22)	100%	(995)
Wellington	53%	(242)	44%	(203)	3%	(12)	100%	(457)
Totals	58%	(3009)	40%	(2072)	2%	(106)	100%	(5187)

Table A-14: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by region

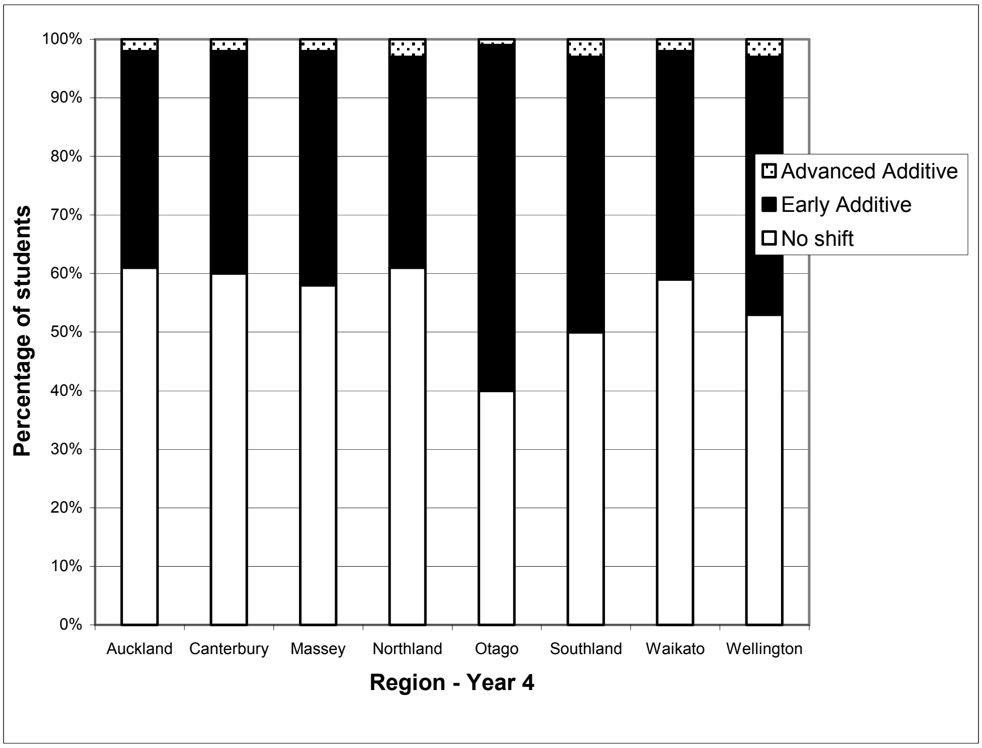

Figure A-14: Year 4: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by region

Region	No shift		Early additive		Advanced additive		Totals	
Auckland	58%	(370)	40%	(254)	2%	(16)	100%	(640)
Canterbury	53%	(408)	44%	(339)	3%	(23)	100%	(770)
Massey	50%	(187)	43%	(162)	7%	(27)	100%	(376)
Northland	49%	(168)	46%	(160)	5%	(16)	100%	(344)
Otago	28%	(50)	65%	(115)	7%	(13)	100%	(178)
Southland	49%	(39)	47%	(37)	4%	(3)	100%	(79)
Waikato	52%	(377)	44%	(323)	4%	(28)	100%	(728)
Wellington	51%	(110)	45%	(97)	4%	(10)	100%	(217)
Totals	51%	(1709)	45%	(1487)	4%	(136)	100%	(3332)

Table A-15: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by region

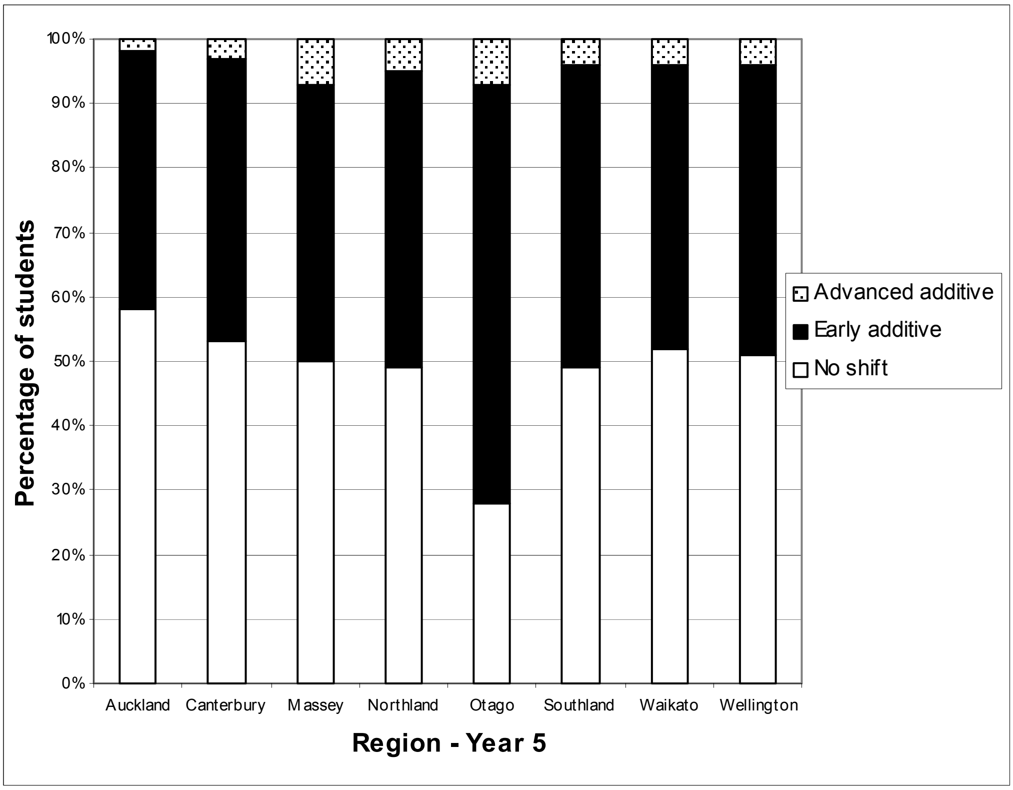

Figure A-15: Year 5: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by region

Region	No shift	Early additive	Advanced additive	Totals
Auckland	47% (283)	47% (286)	6% (39)	100% (608)
Canterbury	44% (275)	53% (333)	3% (22)	100% (630)
Massey	38% (102)	52% (139)	10% (27)	100% (268)
Northland	40% (110)	54% (148)	6% (18)	100% (276)
Otago	40% (52)	45% (59)	15% (19)	100% (130)
Southland	40% (27)	50% (34)	10% (7)	100% (68)
Waikato	44% (256)	49% (284)	7% (42)	100% (582)
Wellington	48% (80)	49% (83)	3% (5)	100% (168)
Totals	43% (1185)	50% (1366)	7% (179)	100% (2730)

Table A-16: Year 6: Addition and subtraction – shift of those initially using counting-based strategies to part-whole strategies – final status by region

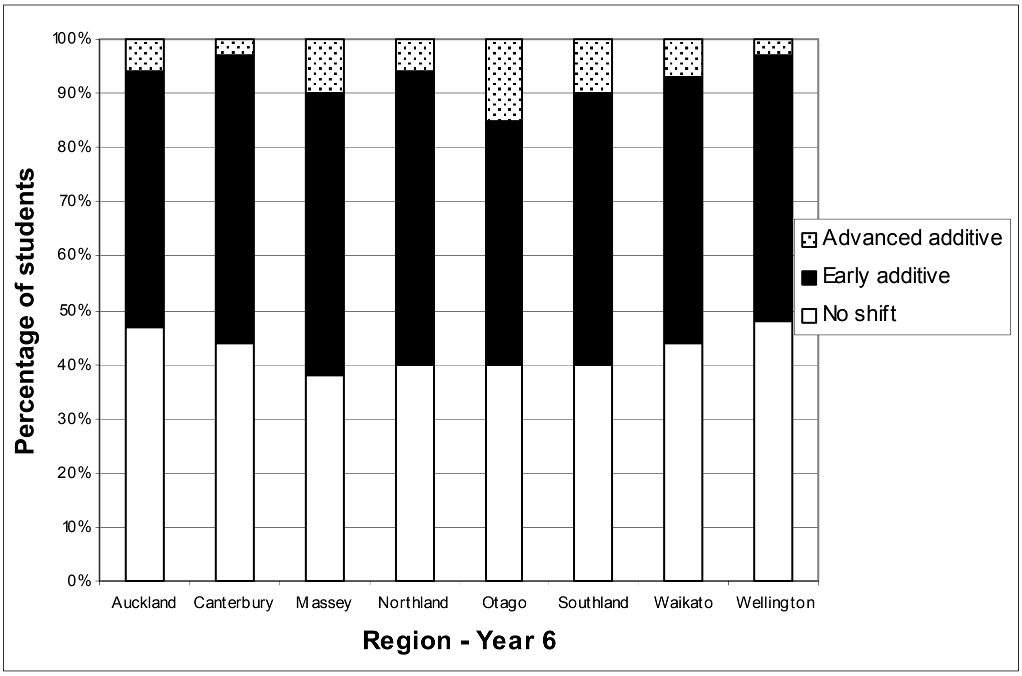

Figure A-16: Year 6: Addition and subtraction – final status by region

Region	No shift	Early additive	Advanced additive	Totals
Auckland	57% (1426)	40% (1010)	3% (84)	100% (2520)
Canterbury	53% (1217)	44% (1008)	3% (59)	100% (2284)
Massey	52% (766)	43% (631)	5% (66)	100% (1463)
Northland	51% (495)	45% (435)	4% (43)	100% (973)
Otago	36% (202)	58% (323)	6% (35)	100% (560)
Southland	47% (143)	48% (144)	5% (15)	100% (302)
Waikato	53% (1222)	43% (991)	4% (92)	100% (2305)
Wellington	51% (432)	46% (383)	3% (27)	100% (842)
Totals	52% (5903)	44% (4925)	4% (421)	100% (11249)

Table A-17: All years: Addition and subtraction - final status by region

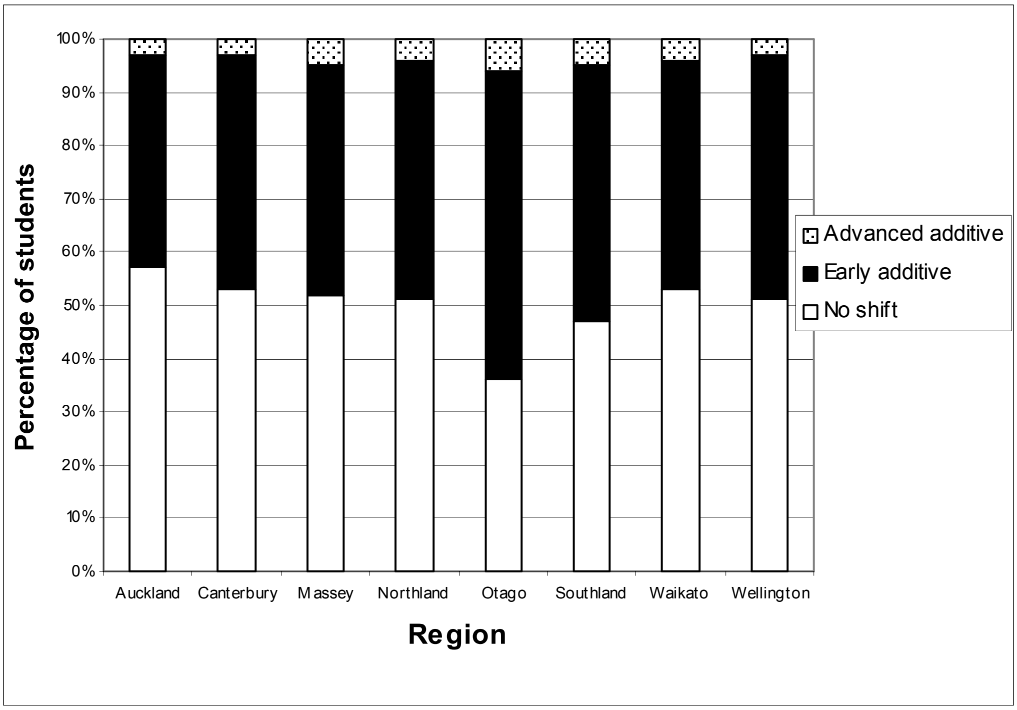

Figure A-17: All years: Addition and subtraction - final status by region