Exploring Issues in
Mathematics Education

An Evaluation of the Advanced Numeracy Project 2001

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Executive Summary

In 2001, the Ministry of Education offered New Zealand schools an opportunity to improve the teaching and learning of number concepts and skills through the Advanced Numeracy Project, a professional development programme for teachers.

The overall aim of the Advanced Numeracy Project was to develop the teachers' knowledge of number concepts, student strategies and instructional practice in order to improve student achievement in years 4 to 6. A key part of the project was presenting teachers with a framework of broad stages describing students' mathematical thinking. Each stage is characterised by the range of strategies that students use to solve problems. Teachers were involved in using a diagnostic interview to assess the stages of their students' thinking. The results of the diagnostic interview form the basis of students' instructional groups. Teachers were also introduced to problem-solving strategies, activities and equipment to use when working with students. There was a particular emphasis on ways of developing increasingly sophisticated strategies for solving number problems.

This report evaluates the impact of the Advanced Numeracy Project on approximately 10,000 students, 500 teachers and 70 principals from 70 schools across 10 regions comprising Auckland, Waikato, Bay of Plenty, Hawke's Bay, Taranaki, Wellington, Nelson, Canterbury, Otago and Southland. The evaluation report identifies changes in student achievement, teacher knowledge and practice that can be attributed to the professional development provided by the Advanced Numeracy Project.

The research addressed eight main questions:

- In what ways does the Advanced Numeracy Project impact on facilitators' subject and pedagogical content knowledge?
- What characterises "effective" facilitation in the Advanced Numeracy Project?
- In what ways does the Advanced Numeracy Project impact on teachers' subject and pedagogical content knowledge?
- How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?
- What was it that the facilitators did that had most impact on improving teachers' classroom practice?
- What progress do students make on The Number Framework?
- How is progress linked to age, ethnicity, gender, geographical region or school decile level?
- To what extent and in what way has student performance in number improved over the duration of the Advanced Numeracy Project in 2001?

Key Findings

- One of the key results of the Advanced Numeracy Project was an increase in student achievement in number. There was clear growth across the six aspects of number assessed. This growth was irrespective of students' age, gender, ethnicity, school region or decile.

- Most students became more sophisticated in their mathematical thinking as a result of taking part in the Advanced Numeracy Project. This was manifested in advancing to part/whole thinking by the final assessment.

- The average gains of Asian and European students were greater than gains of Māori and Pacific Island students taking part in the project.

- The diagnostic interview was identified by facilitators as an important factor in enhancing teacher content and pedagogical content knowledge. It enabled teachers to more closely match the task to the learner by giving the teachers more detailed knowledge. The diagnostic interview provided principals with hard data for setting school expectations of student achievement and for use in their self-review process.

- In-class support provided by facilitators during the project was pivotal in shifting classroom practice by refining teacher questioning and their ability to articulate the reasons for their actions. This support was contextualised in the practice setting.

- Communication about mathematics teaching and learning was enhanced across school transition points through the use of a common language, which arose from The Number Framework. This provided a basis for collaborating on mathematics teaching within and between schools (for example between primary and intermediate).

- Communication to parents about mathematics teaching and learning was more specific in the Advanced Numeracy Project through the use of the student results in reporting to parents.

- Teacher attitudes towards mathematics and the teaching of it were improved as a result of the knowledge they gained from participating in the project, such as their ability to identify barriers to student's success.

- Student attitudes towards learning mathematics (as judged by teachers, principals, and facilitators) were improved as a result of their teachers participating in the Advanced Numeracy Project.

- Policy development at the school level and the fostering of communities of practice (or professional learning communities) appear likely to ensure that advances made as a result of the Advanced Numeracy Project are sustained within participating schools.

Recommendations

- That schools be offered continued guidance and support in becoming communities of practice or professional learning communities through participation in the project

- That there be continued investigation into ways of raising Māori and Pacific Island student achievement in particular

- That the key ideas of the project be incorporated into courses so that as many student teachers and practising teachers as possible have the opportunity to enhance their practice and develop their content knowledge of mathematics

The Advanced Numeracy Project enhanced numeracy work in participating schools and participants hoped that the project would continue in the future and be made available to all New Zealand schools.

Chapter 1: Introduction

This report examines the impact of one aspect of the numeracy policy on schools, teachers, students and facilitators. It comments on changes in student achievement, teacher knowledge and practice and emerging facilitator practice arising out of participation in the Advanced Numeracy Project (ANP) in 2001. In particular, it discusses the role of the facilitator in working with teachers and schools to implement the project.

Consideration of the sustainability of the project in participating schools will be of special concern to any government. The sustainability is related to the extent to which a school can integrate the changes arising from the ANP into existing policy and practice. A key part of this is to develop school policy that will support the new practices. Fullan (1999) identified key characteristics of "collaborative cultures for complex times" that might explain why some schools are more successful than others in implementing new projects. One of these key characteristics relates to how schools respond to new policy. Fullan argued that "collaborative schools are not necessarily the most innovative – they are *selectively* innovative" (page 39). He explained that "when new ideas or policies come along they ask not only whether the idea is potentially good for them, but also how they can integrate it with what they are already working on" (page 39). This process, he suggested, achieved coherence between existing practices and the new ideas. This report examines the ways in which schools participating in the Advanced Numeracy Project were able to achieve this coherence.

Since the Third International Mathematics and Science Studies (TIMSS) results, governments in many Western English-speaking countries have developed policy relating to numeracy. New Zealand's development work in numeracy has been guided by the recommendations of the Mathematics and Science Taskforce (Ministry of Education, 1997). This report begins by providing the background that places the Advanced Numeracy Project in the wider educational context.

New Zealand Policy Setting for Numeracy Developments

The Advanced Numeracy Project fits within the Government's Literacy and Numeracy Strategy. A definition of what constitutes a numerate person has been developed as part of the strategy and highlights the important links between literacy and numerical literacy. Related Government strategies include the ICT Strategy, which aims at developing ICT literate students, the Principal Induction (for new principals) and Development Strategy (for principal leadership), the Assessment Strategy, the Arts professional development supporting the new curriculum document and the Māori Language development. It is important to note the nested nature of these strategies and the effect of this for implementation within the primary sector where one teacher is faced with concurrently implementing different policy statements. A feature of the professional development that supports the numeracy projects has been the way it highlights for teachers links between the government strategies.

Key themes of the Literacy and Numeracy Strategy include clarifying expectations for student achievement, developing professional capability and involving the community. The clarification of expectations for student achievement includes both standards of achievement and rates of progress. The themes of the Literacy and Numeracy Strategy are nested in the requirement set by the modified National Administration Guidelines (NAGs) that emphasise literacy and numeracy

(Ministry of Education, 2000). The Assessment Strategy in particular links to the Literacy and Numeracy Strategy as illustrated in a statement of intent:

> There is consistent evidence to show strong connections between clear goals, motivation, and improvement. Clear expectations focused on educationally significant learning and appropriately challenging goals raise achievement. The commitment generated through setting goals is the glue that holds professional learning communities together. Goals promote conversations focused on learning, they provide the basis for decision making, and they enable students and teachers to gauge their own success. (Ministry of Education, 2000, page 2)

The evaluations of the 2000 Count Me In Too pilot project (Thomas and Ward, 2001) and of the 2000 Year 4–6 Numeracy Exploratory Study (Higgins, 2001a) bear this out. Thomas and Ward reported that:

> The teachers changed their mathematics programmes to include a greater focus on number and, in particular, the number strategies that students use to solve addition and subtraction problems. Many of the teachers had higher expectations of students, felt more able to identify learning needs, and grouped their classes more effectively. … Central to the success of the project was the adopted model of school-based and classroom-based professional development. The teachers met regularly with each other and the facilitator to share experiences and insights and to discuss appropriate learning activities for the students. (Thomas and Ward, 2001, page 51)

Both evaluations highlight the importance of assessing students' understanding of number through a diagnostic interview. Information gained through the interview enabled teachers to have more specific expectations of students' learning.

> An added benefit of the interview was that it also provided teachers with information and a language to describe students' progress both to other teachers and to parents. … Having a language to describe the specific stages of children's progress, rather than the global statements often used, enables teachers to pinpoint children's progress more accurately in reporting to the children themselves, to other teachers, and to parents. … Reporting then becomes another form of parent education. (Higgins, 2001a, page 37)

The very favourable student results reported in the Count Me In Too pilot project helped raise the expectations for the Advanced Numeracy Project to deliver similar gains in student achievement.

Number Acquisition

Number is an important part of primary mathematics. One of the key debates around instruction in number is the teaching of formal algorithms. Increasingly, commentators (for example, Kamii, 1985; Treffers, 1991; Wright, 2000) have warned of the pitfalls of the early introduction of the algorithm of column arithmetic. Recently, a number of writers have advocated greater use of mental strategies in the teaching of place value. The work of Dutch mathematics educators has been influential in promoting the use of mental strategies with numbers greater than 10 (Beishuizen, 1993). With this shift in thinking about the teaching and learning of number, there continues to be concentrated development of theoretical models that explain students' acquisition of number concepts (see Jones et al., 1996; Fuson et al., 1997; Wright, 1998; Young-Loveridge, 1999). These frameworks attempt to capture the complexities of place value and multi-digit understanding for instructional purposes through identifying the different conceptual structures or key constructs in number development.

The complexities arise from the interrelationships between the key constructs of counting, number knowledge, grouping and partitioning. It is generally accepted that the constructs fall into two broad groups of understanding. One is based on counting, including number relationships, and the other is based on collections or groups (Cobb and Wheatley, 1988). As Young-Loveridge (2001) pointed out, "there seems to be reasonable agreement that, although counting provides an important first step towards understanding numbers, the emphasis on counting must shift to a focus on part/whole relationships among numbers, if a more advanced understanding about numbers is to be achieved" (pages 72–73). She commented that "unfortunately, some children do seem to get stuck on counting, and find it hard to make the transition to part/whole strategies. As school mathematics becomes increasingly challenging, their dependence on counting is likely to be more and more of an obstacle to their mathematics learning" (page 73).

The part/whole idea or the idea of partitioning numbers represent an important conceptual shift for students in early number learning. Partitioning is the ability to think about numbers as ideas and understand that numbers can be split or partitioned in multiple ways. The way in which the partitioned numbers are put back together removes the need to depend on counting to solve a problem (Young-Loveridge, 2001). The aim is for students to develop increasingly sophisticated and efficient ways of partitioning numbers that suit the problem being solved.

The Number Framework

The Number Framework was developed from the late 1990s on and underpins the Early and Advanced Numeracy Projects trialled in New Zealand schools in 2001. Wright (2000) saw the development of number frameworks as a way of prioritising knowledge about numbers and how to use them. In comparing curriculum statements and number frameworks, he highlighted the difference in intent of each. He described number frameworks as being "developed primarily for teachers" and being "more tightly focused in terms of both content and ages of students and therefore … more detailed and specific than curriculum statements can be" (page 4). Wright also noted that the development of a number framework was an opportunity to incorporate research evidence from the early 1990s (the publication of the curriculum statement), which has contributed to further understanding of early number acquisition. Key research evidence incorporated into The Number Framework emphasises mental strategies and delaying the introduction of the written algorithm:

> Written recording is seen primarily as a means to "think through" calculations **so that students are not exposed to standard vertical algorithms[1] until mental strategies are sufficiently advanced**. The informal jottings of students are to be encouraged as a way to capture their mental processes so that their ideas can be shared with others. (Ministry of Education, 2002b, page 9)

Knowledge and Strategies

The Number Framework makes a distinction between knowledge and strategy. Young-Loveridge (2001) commented on this distinction:

> Strategies are the ways that children solve number problems, in particular, the mental processes they use. Knowledge includes the key information which children need to have in order to apply particular strategies. These are seen as mutually supportive, with *strategies* and their use leading to the creation of new knowledge, and *knowledge* providing the foundation for strategies. (page 74)

[1] An algorithm is "a standard procedure for performing a task or solving a problem" (Ministry of Education, 1992, page 210).

The Number Framework (Ministry of Education, 2002b) emphasises how important it is for students to make progress in both the knowledge and strategy sections of the framework and states:

> Strong knowledge is essential for students to broaden their strategies across a full range of numbers, and knowledge is often an essential prerequisite for the development of more advanced strategies. For example, a student is unlikely to solve 9 + 6 as 10 + 5 if he or she does not know the "ten and" structure of teen numbers. Similarly, using more advanced strategies helps students to develop knowledge. For example, a student who uses doubling of the multiplication by two facts [the 2 times table] to work out the multiplication by four facts [the 4 times table] will soon learn these facts through appropriate repetition. (page 1)

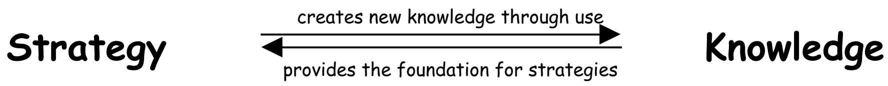


Figure 1.1: The Strategy/Knowledge Relationship (Ministry of Education, 2002b, page 1)

The framework promotes the teaching of mental strategies as a pathway to part/whole thinking of number. The strategy stages identified in the framework describe increasingly sophisticated ways of thinking about numbers. As Wright (2000) put it, "a student's strategies on various problems can be explained in terms of the types of unit they have scheme for and are able to co-ordinate" (page 6). In the framework, the strategies "are grouped by genre. Underlying each genre of strategies is the complexity of unit structure being used" (page 7).

Of particular relevance to this study is the development of part/whole strategies. The framework describes less sophisticated part/whole thinking as early additive part/whole, where students split and join numbers in one or two ways. For instance, the student may compensate from known facts as in "7 + 8: 7 + 7 is 14, so 7 + 8 is 15" or use standard place value partitioning as in "43 + 35 is (40 + 30) + (3 + 5) = 70 + 8" (Ministry of Education, 2002b, page 4).

More sophisticated part/whole thinking is described as advanced additive part/whole where students split and join numbers using a wide repertoire of part/whole strategies. For instance:

> They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining. Simultaneously, the efficiency of these students in addition and subtraction is reflected in their ability to derive multiplication answers from known facts. These students can solve fraction problems using a combination of multiplication and addition-based reasoning. (Ministry of Education, 2002b, page 5)

Two further stages of advanced multiplicative part/whole and advanced proportional part/whole represent increased sophistication in thinking about numbers. At the advanced multiplicative stage, students are:

> … learning to choose appropriately from a range of part/whole strategies to estimate answers and solve problems involving multiplication and division. Some writers describe this stage as "operating on the operator". This means that one or more of the numbers involved in a multiplication or division is partitioned and then recombined. … A critical development at this stage is the use of reversibility, in particular, solving division problems using multiplication. Advanced Multiplicative Part/whole students are also able to estimate answers and solve problems with fractions using multiplication and division. (Ministry of Education, 2002b, page 6)

At the advanced proportional part/whole stage, the students' part/whole repertoire includes strategies that involve fractions, proportions and ratios, with the specific inclusion of strategies for multiplying decimals and calculating percentages. (See Appendix A for the framework as used by teachers in this study in 2001). Young-Loveridge (2001), based on the results of an investigation into students' part/whole thinking, pointed out "the need to strengthen students' part/whole understanding and to help students learn to use part/whole strategies across a range of different problem types" (page 75).

The framework is useful to teachers as it allows them to identify what stage a student is at. However, this is not always clear-cut as "students are sometimes between stages. That is, they display characteristics of one stage given a certain problem but use a more-or-less advanced strategy given a different problem. Gaps in key knowledge are usually the reason" (Ministry of Education, 2002b, page 2).

Despite this, teachers are able to:

> describe their students by global developmental stage and anticipate with considerable reliability how their students are likely to solve a range of number problems across various threads of the number strand. Progression through the stages will clearly define directionality of teaching resulting in learning experiences, which can be closely matched to student needs. (Wright, 2000, page 7)

The evaluation of the Count Me In Too pilot project (Thomas and Ward, 2001) mentioned that "many of the teachers commented on the usefulness of a **framework** which clearly sets out the progressions in a student's understanding of number" (page 51).

A teaching model is a useful way of conceptualising how to promote students' strategic thinking as opposed to knowledge development. The teaching model (Ministry of Education, 2002a) is based on Pirie and Kieren's (1992) model of the development of students' understanding of mathematical ideas. In that model, imaging becomes a bridging technique between the concrete and the abstract. The teaching model as shown below guides the explicit teaching of strategies.

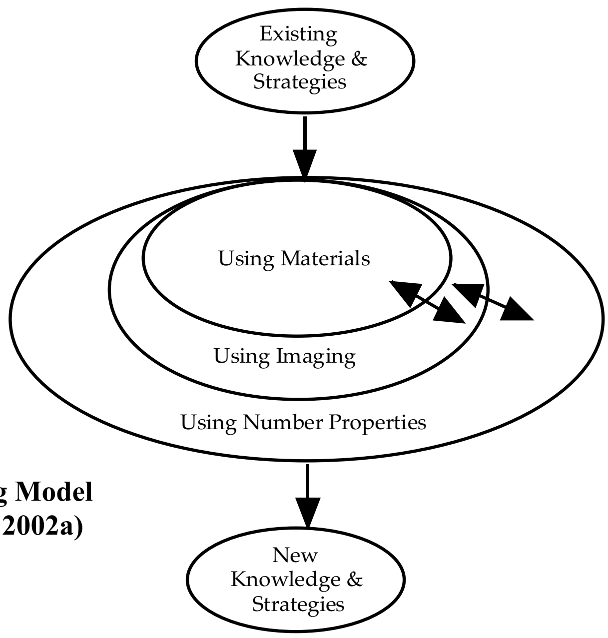


Figure 1.2: The Teaching Model (Ministry of Education, 2002a)

The teaching model highlights the potential difficulties in abstracting mathematical ideas from equipment. More traditional approaches to mathematics teaching use equipment to represent mathematical ideas. Cobb (1987) in particular has commented on the problems that arise from such an approach, terming it a "learning paradox" (Bereiter, 1985, cited in Cobb, 1987). To "see" mathematical relationships being represented in equipment, one needs to first understand the relationship, and for this reason, as an instructional approach, it tends to break down. As Cobb put it "those that have got it get it, and those that haven't, don't" (page 28). An alternative approach is that in which "models stimulate and support students' mathematical thinking rather than … the extent to which the models illustrate the steps of a procedure used to solve a number problem. The metaphors of 'working' or 'thinking' models of mathematical ideas epitomise this distinction" (Higgins, 2001b, page 37). The key point is the way in which any piece of equipment is used rather than the piece of equipment itself. (For a full discussion, see Higgins, 2001b). Having said that, some pieces of equipment lend themselves more easily than others to becoming "thinking models". (See Young-Loveridge, 1998, for a useful discussion of different pieces of equipment in promoting part/whole understanding.)

Developing Professional Capability

The second prong to the Literacy and Numeracy strategy is about developing professional capability; that is about bridging between improved practice and more successful learning by establishing effective teaching/learning models that have been informed by sound content knowledge of mathematics. Parsons (2001) in a policy paper addressing improving teaching capability through professional development stated that "implementation strategies need to support teacher learning. Successful, sustained change at the classroom level is the result of teachers who are confident and committed to an ongoing concept of professional development" (page 2).

Strategies to address an "implementation dip" (Fullan, 2001) are an important consideration with any large-scale project or innovation. Fullan defined the implementation dip as "literally a dip in performance and confidence as one encounters an innovation that requires new skills and new understandings" (page 40). A consequence of innovations is the impact on people's beliefs and values shaping their behaviour. Fullan argued that "All innovations worth their salt call upon people to question and in some respects to change their behavior and their beliefs – even in cases where innovations are pursued voluntarily" (page 40). This impact is characterised by the questions:

> What happens when you find yourself needing new skills and not being proficient when you are used to knowing what you are doing (in your own eyes, as well as in those of others)? How do you feel when you are called upon to do something new and are not clear about what to do and do not understand the knowledge and value base of new belief systems? (page 40)

Fullan summarised these experiences in the dip as two kinds of problems: "the social-psychological fear of change, and the lack of technical know-how or skills to make the change work" (page 41). He suggested that leaders who understand such feelings use a combination of leadership styles to ease the organisation through the dip.

Parsons (2001) highlighted the importance of a school becoming a professional learning community to support such cultural change.

> The notion of a professional learning community is central to school improvement. It is through the activities of the professional learning community that cultural change occurs. A professional learning community is focused on student performance. Teachers pursue a clear purpose. They engage in collaborative activity to achieve that purpose. Improvement in student performance is a result of the professional learning community's focus on high quality assessment and pedagogical practice. (page 3)

In the case of the Advanced Numeracy Project, the professional learning community extends beyond the school to incorporate the facilitator and the clusters with which individual schools are working. The nested community of teachers and facilitators is a central aspect of the context in which the transformation of common practices becomes possible (Remillard and Rickard, 2001). The processes used by a community are of particular interest. Fullan (2001) termed transforming the culture or "the way we do things around here" as reculturing (page 44). A key to this transformation is the establishment of a collaborative work culture. Fullan explained that "… it is a particular kind of reculturing for which we strive: one that activates and deepens moral purpose through collaborative work cultures that respect differences and constantly build and test knowledge against measurable results – a culture within which one realizes that sometimes being off balance is a learning moment" (page 44). He commented, "the paradox is that transformation would not be possible without accompanying messiness" (page 31).

The situating of the Advanced Numeracy Project in the middle school further impacts on the transformation of practice. The middle school context stands in contrast to that of the junior school in which the Early Numeracy Project is being implemented. I have previously argued that:

> These two very different contexts are evident in the discourses of junior primary and the standard classes of the middle school in terms of mathematics education and models of learning and teaching. This is often signalled by greater emphasis on written recording, desk-work, formal algorithms, written assessment, and learning and testing of the addition basic facts and the multiplication tables. There is often less emphasis on equipment and "hands-on" activities. (Higgins, 2001b, page 30)

This contrast in context presents very different challenges to facilitators and school communities in their attempts to transform practice. Hence the analysis that informs this evaluation is based on a teacher-centred contextualised model of professional development, which is fully discussed in the next chapter explaining the methodology used.

The Structure of This Report

Chapter 2 discusses the methodology. Chapter 3 presents the analysis of student results from the initial and final diagnostic interviews. Chapter 4 provides evidence from the questionnaires on the perspectives of adult participants on the impact of the Advanced Numeracy Project on all participants. It discusses the extent to which teachers' professional knowledge and attitudes have been enhanced through participation in the project. Chapter 5 focuses on facilitation and in particular examines factors that lead to facilitation being effective in enhancing classroom programmes, developing teacher knowledge and supporting schools in developing policies to ensure the sustainability of the changes. It suggests that the key factor in effective facilitation is the extent to which a facilitator can act as a mediator between teachers' and schools' old and new practices.

Chapter 2: Methodology

Contextualising Professional Development: A Teacher-centred Model

The ability of a facilitator to establish the professional development within the teachers' context of practice is important. Fennema and Franke's (1992) model of the context-specific nature of teacher knowledge is a useful analytical tool for identifying and interpreting contextual factors that impact on the development of teachers' knowledge and classroom practice of number concepts. Their model highlights "the interactive and dynamic nature of teacher knowledge" which they saw as including "the components of teacher knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students' cognitions and teacher beliefs" (page 162). They explained the knowledge and beliefs as follows:

> The context is the structure that defines the components of knowledge and beliefs that come into play: Within a given context, teachers' knowledge of content interacts with knowledge of pedagogy and students' cognitions and combines with beliefs to create a unique set of knowledge that drives classroom behavior. (page 162)

For the purposes of this study, the Fennema and Franke model has been adapted to highlight the complexities or multiple layers of context within which a facilitator works with teachers and schools. The teachers' context of practice includes aspects such as school policies and structures and student backgrounds. This is shaped by teachers' pedagogical knowledge, teachers' knowledge of learners' cognitions in mathematics and teachers' content knowledge.

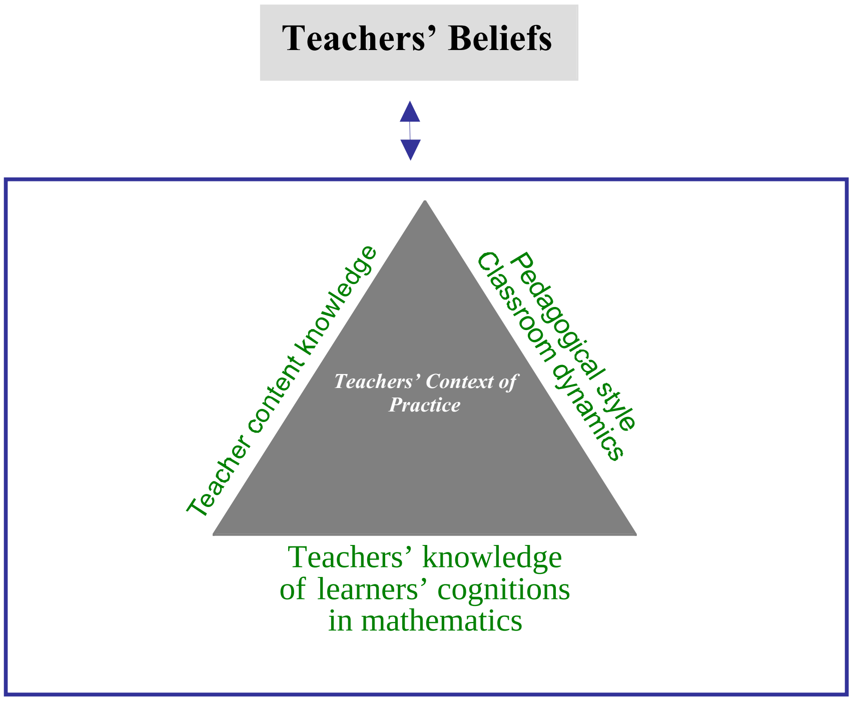
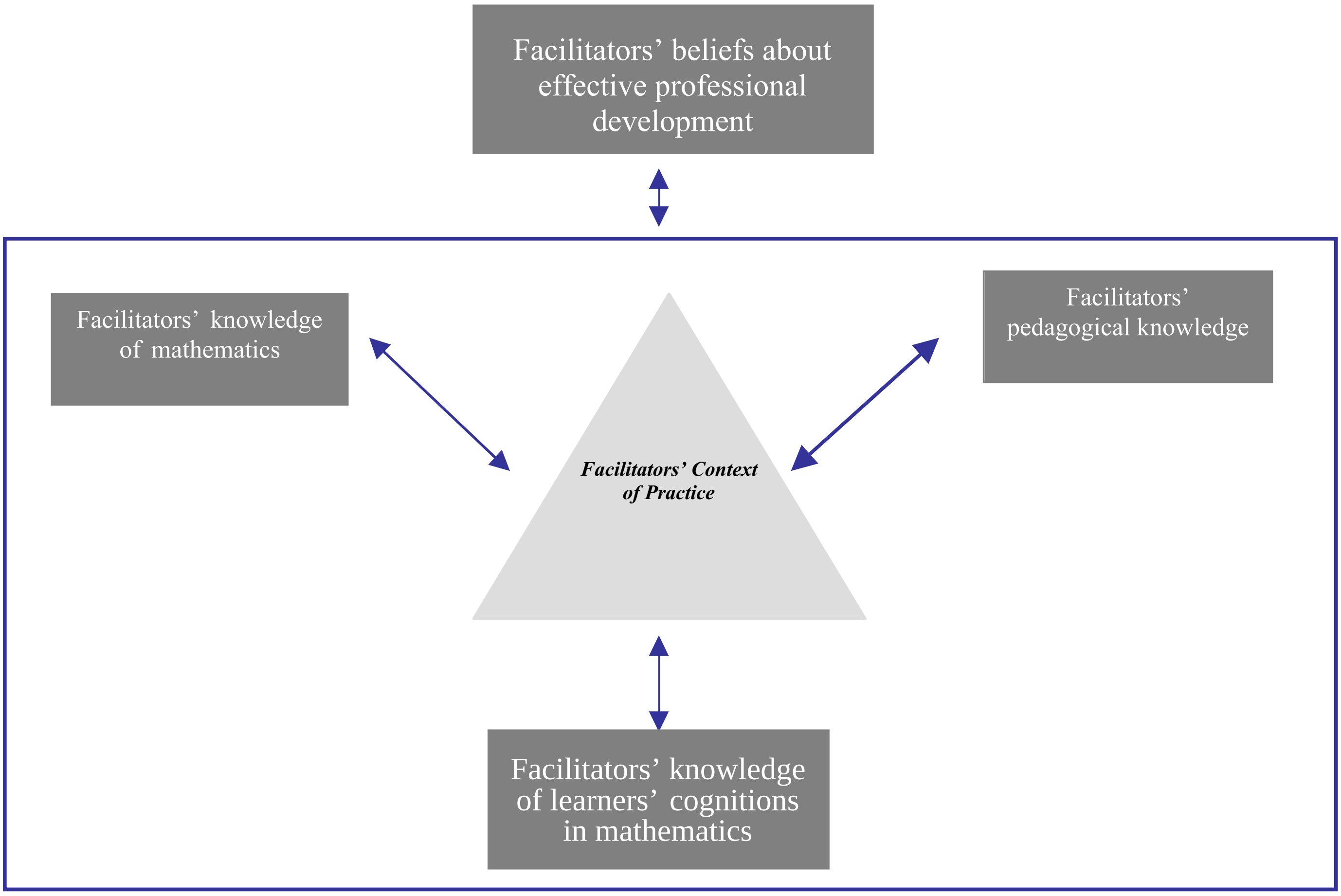


Figure 2.1: Teachers' Context of Practice

The facilitator's understanding and orientation to change is critical to their role of working with teachers in schools. This work might include presenting teachers with compelling reasons for them to change their practice so that they incorporate pedagogical strategies, such as teaching a group or a class some aspects of mathematics that the teachers themselves do not believe are possible to teach within that practice setting.

A further adaptation of the model incorporates the role that a facilitator plays in the process of changing teaching practice to provide students with greater opportunities to develop a sound understanding of number concepts. This additional layer can be thought of as the facilitator's context of practice and subsumes the teachers' context of practice as illustrated in Figure 2.2. Within this layer, the facilitator's knowledge and beliefs about teacher learning as well as their knowledge of content and classroom pedagogy and students' cognitions provide dynamic additional components that interact with and overlay the unique set of knowledge that drives any teacher's classroom behaviour as described by Fennema and Franke.



Source: Adapted from Fennema and Franke (1992, page 162)

Figure 2.2: Facilitators' Context of Practice

The work of Remillard and Rickard (2001) highlights that the place of the community is central to the context of practice. Seen in terms of the Fennema and Franke (1992) model, the context can be thought of as the community of the respective practitioners, be it the community of facilitators or the community of teachers or the joint communities of primary level educators. Remillard and Rickard (2001) suggested that examining "whether and how a community of practice negotiates and takes on practices that do not reflect the traditions of its trade" is useful in thinking about teacher learning. Their key focus was "understanding the extent to which transformation of common practices is possible and how it happens within the community [of practice]" (page 2). They proposed that a stance of critique and inquiry be set up to explain the "kinds of learning opportunities constructed by the group" (page 2) and saw this stance as being underpinned by the assumption that "learning to teach mathematics differently involves repositioning one's self with respect to one's own knowledge and how it is learned" (page 2). Three dimensions of this stance that were identified included "the relationships learners develop with new ideas", "the process of learning" and "the context in which learning takes place" (page 3). Remillard and Rickard argued that the notion of a community of teachers is a central aspect of the context in which transformation of common practices becomes possible through a community process of negotiation and adoption of new practices.

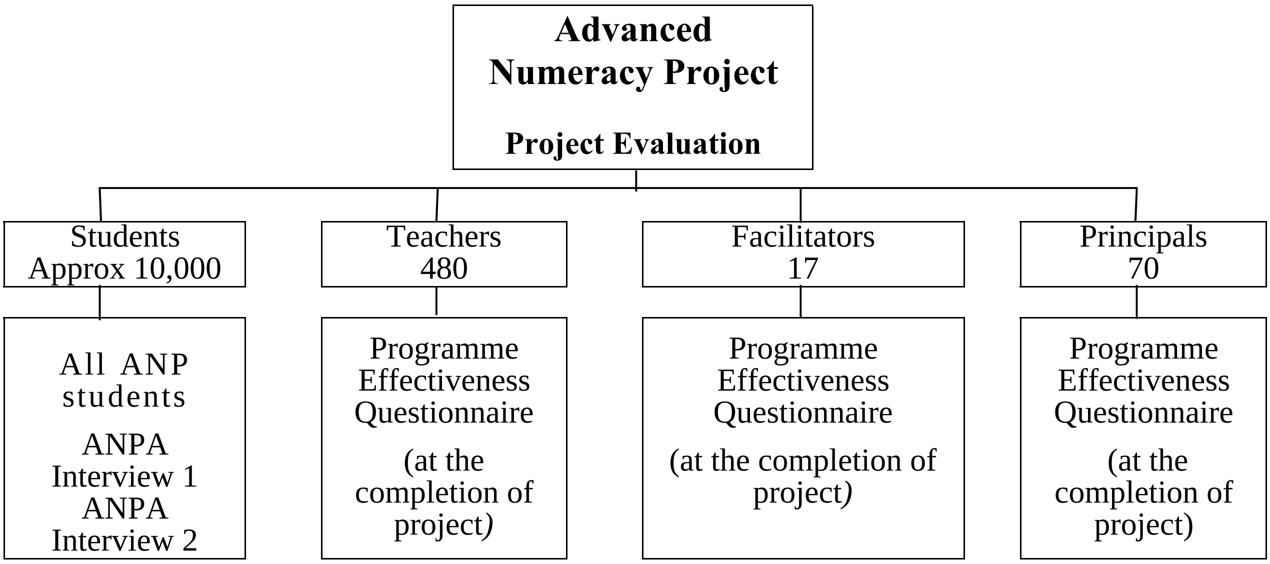


Figure 2.3: Evaluation of ANP – Impact on Participants

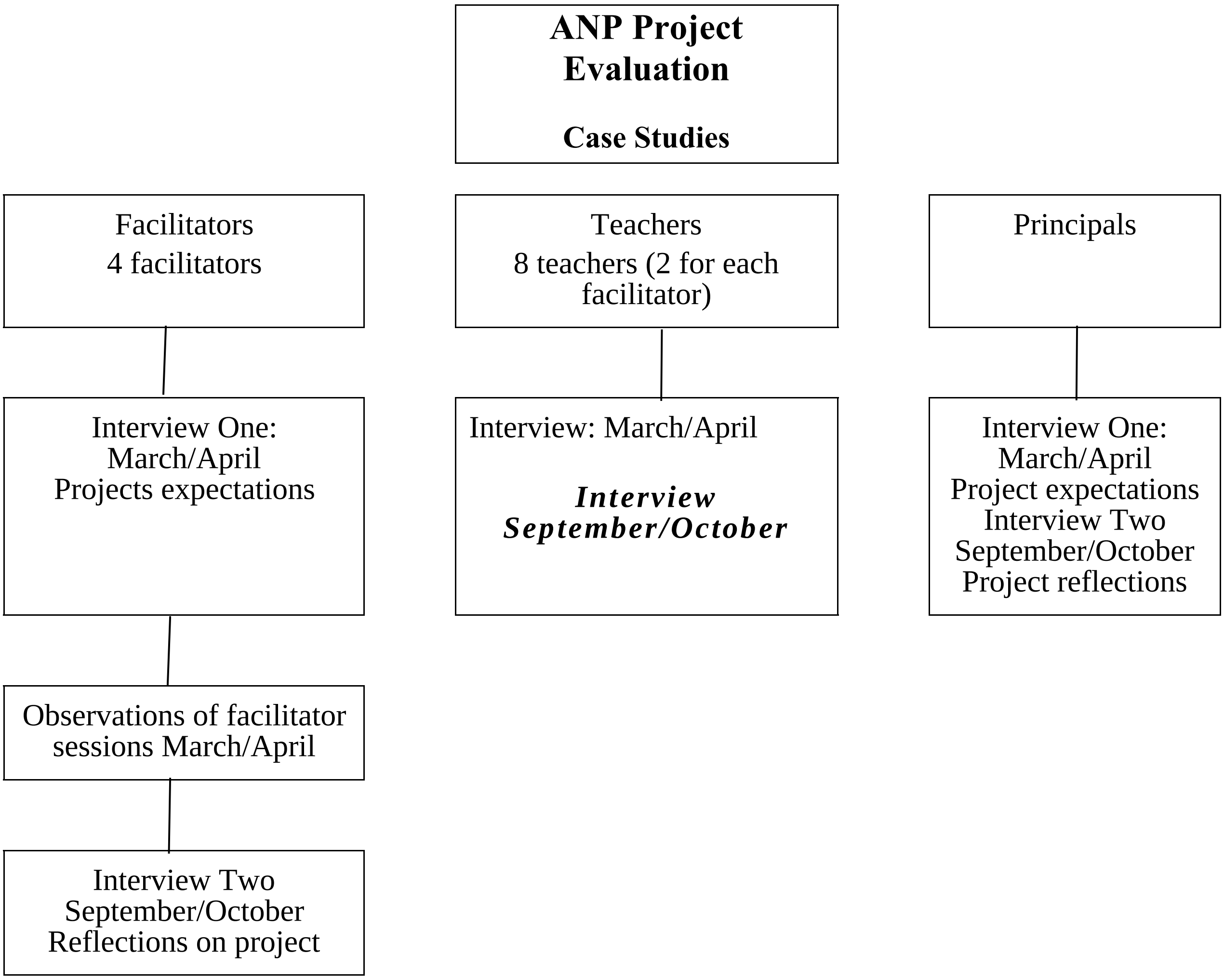


Figure 2.4: Effective Facilitation of ANP – Case Studies

Aim of the Investigation

This project complements the evaluation of the Early Numeracy Project (Thomas and Ward, 2002). The investigation examined the impact of the Advanced Numeracy Project on participating teachers and students, with an increase in teachers' professional knowledge and increased levels of numeracy for students as intended outcomes of the project. This investigation specifically examined characteristics of effective facilitation of professional development. The three broad areas of the investigation are outlined below.

Facilitators

The first broad area investigated the characteristics of effective facilitation by looking at the ways in which the Advanced Numeracy Project impacted on facilitators' knowledge, and the effectiveness of the facilitator training. The characteristics of effective facilitation were examined using a case-study approach of four facilitators and the impact these facilitators had on two teachers with whom each had been working. This is not intended to suggest that facilitators' effectiveness can be judged against a set of criteria but that there are key principles that transcend contextual factors of any one situation within which a facilitator is working with teachers and that the identification of such principles will inform the facilitation process for others. Hence the facilitators in the case studies were chosen as reflecting a range of contexts across different regions in New Zealand.

The project was designed to answer the following research questions:

1. In what ways does the Advanced Numeracy Project impact on facilitators' subject and pedagogical content knowledge?
2. What characterises "effective" facilitation in the Advanced Numeracy Project?

Teachers

The second broad area investigated the impact of the Advanced Numeracy Project on teachers' subject and pedagogical content knowledge and classroom practice. The investigation examined in detail the impact of the facilitation of the programme on teachers' practice through an interview at the beginning of the project and another at the completion of the project. Specific questions addressed were:

1. In what ways does the Advanced Numeracy Project impact on teachers' subject and pedagogical content knowledge?
2. How do teachers see changes in their subject and pedagogical content knowledge impacting on their classroom practice?
3. What was it that the facilitators did that had most impact on improving teachers' classroom practice?

Students

The third broad area investigated the impact of the Advanced Numeracy Project on students' understanding of number concepts as detailed on The Number Framework. The following research questions were addressed:

1. What progress do students make on The Number Framework?
2. How is progress linked to age, ethnicity, gender, geographical region or school decile level?
3. To what extent and in what way has student performance in number improved over the duration of the Advanced Numeracy Project in 2001?

Design and Methodology

The data set comprised three main components: the quantification of student progress using The Number Framework, questionnaires to teachers and facilitators on the effectiveness of the teacher professional development programme, and an examination by case study of how effective the facilitation of the programme was in terms of developing teachers' classroom practice in number. Prior to the commencement of data gathering, ethical approval was sought from the Wellington College of Education Ethics Committee, which operates under the NZARE Code of Ethics. To protect their privacy, the names of the teachers and schools who took part in the evaluation have been changed in this report.

Advanced Numeracy Project: All Participants

The Ministry of Education offered the Early and Advanced Numeracy Projects to schools throughout New Zealand with a particular emphasis on those classified as lower decile. The Advanced Numeracy Project involved approximately 17 facilitators, 70 principals, 480 teachers and 10,000 students in 2001.

The facilitator training included participation in three training days in December 2000 and another 3 days in February 2001. In addition, there were two further regional seminar days conducted during the year and the national co-ordinator of the project visited and was in regular communication with the facilitators. The facilitators in each region were invited to complete a questionnaire of open-ended questions on aspects of the Advanced Numeracy Project. (See Appendix C). The questionnaires were completed anonymously, and a stamped addressed envelope was included for the return of the questionnaire to ensure confidentiality of those participating in the study. The findings are presented in terms of emerging trends for qualitative data and in summary tables for quantitative data.

The approximately 480 teachers who participated in the Advanced Numeracy Project were working with 17 facilitators in 10 regions around New Zealand. Those schools who had previously participated in the Count Me In Too pilot project were given priority to participate in this project. Each facilitator worked with up to 30 teachers in total. All participating teachers were organised into clusters comprising a few schools in an area for after-school meetings. The teacher development within any school has a syndicate focus. Building on the approach adopted in the Count Me In Too pilot project, the teacher development had a prime focus of the Advanced Numeracy Project Assessment (ANPA) diagnostic tool. Teachers video-taped themselves interviewing students and these videos were used to explore The Number Framework. At the conclusion of the project, the 480 teachers were invited to complete a questionnaire on its effectiveness. In particular, the analysis focused on possible ways in which the project might be improved in subsequent years. This approach follows that of Dr Janette Bobis' evaluation of the New South Wales implementation of Count Me In Too (Bobis, 1999).

The teachers assessed students twice during the project, using the Advanced Numeracy Project Assessment (ANPA) diagnostic tool, first at the start of the project after teachers had received their initial training, and then again after 15 weeks teaching at the completion of the project. On the basis of the initial results, facilitators worked with teachers to plan their instructional programme. These results were used to track the achievement of students using the website set up especially for the project. Teachers completed a profile for each student including their date of birth, their year level and their ethnicity (the ethnic group with which they mostly identify).

Overview of Student Participants

			Decile Band			Total
			Low (1–3)	Middle (4–7)	High (8–10)	
Ethnicity	**Asian**	**N**	124	149	210	483
		%	26%	31%	43%	100%
	European	**N**	1466	2273	1397	5136
		%	29%	44%	27%	100%
	Māori	**N**	928	465	78	1471
		%	63%	32%	5%	100%
	Pacific Islands	**N**	535	183	51	769
		%	70%	24%	6%	100%
	Other	**N**	60	116	55	231
		%	26%	50%	24%	100%
Total		**N**	3113	3186	1791	8090
		%	39%	39%	22%	100%

Table 2.1: Frequencies of Students by Ethnicity and Decile

			Region										Total
			Auckland	**Waikato**	**Bay of Plenty**	**Hawke's Bay**	**Taranaki**	**Wellington**	**Nelson**	**Canterbury**	**Otago**	**Southland**	
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
Gender	**F**	**N**	1487	331	189	280	265	282	199	401	260	229	3923
		%	38%	8%	5%	7%	7%	7%	5%	10%	7%	6%	100%
	M	**N**	1586	364	235	260	252	310	227	442	258	233	4167
		%	38%	9%	6%	6%	6%	7%	5%	11%	6%	6%	100%
Total		**N**	3073	695	424	540	517	592	426	843	518	462	8090
		%	38%	9%	5%	7%	7%	7%	5%	10%	6%	6%	100%

Table 2.2: Frequencies of Students by Region

Effective Facilitation: Case Studies

The case studies examined the effectiveness of the programme's facilitation. This involved inviting four facilitators in four different regions to participate in pre- and post-project interviews that sought to establish their pedagogical content knowledge of mathematics, their beliefs and knowledge of teacher development and their approach to curriculum and resources (including The Number Framework). The overarching question was "What are the characteristics of effective facilitation?" Two teachers working with each facilitator had also been invited to participate in pre- and post-project interviews. The purpose of these interviews was to find out what the facilitator did that helped them the most in improving their classroom practice, and how any professional knowledge gained on the project impacted on their classroom practice. In addition, biographical details of case-study participants, details of the facilitator training, records of individual facilitator's work with the teachers concerned and an informal interview with the principal of each school formed part of this set of data.

Chapter 3: Impact on Students

The Advanced Numeracy Project, like the other numeracy projects (the Early Numeracy Project and the Years 7–10 Numeracy Exploratory Study), aims to raise student achievement in number. As part of this project, the students were assessed across six number aspects of:

- whole number identification, order and sequence (WNIDS)
- fraction, decimal and percentage identification, order and sequence (FRIDS)
- grouping (Group)
- addition and subtraction/place value (Plus/Min)
- multiplication and division (Mult/Div) and
- fractions (Fractions).

These assessments took the form of diagnostic interviews before and after an instructional period of approximately 15 weeks. As explained in Chapter 1 the number aspects split into those focusing on knowledge aspects of number (WNIDS, FRIDS, and Group) and those focusing on strategies (Plus/Min, Mult/Div and Fract). There are two sections to this chapter. The first section deals with the mean stage gains, with the aim of presenting an overview of patterns of improvement in student results. The second section deals with pre- and post-instructional comparisons. In reporting the results, both sections consider the impact that ethnicity, school decile, gender and region of the students may have had on the results.

The size of the sample for the 2001 pilot was close to 10,000, the same as for the Count Me In Too pilot project in 2000. Following the approach taken by Thomas and Ward (2001) in their evaluation of the Count Me In Too pilot project, the results are reported in terms of differences in a practical rather than statistical sense. Thomas and Ward's rationale was:

> Because data was collected on close to 10,000 students, there are large numbers in each sample even when samples are categorised according to the variables of students' age and ethnicity and school region and decile. Sample size raises issues related to practical versus statistical significance in the analysis and reporting of the results. With such large samples, even the smallest differences can be statistically significant. For example, a difference of 0.11 in the mean gain of two subgroups is significant at the 0.001 level. While the 0.11 difference is statistically significant, a difference of a tenth of a stage is not meaningful in any practical sense. (page 13)

The Advanced Numeracy Project also aimed to impact on teachers' professional knowledge. This professional knowledge encompasses content knowledge, knowledge of how students learn mathematics and knowledge of how to teach mathematics. The instruction that the students received impacts on their achievement, and an analysis of student achievement should make links to this instruction.

Overview of the Findings

The student results of the pilots of Count Me In Too and the Early Numeracy Project for 2000 and 2001 (Thomas and Ward, 2001; 2002) were impressive. Similarly the overall picture for the 2001 pilot of ANP reported here clearly demonstrates a pattern of improvement. This is shown in two different ways: mean stage gains and pre- and post-instructional improvements.

Intervention programmes of the nature of the Advanced Numeracy Project encounter challenges in measuring changes in student achievement both in terms of the measures used and in terms of the duration of the data gathering process. To show gains made, it is helpful to be able to track shifts in student achievement over time. Evidence of such shifts is constrained by the duration of the 2001 Advanced Numeracy Project being a 15-week teaching period. Longitudinal analysis, such as has been conducted for the Early Numeracy Project, will be possible in subsequent years of the implementation of the Advanced Numeracy Project. The way in which advances in student understanding are recorded in the Advanced Numeracy Project against the stages of The Number Framework presents special challenges for the evaluation process. The Number Framework was not intended to be an interval scale representing equal steps in number understanding, but was designed so that classroom teachers and schools could easily track the progress of their students. This design supports the professional development component of the Advanced Numeracy Project, which aims to develop teachers' knowledge of number concepts, student strategies and instructional practice in order to raise student achievement. With this practical purpose in mind, the stages at the lower levels of the framework are much smaller than the stages at the upper levels[2]. This design acknowledges the importance of assessing (and if necessary intervening) in the early stages of learning number concepts so that a sound foundation is laid. If the foundation is not secure, higher levels of understanding and achievement will not be reached. From a mathematical point of view, an interval scale would have been desirable for analysis and interpretation in terms of reliability and validity. However, from an educational point of view it is really more important that we have more milestones or benchmarks at the lower levels than the higher ones.

One way to marry the mathematical and educational perspectives on measuring gains is to highlight the context in which teachers use the framework. The use of context is consistent with the theoretical model presented in Chapter 2, which underpins the qualitative analysis of this report. While we can say with confidence that the gains in achievement are real and occur across all sub-groups, more definitive statements cannot yet be made about the links between the professional development and the achievement shown. In determining the reasons for the differential gains, it will be important to establish firm links between particular actions of facilitators, specific components of the programme and gains made in mathematical achievement in students whose teachers were involved in the programme. For instance, while there are gains in all aspects, it is more difficult to establish why some gains are better than others.

Our knowledge of the processes underpinning this pattern of improvement is relatively sketchy. The qualitative data reported in later chapters of this report point to a number of key factors. All participants identified a shift towards more focused teaching as one of these factors. Teachers have identified mental strategies in particular as being a key part of their shift in practice. This shift in practice is discussed in detail in chapters 4 and 5. Further work is being done to determine the importance of the improvement shown in each of the number aspects in relation to the factors identified in the qualitative analysis.

Mean Stage Gains: Total Sample

The results are first discussed in terms of mean stage gains. Mean stage gains are a useful way of examining broad trends in achievement across the sample. However, in this analysis, where

[2] See Young-Loveridge and Wright (forthcoming) for a paper that addresses the validation of The Number Framework.

the steps on the interval scale are uneven, the real gains made in overall achievement need to be interpreted carefully. The number of available stages limits the maximum possible gain. Stage 3 is the lowest possible stage and includes all students operating at a level below Stage 4 (Advanced Counting). The maximum possible gain is three stages for addition and subtraction, four stages for multiplication and division, and five stages for fractions. Students who are at the penultimate stage can only gain one stage, and this will contribute to lowering the average gain for those students included in the analysis. Those students reaching the top stage in any number aspect in the initial diagnostic interview prior to instruction were excluded from the mean gain analysis.

The mean stage gains were not uniform across the different aspects of number. Table 3.1 shows that the least gain (0.62 of a stage) was in whole number identification (WNIDS). The probable reason for this is that the students' knowledge of WNIDS was high to start with given that middle school mathematics programmes have typically emphasised knowledge in whole number at the expense of strategy. In terms of the other aspects of knowledge, the areas of fraction knowledge (FRIDS) (0.86 of a stage) and grouping (Group) (0.80 of a stage) showed the greatest mean stage gains.

Other strategy aspects of addition and subtraction/place value (Plus/Min) (0.65), multiplication and division (Mult/Div) (0.7) and fractions (Fractions) (0.67) fell in the middle and were similar in terms of mean stage gains. The slightly lower gains for addition and subtraction/place value are unsurprising given the greater emphasis that this topic has typically had in mathematics programmes in the middle school.

Thus it would seem that the greater the student's knowledge before intervention with the Advanced Numeracy Project, the less gain they will make along the stages as they proceed through the project.

		WNIDS	FRIDS	Group	Plus/Min	Mult/Div	Fractions
Total	**N = 8059**	**0.62**	**0.86**	**0.80**	**0.65**	**0.70**	**0.67**

Table 3.1: Mean Stage Gains: Total Adjusted Sample

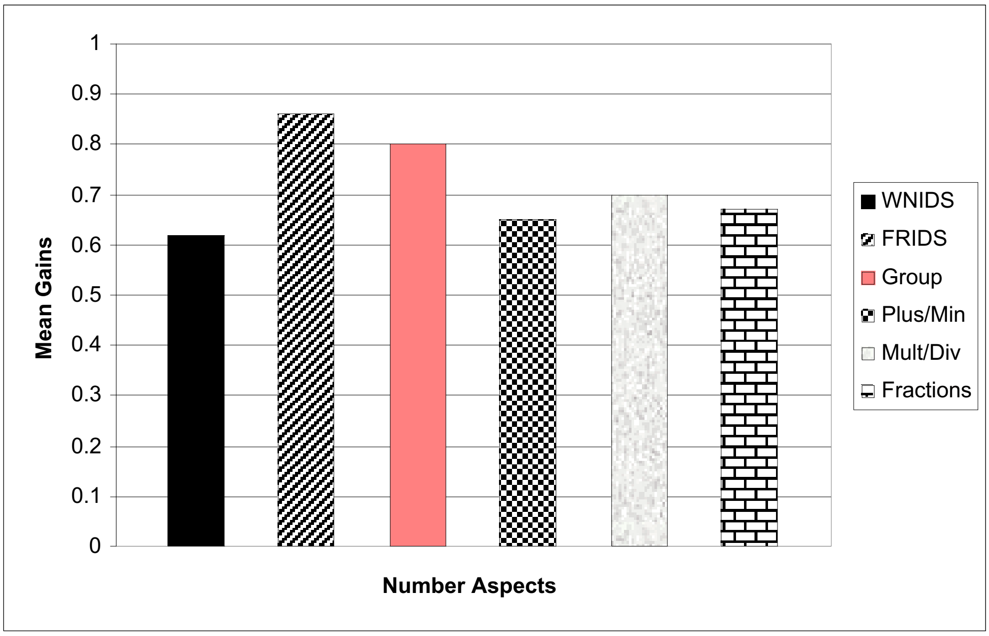


Figure 3.1: Mean Stage Gains: Total Adjusted Sample

Mean Stage Gains by Age Group

The next section examines mean stage gains by age group. The middle school normally has students who are 8, 9 and 10 years old. The table also includes results for the few 7-year-olds and 11-year-olds who are classified as Year 4 and 6 students respectively. This information is presented in both table and figure formats. Table 3.2 shows that mean stage gains across all age groups are similar. Disregarding the sub-groups of 7-year-olds and 11-year-olds (where the number of students is quite small) and comparing the 8-, 9- and 10-year-olds, the patterns varied across the number aspects. There was less than 0.07 of a stage difference between age groups with the exception of fractions and grouping. The 10-year-olds in particular made slightly higher mean stage gains in the two knowledge aspects of fraction knowledge (FRIDS) and grouping (Group).

AGE		WNIDS	FRIDS	Group	Plus/Min	Mult/Div	Fractions
7	**N = 198**	**0.61**	**0.84**	**0.72**	**0.67**	**0.68**	**0.67**
8	**N = 2436**	**0.61**	**0.80**	**0.71**	**0.64**	**0.72**	**0.68**
9	**N = 2586**	**0.59**	**0.83**	**0.79**	**0.63**	**0.67**	**0.63**
10	**N = 2639**	**0.66**	**0.95**	**0.90**	**0.68**	**0.71**	**0.70**
11	**N = 200**	**0.62**	**0.86**	**0.75**	**0.66**	**0.73**	**0.57**
Total	**N = 8059**	**0.62**	**0.86**	**0.80**	**0.65**	**0.70**	**0.67**

Table 3.2: Mean Stage Gains by Age

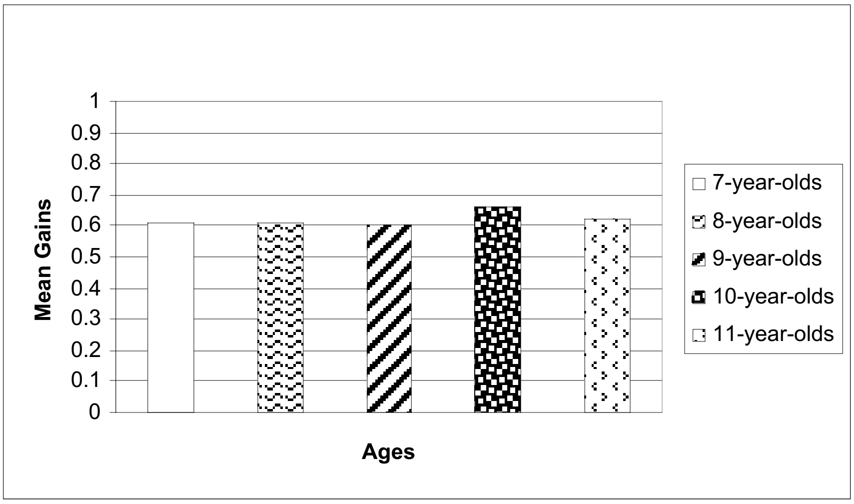


Figure 3.2: Mean Stage Gains on Whole Number Knowledge

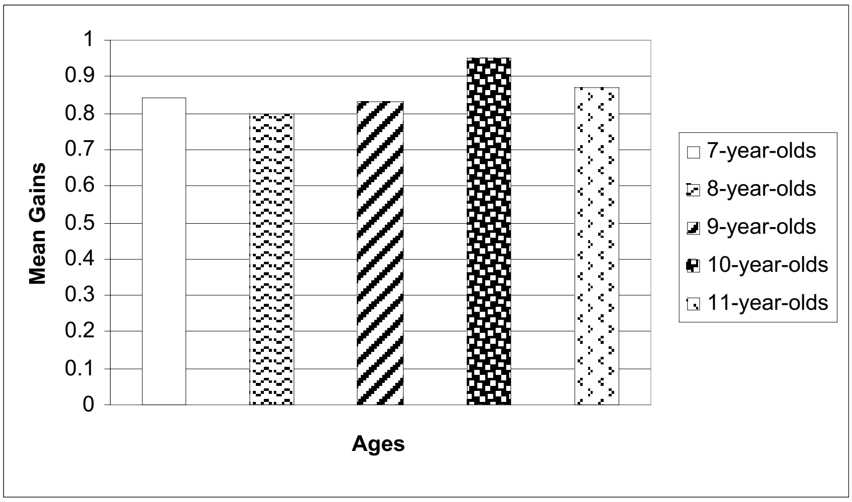


Figure 3.3: Mean Stage Gains on Fraction Knowledge

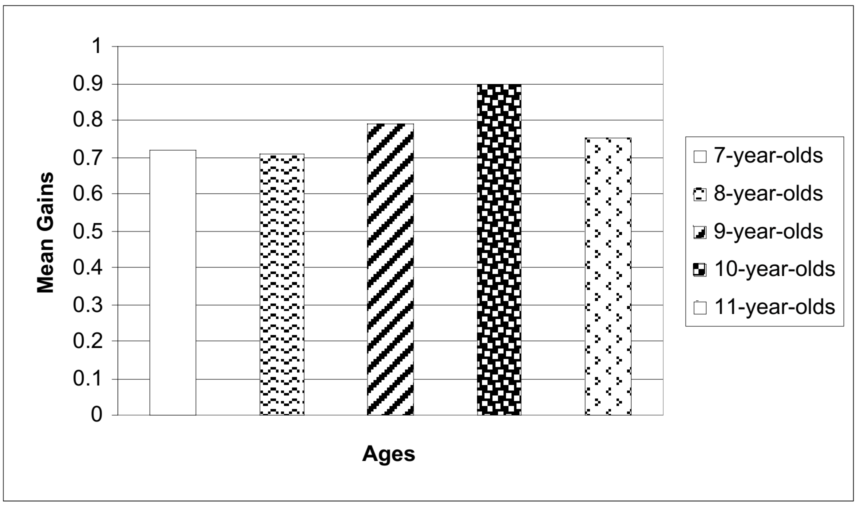


Figure 3.4: Mean Stage Gains on Grouping

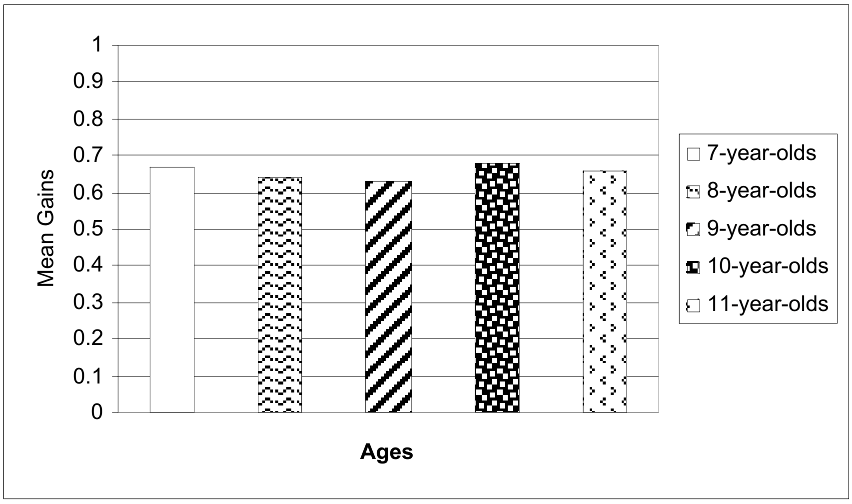


Figure 3.5: Mean Stage Gains on Addition and Subtraction

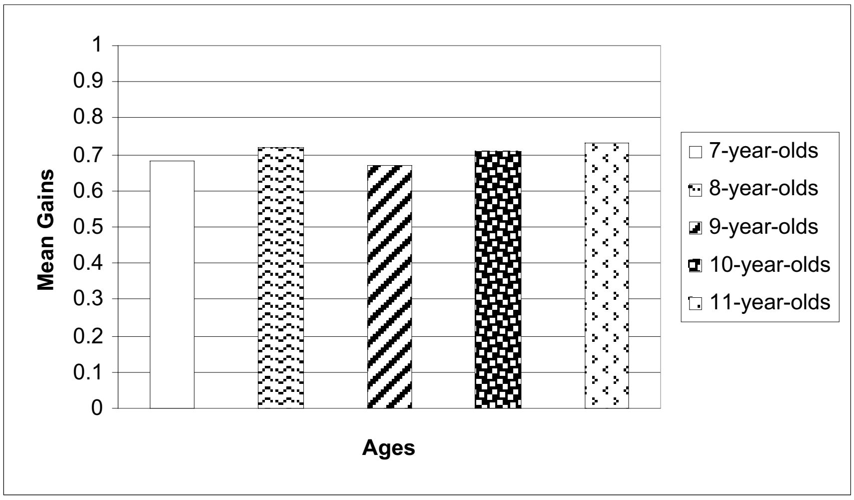


Figure 3.6: Mean Stage Gains on Multiplication and Division

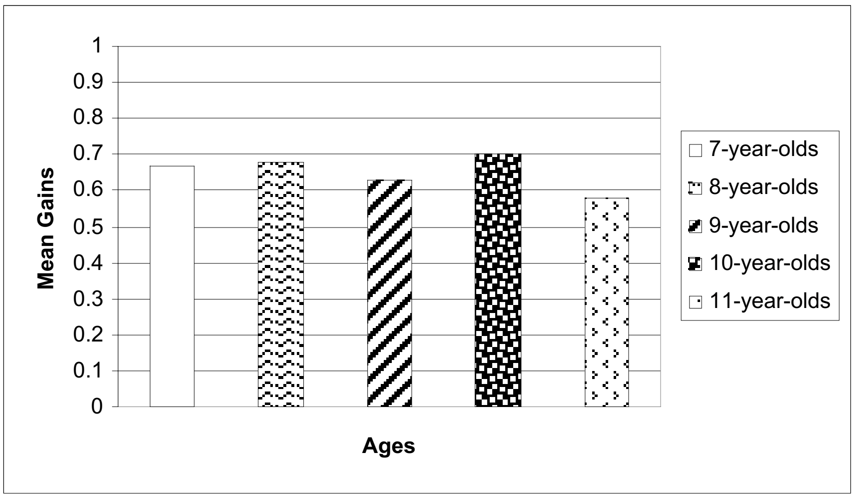


Figure 3.7: Mean Stage Gains on Fractions

Relationship between Decile Rating and the Mean Stage Gains across Age Groups

It is important to consider the association between decile and students' achievements. The decile ratings have been aggregated into three broad groups for the purposes of analysis. The low group comprises deciles 1 to 3, the middle group deciles 4 to 7 and the high group deciles 8 to 10. Table 3.3 shows that the mean stage gains across the decile groupings are relatively similar. There was no more than 0.03 of a stage separating the different groups with the exception of fraction knowledge (FRIDS) (0.08) and grouping (Group) (0.06). These results suggest that there is no overall effect of school decile on student achievement, but this may be due to the ceiling effects operating for certain categories and not others, for example, WNIDS and Plus/Min.

DECILE		WNIDS	FRIDS	Group	Plus/Min	Mult/Div	Fractions
Low	N = 3107	0.62	0.82	0.80	0.64	0.70	0.68
Middle	N = 3173	0.62	0.87	0.78	0.65	0.68	0.65
High	N = 1779	0.60	0.90	0.84	0.65	0.71	0.67
Total	N = 8059	0.62	0.86	0.80	0.65	0.70	0.67

Table 3.3: Mean Stage Gains by Decile Rating

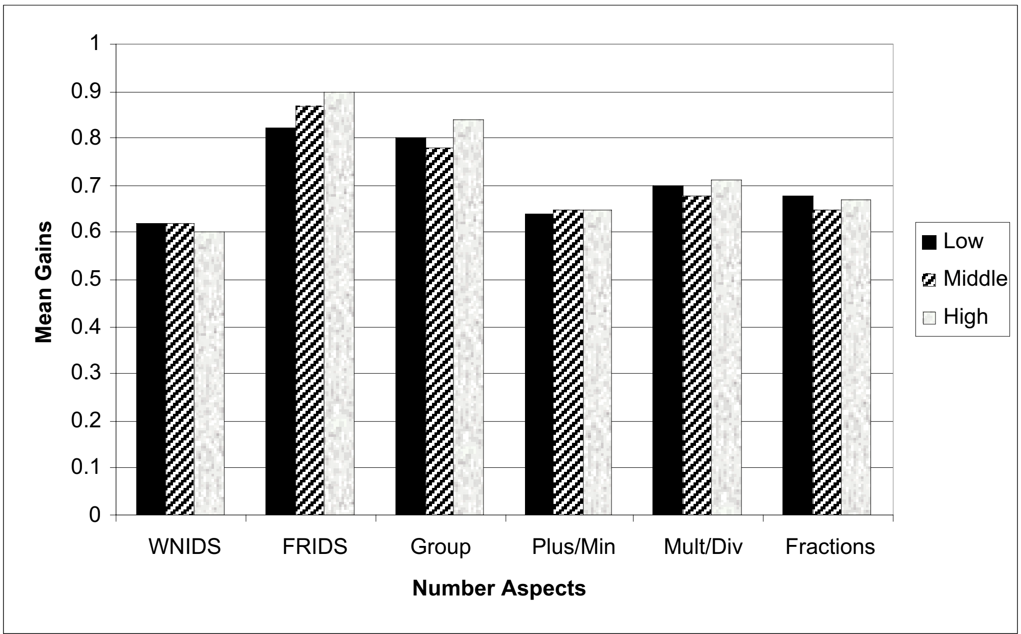


Figure 3.8: Mean Stage Gains by Decile Rating

Relationship between Ethnicity and the Mean Stage Gains across Age Groups

It is also important to consider the extent to which ethnicity is related to student achievement. There are currently concerns about Māori and Pacific Island students' achievement in particular. Table 3.4 below shows some variation across all ethnic groups although all students benefited from ANP regardless of ethnicity. As illustrated in Table 3.4, Pacific Island students made the least gains across all number aspects. This confirms the findings of Forbes et al. (1989). Māori student gains were similar to those of other ethnic groups. European students made notable gains for fraction knowledge (FRIDS) (0.92 of a stage) and grouping (Group) (0.84 of a stage). Asian students made slightly higher gains for grouping (Group) (0.89 of a stage).

ETHNICITY		**WNIDS**	**FRIDS**	**Group**	**Plus/Min**	**Mult/Div**	**Fractions**
Asian	N = 476	0.59	0.78	0.89	0.67	0.69	0.71
European	N = 5118	0.62	0.92	0.84	0.66	0.72	0.68
Māori	N = 1469	0.66	0.79	0.73	0.64	0.66	0.66
Pacific Island	N = 768	0.56	0.65	0.62	0.60	0.63	0.59
Other	N = 228	0.66	0.82	0.76	0.64	0.69	0.71
Total	N = 8059	0.62	0.86	0.80	0.65	0.70	0.67

Table 3.4: Mean Stage Gains by Ethnicity

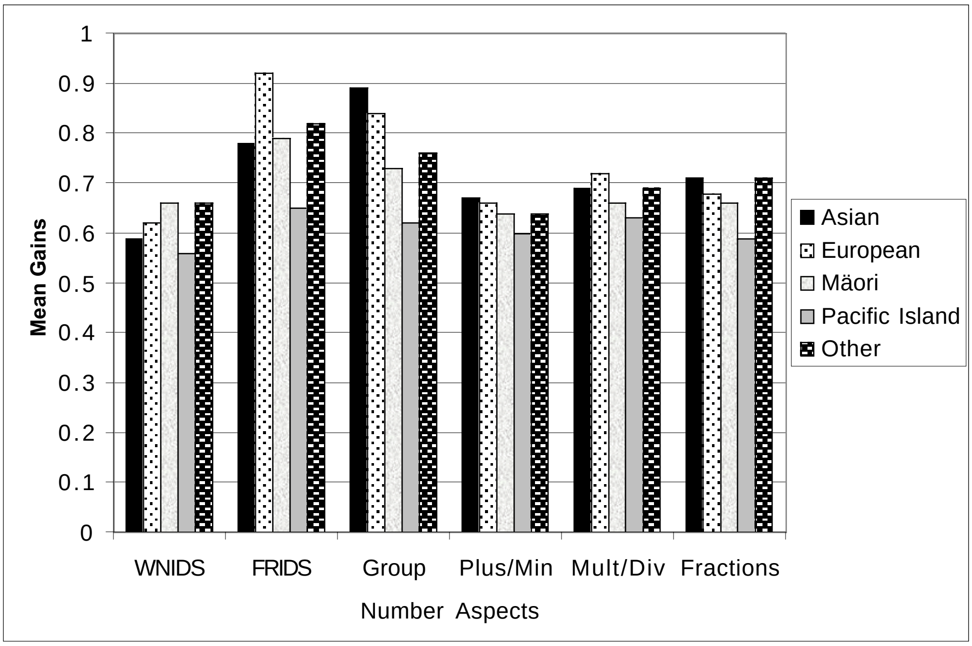


Figure 3.9: Mean Stage Gains by Ethnicity

Relationship between Gender and the Mean Stage Gains across Age Groups

Gender appeared to have very little impact if any on student achievement. Mean stage gains of less than 0.05 of a stage separated the gains on the six number aspects monitored. Similar findings of less than 0.03 of a stage difference were reported by Thomas and Ward (2001) in the evaluation of the Count Me In Too pilot project.

GENDER		WNIDS	FRIDS	Group	Plus/Min	Mult/Div	Fractions
Female	N = 3915	0.60	0.85	0.80	0.63	0.67	0.66
Male	N = 4144	0.63	0.87	0.80	0.66	0.72	0.68
Total	N = 8059	0.62	0.86	0.80	0.65	0.70	0.67

Table 3.5: Mean Stage Gains by Gender

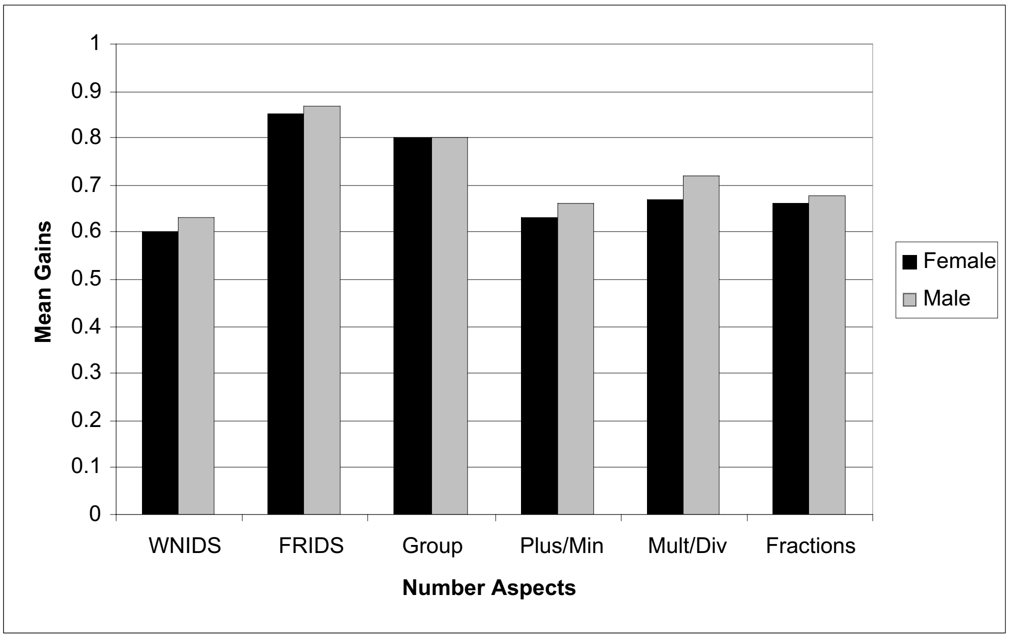


Figure 3.10: Mean Stage Gains by Gender

Mean Stage Gains Adjusted by Age and Stage

The next section examines the gains made when taking into account the stage at which the students started. As Table 3.6 shows, the pattern that emerges is that students starting at the lowest stages made the greatest gains and conversely those starting at the highest stages make the least gains. These patterns of gain hold across ages (with a slight exception of 11-year-olds), ethnicity, decile ratings and the strategy aspects of number monitored. It is important to note that the gains made by Pacific Island students, and to a lesser extent Māori students at the more advanced stages, are lower than the other groups for all strategy aspects of number. The set of tables and graphs for Addition and Subtraction follow. The other strategy aspects are included in Appendix D.

AGE		**Count All**	**Advanced Counting**	**Early Additive Part/Whole**
7-year-olds	**N = 198**	0.90	0.76	0.46
8-year-olds	**N = 2436**	1.00	0.69	0.42
9-year-olds	**N = 2586**	1.10	0.72	0.44
10-year-olds	**N = 2639**	1.14	0.83	0.51
11-year-olds	**N = 200**	0.50	0.89	0.44
Total	**N = 8059**	1.05	0.74	0.46

Table 3.6: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Age

ETHNICITY		**Count All**	**Advanced Counting**	**Early Additive Part/Whole**
Asian	N = 476	1.05	0.82	0.51
European	N = 5118	1.04	0.78	0.48
Māori	N = 1469	1.05	0.72	0.40
Pacific Islands	N = 768	1.12	0.56	0.43
Other	N = 228	0.82	0.77	0.46
Total	N = 8059	1.05	0.74	0.46

Table 3.7: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Ethnicity

DECILE		**Count All**	**Advanced Counting**	**Early Additive Part/Whole**
Low	N = 3107	1.05	0.70	0.45
Middle	N = 3173	1.04	0.75	0.45
High	N = 1779	1.07	0.83	0.50
Total	N = 8059	1.05	0.74	0.46

Table 3.8: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Decile Rating

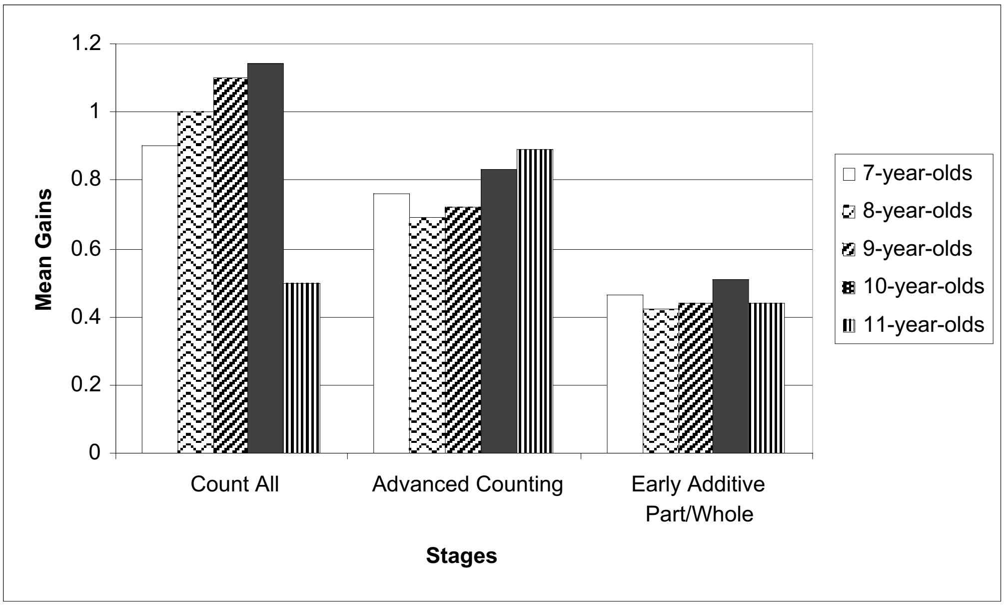


Figure 3.11: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Age

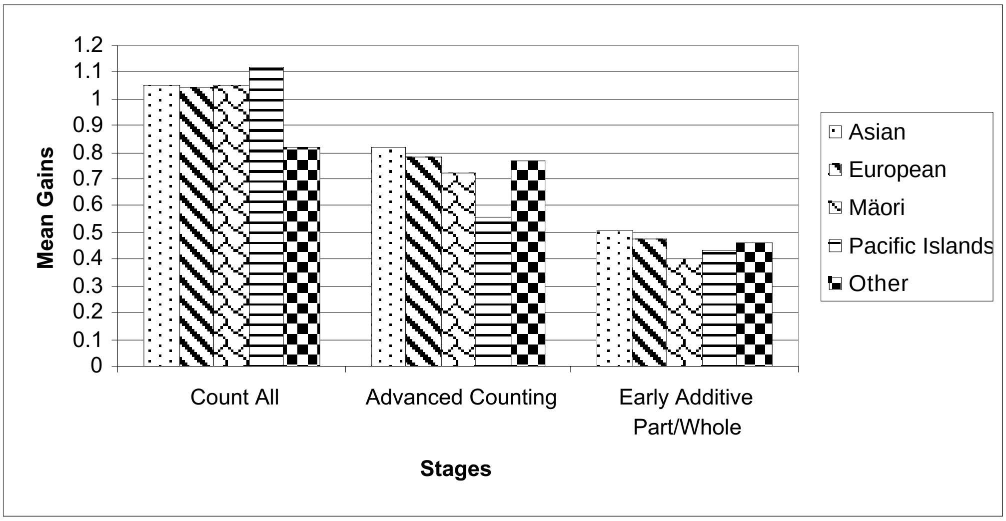


Figure 3.12: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Ethnicity

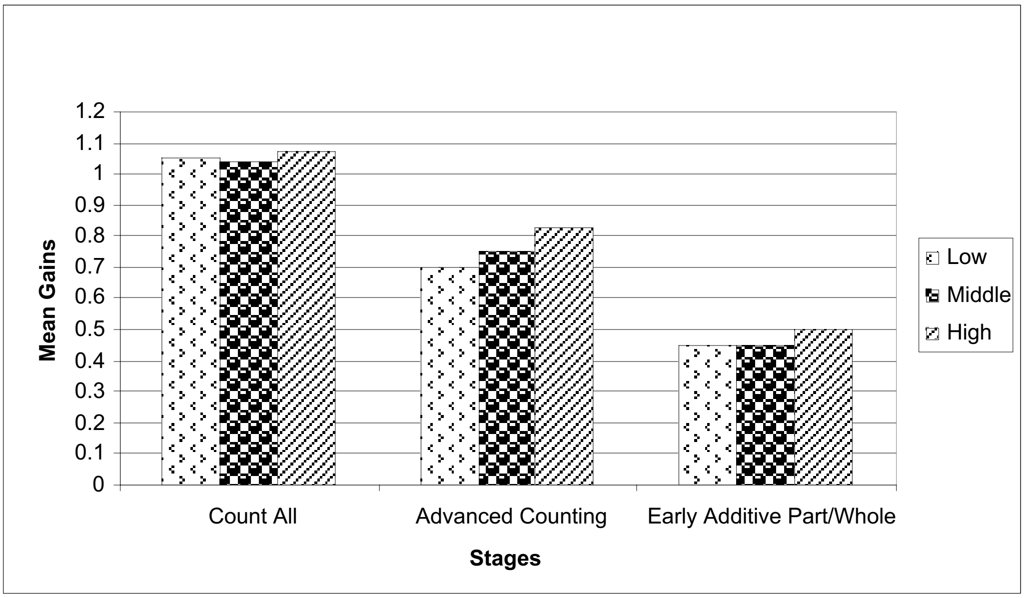


Figure 3.13: Mean Stage Gains for Addition and Subtraction/Place Value: Stages by Decile Rating

Pre- and Post-instruction Comparisons

Pre- and post- instruction comparisons are useful for viewing progress as a change in the percentage of students at a particular stage before and after instruction. The following commentary will focus on the strategy aspects of number monitored, as these indicate the level of sophistication of students' problem-solving strategies in number. The knowledge aspects, as discussed in Chapter 1, are "an essential prerequisite for the development of more advanced strategies" (Ministry of Education, 2002b, page 1) (the pre- and post-instruction comparisons for the knowledge aspects can be found as Appendix E).

Tables 3.9–3.11 show that the majority of students started at stage 4 or stage 5 for addition and subtraction/place value, multiplication and division, and fractions prior to instruction. Stage 4 represents advanced counting, where students rely on counting to solve problems. Stage 5 represents early additive part/whole. This is the least sophisticated form of part/whole thinking in which students split and join numbers in one or two ways.

The patterns of achievement in post-instruction show a clear movement of the majority of the sample to the next stage; in other words the majority of students finished the project sitting at stage 5 (early additive part/whole) or above for each of the strategy aspects of number monitored. The results for addition and subtraction show that 79% of students had part/whole strategies after instruction. Nearly as many students were at stage 5 (early additive part/whole) as were at stage 6 (advanced additive part/whole). For multiplication and division 72% of students had part/whole strategies after instruction. For fractions, 69% of students had part/whole strategies after instruction.

STAGE	Pre-Instruction	Post-Instruction
Count All	7% (588)	2% (131)
Advanced Counting	41% (3308)	19% (1548)
Early Additive Part/Whole	37% (2951)	42% (3409)
Advanced Additive Part/Whole	15% (1243)	37% (3002)
Total	100% (8090)	100% (8090)

Table 3.9: Addition and Subtraction/Place Value: Pre- and Post-Instruction

STAGE	Pre-Instruction	Post-Instruction
Count All	15% (1220)	4% (321)
Advanced Counting	38% (3068)	24% (1915)
Early Additive Part/Whole	29% (2324)	32% (2689)
Advanced Additive Part/Whole	14% (1151)	26% (2067)
Advanced Multiplicative Part/Whole	4% (327)	14% (1098)
Total	100% (8090)	100% (8090)

Table 3.10: Multiplication and Division: Pre- and Post-Instruction

STAGE	Pre-Instruction	Post-Instruction
Count All	21% (1707)	7% (538)
Advanced Counting	33% (2654)	24% (1930)
Early Additive Part/Whole	33% (2650)	38% (3071)
Advanced Additive Part/Whole	9% (692)	18% (1454)
Advanced Multiplicative Part/Whole	4% (356)	11% (921)
Advanced Proportional Part/Whole	0% (31)	2% (176)
Total	100% (8090)	100% (8090)

Table 3.11: Fractions: Pre- and Post-Instruction

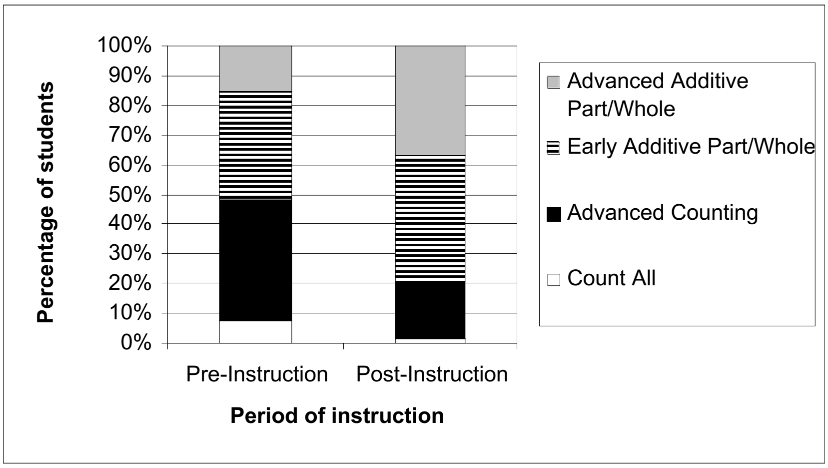


Figure 3.14: Addition and Subtraction/Place Value: Pre- and Post-Instruction

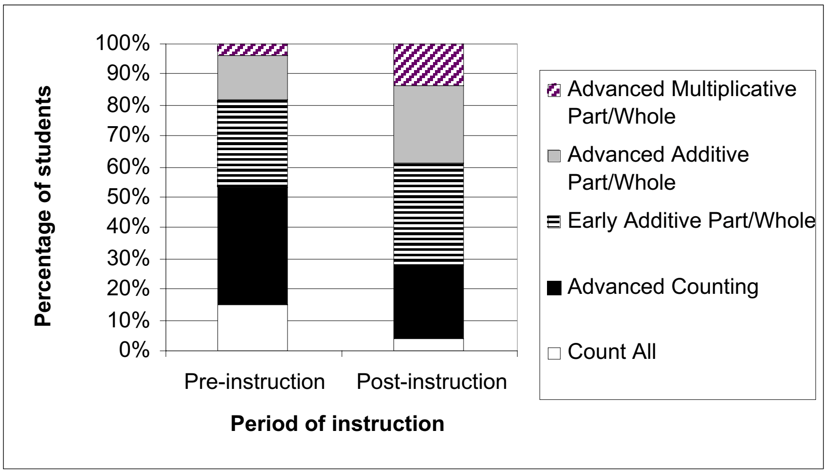


Figure 3.15: Multiplication and Division: Pre- and Post-Instruction

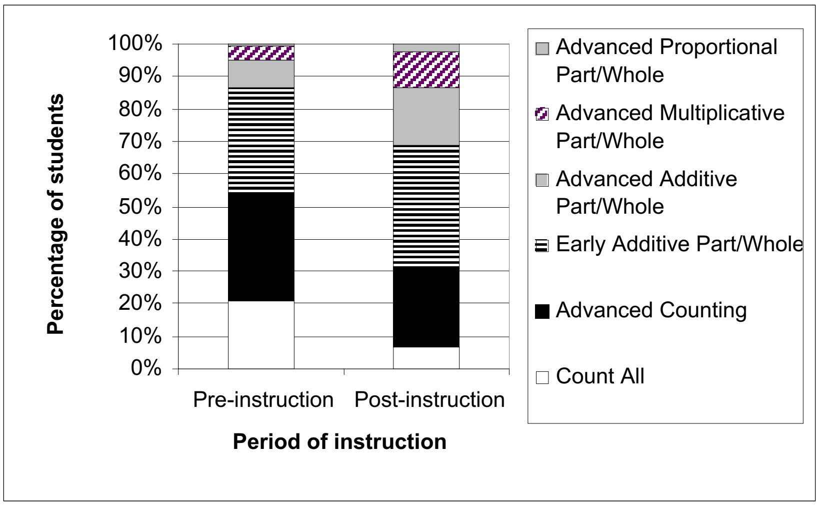


Figure 3.16: Fractions: Pre- and Post-Instruction

When adjusted for age the data show that across all the strategy aspects of number, 7- and 8-year-olds make up the greatest proportion of students starting at stages 3 and 4 (counting-based strategies). The greatest proportion of students starting at the highest stage (more sophisticated part/whole-based strategies) for each aspect are 10- and 11-year-olds.

Of concern overall is the small proportion of students who make the transition to advanced part/whole thinking, as shown below. A high proportion of 10-year-olds made this transition. This is particularly so for multiplication and division and fractions with only a small proportion achieving this transition. (See appendices E and F.)

STAGE	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	10% (20)	11% (275)	7% (188)	4% (97)	4% (8)	7% (588)
Advanced Counting	50% (98)	50% (1216)	40% (1033)	33% (880)	40% (81)	41% (3308)
Early Additive Part/Whole	30% (59)	31% (757)	38% (980)	41% (1080)	37% (75)	37% (2951)
Advanced Additive Part/Whole	11% (21)	8% (189)	15% (391)	23% (604)	19% (38)	15% (1243)
Total	100% (198)	100% (2437)	100% (2592)	100% (2661)	100% (202)	100% (8090)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table 3.12: Addition and Subtraction/Place Value Adjusted by Age (pre-instruction)

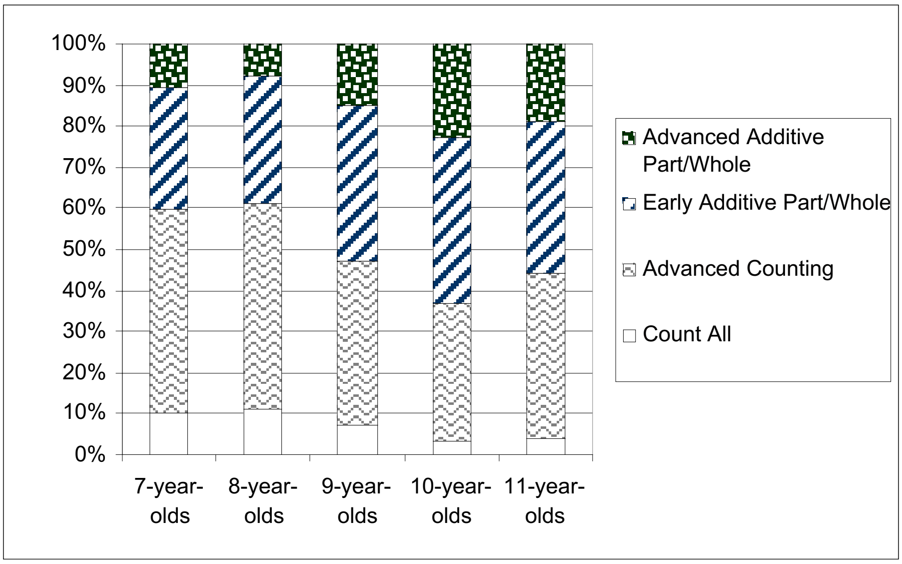


Figure 3.17: Addition and Subtraction/Place Value Adjusted by Age (pre-instruction)

STAGE	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	3% (6)	2% (59)	2% (44)	1% (18)	2% (4)	2% (131)
Advanced Counting	22% (44)	27% (645)	19% (493)	13%(336)	15% (30)	19% (1548)
Early Additive Part/Whole	46% (90)	46% (1120)	43% (1111)	38%(1008)	40% (80)	42% (3409)
Advanced Additive Part/Whole	29% (59)	25% (613)	36% (944)	49%(1298)	44% (88)	37% (3002)
Total	100%(199)	100% (2437)	100%(2592)	100%(2660)	100%(202)	100% (8090)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table 3.13: Addition and Subtraction/Place Value Adjusted by Age (post-instruction)

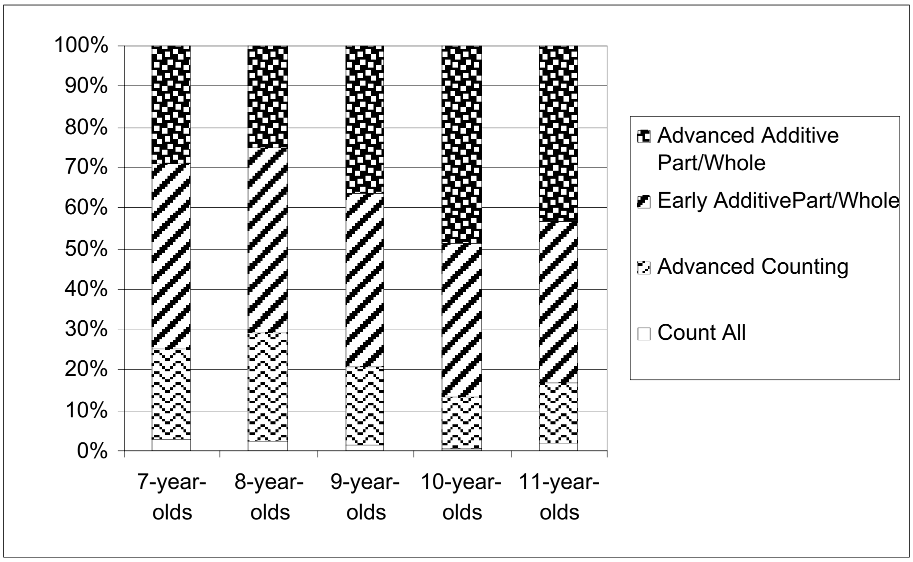


Figure 3.18: Addition and Subtraction/Place Value Adjusted by Age (post-instruction)

Figure 3.19 shows a pattern of improvement across the age groups. The graph shows the improvements that occurred in the stages for each age level. For instance, for 8-year-old students the greatest shift showed between the Advanced Counting and the Early Additive Part/Whole and Advanced Additive Part/Whole stages – the number of students at the Advanced Counting stage decreased markedly from 50% to 27% (a reduction of 23%), while the number of students at the Early Additive Part/Whole stage increased by 15% from 31% to 46% and the number of students at the Advanced Additive Part/Whole stage increased by 16% from 8% to 25%. By contrast, 10-year-olds made a smaller shift at the Early Additive Part/Whole stage – a decrease of 41% to 38%, but they showed a marked increase at the Advanced Additive Part/Whole stage – leaping from 23% to 49%. (See tables for the other strategies in Appendix H.)

As reported by Thomas and Ward (2002) in their evaluation of the Early Numeracy Project, this figure also shows that the younger students, after participation in ANP achieved more highly than older students before participation. For instance, the final assessment of 7-year-olds can be compared with the initial assessment of 8-year-olds and so on. As for the Early Numeracy Project, the comparisons indicate that all students benefit to some extent from participating in the project.

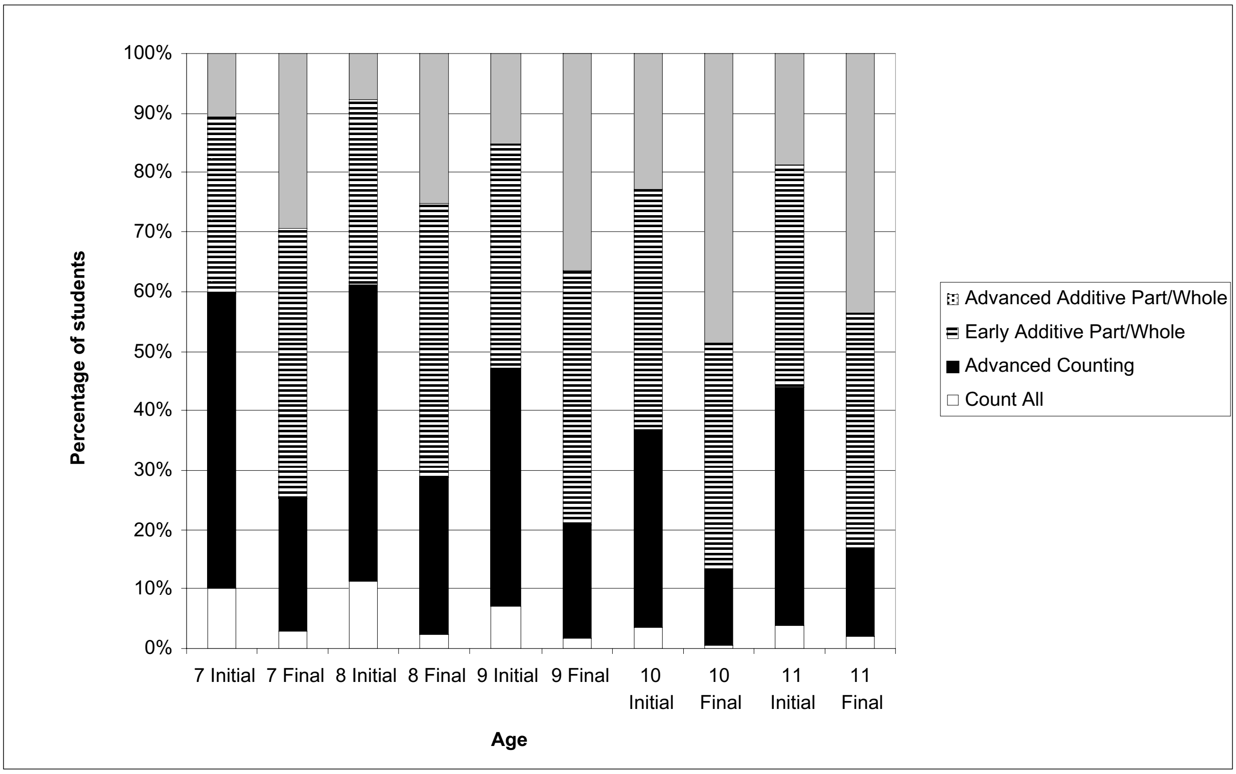


Figure 3.19: Patterns of Improvement in Addition and Subtraction/Place Value by Age

The key aim of the Advanced Numeracy Project is to shift students from counting- based to part/whole based strategies. This shift is critical to later success in mathematics (Thomas and Ward, 2001; Wright, 1998; Young-Loveridge, 2001), and it is therefore very useful to know which students make this shift and which students make no change. It is worth noting here that this is a significant shift in student understanding and is the culmination of many years of early development in number related activities. The stages of learning as represented on The Number Framework are not intended to represent even-sized steps, and this is particularly the case with the shift from Advanced Counting to Early Additive Part/Whole (labelled respectively as stages 4 or 5 on The Number Framework) as already noted.

The table below again shows that the majority of those students who were initially advanced counters (that is using counting-based strategies for problem solving) in addition and subtraction by the end of the project had shifted to using part/whole-based strategies. As presented in previous analyses, it is important to note that the strategies most students adopted represented the early additive stage or the stage in which numbers are split or joined in one or two ways. Fewer of the advanced counters shifted to use more sophisticated strategies at the advanced additive stage. A smaller proportion of students who were initially advanced counters made no change at all. There were no marked differences between age groups.

AGE	No Change		Became Early Additive		Became Advanced Additive	
7-year-olds	35%	(34)	55%	(54)	10%	(10)
8-year-olds	40%	(484)	52%	(635)	8%	(99)
9-year-olds	39%	(401)	51%	(524)	10%	(108)
10-year-olds	32%	(285)	52%	(456)	16%	(139)
11-year-olds	32%	(26)	47%	(38)	21%	(17)
Total	37%	(1230)	52%	(1707)	11%	(373)

Table 3.14: Final Status of Students, by Age, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

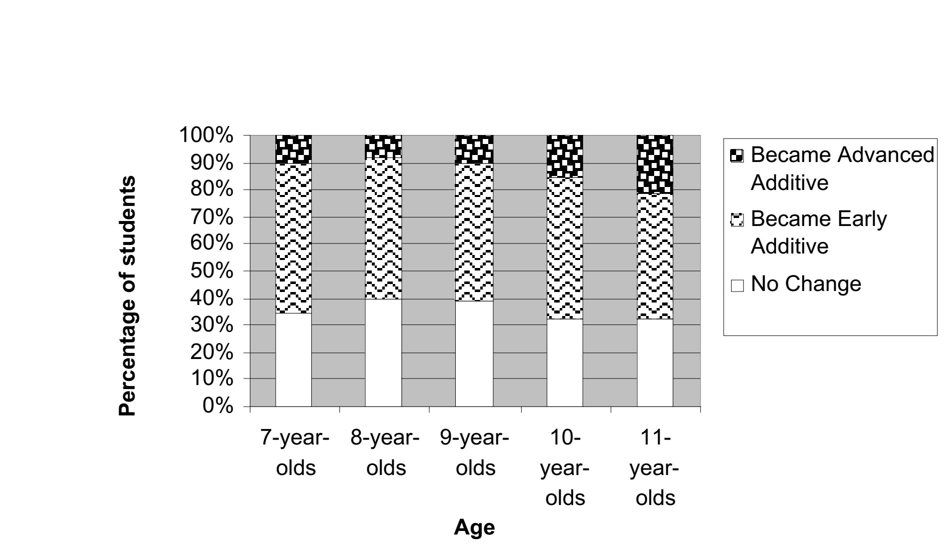


Figure 3.20: Final Status of Students, by Age, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

When these data are analysed against ethnicity, the patterns are similar, with the majority of students adopting part/whole-based strategies. Some minor variations occur. Of particular note is the lower proportion of Pacific Island students who make the shift to part/whole-based strategies.

ETHNICITY	No Change		Became Early Additive		Became Advanced Additive	
Asian	33%	(44)	52%	(70)	15%	(20)
European	34%	(669)	53%	(1038)	13%	(244)
Māori	39%	(285)	50%	(366)	11%	(79)
Other	34%	(27)	54%	(43)	11%	(9)
Pacific Islands	49%	(205)	46%	(190)	5%	(21)
Total	37%	(1230)	52%	(1707)	11%	(373)

Table 3.15: Final Status of Students, by Ethnicity, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

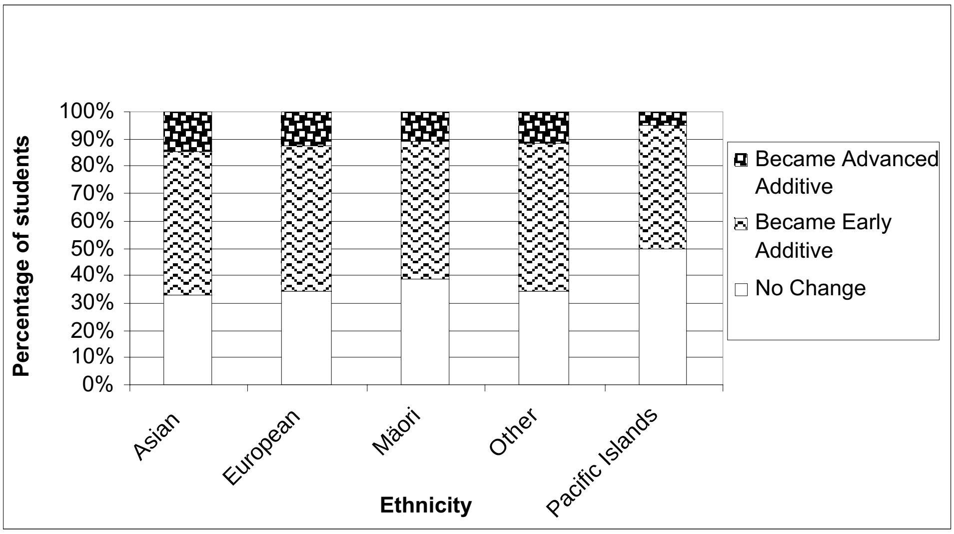


Figure 3.21: Final Status of Students, by Ethnicity, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

Decile ratings when clustered into low, middle and high groupings do not demonstrate a marked effect on the shift to part/whole strategies of students who are advanced counters. However, slightly more students in lower decile schools made no change and fewer shifted to the advanced additive stage from these decile groups. When we look at the deciles in isolation, however, we can see that in the lowest decile schools (decile 1), there was no shift in over half the students who had been advanced counters before instruction with the Advanced Numeracy Project.

Decile	No Change		Became Early Additive		Became Advanced Additive	
Low (1–3)	40%	(592)	50%	(731)	10%	(146)
Middle (4–7)	36%	(466)	52%	(675)	12%	(149)
High (8–10)	31%	(172)	55%	(301)	14%	(78)
Total	37%	(1230)	52%	(1707)	11%	(373)

Table 3.16: Final Status of Students, by Decile, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

DECILE	No Change		Became Early Additive		Became Advanced Additive	
1	62%	(136)	34%	(74)	4%	(8)
2	31%	(183)	54%	(321)	15%	(86)
3	41%	(273)	51%	(336)	8%	(52)
4	36%	(242)	52%	(355)	12%	(82)
5	35%	(35)	47%	(47)	17%	(17)
6	32%	(87)	57%	(154)	11%	(29)
7	42%	(102)	49%	(119)	9%	(21)
8	28%	(67)	52%	(124)	19%	(46)
9	33%	(12)	58%	(21)	8%	(3)
10	33%	(93)	56%	(156)	10%	(29)
Total	37%	(1230)	52%	(1707)	11%	(373)

Table 3.17: Final Status of Students, by Decile, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

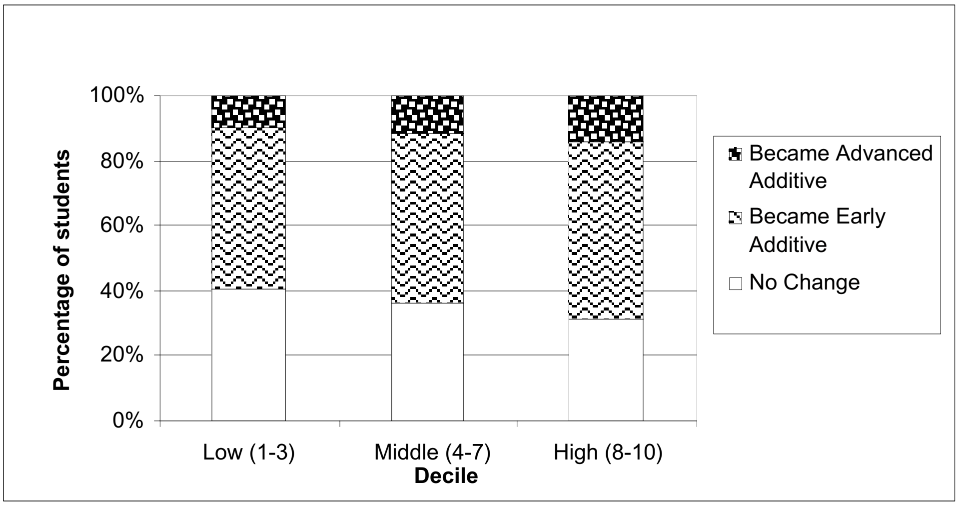


Figure 3.22: Final Status of Students, by Aggregated Decile Rating, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

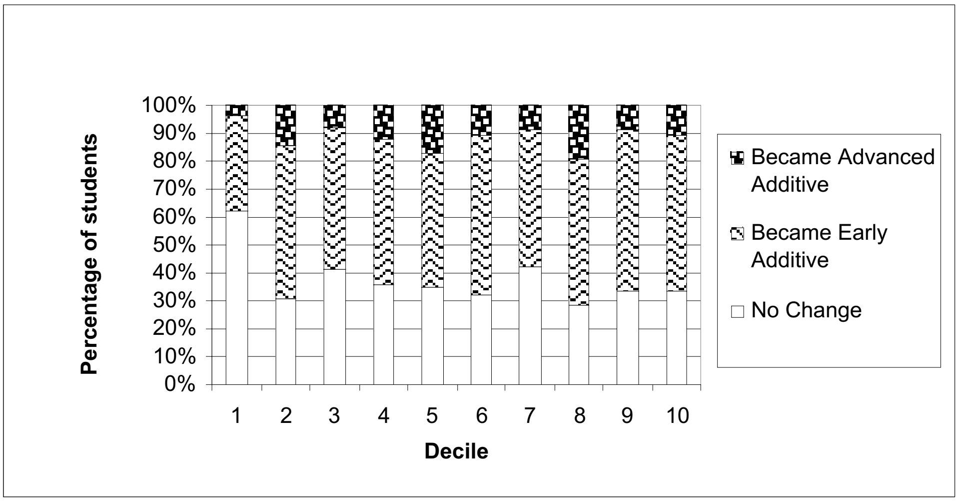


Figure 3.23: Final Status of Students, by Decile Rating, Who Were Initially Advanced Counters for Addition and Subtraction/Place Value

Many more analyses of these data are possible. As noted in the introduction, while there was a clear pattern of improvement, ongoing analysis beyond the scope of this evaluation would shed further light on why some gains are larger than others. More detailed student results can be found in the appendices D–H.

Chapter 4: Perspectives of Adult Participants

The development of teachers' subject and pedagogical content knowledge is important in raising student achievement in mathematics through effective mathematics teaching. The depth of this knowledge underpins the quality of the classroom programme. The project aimed to impact on this knowledge by providing an explanatory framework and by working intensively with teachers. This chapter will consider the impact of the project on teachers and students by drawing on three different sets of data: teacher questionnaires, principal questionnaires and facilitator questionnaires. The emerging themes can be tracked across these multiple data sources and across the different questions in any one return. These themes will again be picked up in the case studies on effective facilitation that can be found in Chapter 5.

Impact on Teachers' Professional Knowledge

All three groups of participants in the project were asked to comment on the development of teachers' professional knowledge. In considering the impact the project had on their professional knowledge, the teachers, principals and facilitators were asked to respond specifically to three aspects: teachers' content knowledge of mathematics, teachers' understanding of how students learn number and teachers' approach to teaching number. The questions asked followed a similar pattern to those asked by Thomas (Thomas and Ward, 2001, 2002) in her evaluations of the Early Numeracy Project and Bobis (1999) in her evaluation of Count Me In Too in New South Wales.

Of the 399 questionnaires sent out, 268 were returned, resulting in a 67% response rate. Of these 268, there were 8 incomplete returns, and 6 arrived too late for inclusion. The following analysis is based on the remaining 254. Of the 70 questionnaires sent out to principals, 65 were returned, a response rate of 93%. Sixteen of the 17 questionnaires sent to facilitators were returned, a 94% response rate.

Category	Details	Percentage (n = 254)
Size of school*	< 100	2
	101–200	9
	201+	80
Region*	Auckland	32
	Waikato	7
	Bay of Plenty	6
	Hawke’s Bay	7
	Taranaki	5
	Wellington	6
	Nelson	4
	Canterbury	11
	Otago	7
	Southland	6
Decile*	1–3	36
	4–7	32
	8–10	22
Age*	20–25	10
	26–35	22
	36–45	26
	46–55	32
	56+	7
Gender*	Female	69
	Male	16
Years of teaching experience*	1–5	27
	6–10	15
	11–15	12
	16–20	16
	21+	28
Level currently being taught*	3/4	24
	4	11
	4/5	11
	5	5
	5/6	34
	6	11
	6/7	2
Years of teaching experience at years 4–6*	1–5	51
	6–10	20
	11–15	11
	16–20	6
	21+	7
Length of time at current school*	1–5 years	60
	6–10 years	17
	11–15 years	15
	16–20 years	4
	21+ years	2
Highest level teaching qualifications*	Diploma of Teaching	31
	Postgraduate Diploma	12
	Bachelor’s Degree	49
	Master’s Degree	3
Undertaking further study*	Yes	27
	No	63

*Not all respondents completed this question.

Table 4.1: Demographic Data of Teacher Respondents

Category	Details	Percentage (n = 65)
Size of school*	< 100	5
	101–200	25
	201+	69
Region*	Auckland	20
	Waikato	6
	Bay of Plenty	6
	Hawke's Bay	9
	Taranaki	3
	Wellington	5
	Nelson	3
	Canterbury	12
	Otago	11
	Southland	9
Decile*	1–3	45
	4–7	34
	8–10	20

*Not all respondents completed this question.

Table 4.2: Demographic Data of Principal Respondents

Category	Details	Percentage (n = 16)
Age	20–25	0
	26–35	25
	36–45	31
	46–55	31
	56+	13
Gender*	Female	81
	Male	13
Years of teaching experience	1–5	13
	6–10	13
	11–15	6
	16–20	13
	21+	55
Years of advisory experience	1–5	50
	6–10	31
	11–15	13
	16+	6

*Not all respondents completed this question.

Table 4.3: Demographic Data of Facilitator Respondents

Teachers' Content Knowledge of Mathematics

Seventy-three percent of teachers reported that their content knowledge of mathematics had been developed in some way as a result of participating in the Advanced Numeracy Project. One teacher commented:

> *My knowledge has improved! And I have studied maths at university!!*

Of the 24% who felt that their content knowledge had not developed, most commented that they already knew the content well. A small number of teachers who answered the questionnaire (3%) did not respond to this question.

Of the 73% of teachers who felt their content knowledge had developed, 79% mentioned mental strategies. Typical comments included:

> *The strategy of using 10 is new to me, but it works.*
>
> *[I am] more aware of the strategies available for children to use to help them manipulate and work with numbers in their head.*
>
> *I use more sophisticated strategies in my own mental computation. I almost never use a calculator now!*
>
> *I am putting numbers together and taking them apart using different methods – I would not have thought to use some of these methods in the past.*

Some teachers (47%) commented on a general expansion and deepening of their knowledge, such as the teacher who said, "Although I'm teaching a younger age-level, for the first time, I believe I now have a greater understanding of how maths works". Other comments made reference to the central place of number in relation to the other strands. As one teacher explained, "The importance of number as being a fundamental base for all maths teaching/learning was emphasised, and now I continually revisit and reinforce those concepts throughout the programme".

Another group of comments (35%) related to the developmental number framework. Teachers' comments included:

> *Now I understand that children go through various "stages" in their mathematical development.*
>
> *I'm more aware of number and its importance, as well as sequential development and strategies.*
>
> *I have a much clearer understanding of the stages children reach and need to move onto – also the strategies they use to solve number problems.*

All facilitators felt that the Advanced Numeracy Project had impacted on teachers' mathematics content knowledge. Specific areas mentioned were knowledge of fractions, decimals and percentages (38%) and knowledge of mental strategies (38%). The facilitators' comments included:

> *Absolutely YES! Most especially in the importance of place value and of the relationships between fractions, decimals, percentages and proportional thinking.*
>
> *Absolutely. Most teachers I work with would not rate their own personal maths at higher than Advanced Additive before the project. Now many see what Advanced Multiplicative/Proportional are about and understand the complexity of developing fraction/decimal concepts.*

They became more aware of the knowledge itself and how it scaffolded the stages of development. They recognised gaps in student knowledge (especially fractions, decimals, percentages) and thus looked more closely at their own knowledge.

The teachers commented that they had to really work on their own strategies and that their skills with the strategies improved as they became more familiar with the various strategies by using them.

	Mental Strategies	More In-depth Knowledge	Fractions, Decimals and Percentages	Developmental Number Framework
Teachers	79%	47%	5%	35%
Facilitators	38%	0%	38%	19%

Table 4.4: Percentages of Respondents Who Commented on Particular Factors Enhancing Teachers' Content Knowledge

It is interesting to note that teachers identified mental strategies more strongly than facilitators did. Teachers talked about their knowledge in general terms while facilitators were more likely to see content knowledge development in the specific area of fractions, decimals and percentages. Teachers rated the developmental number framework as having more impact on their content knowledge than facilitators did. The way in which the developmental number framework serves as an explanatory function of how students learn mathematics is an important feature of the Advanced Numeracy Project and is discussed more fully in Chapter 5.

Teachers' Understanding of How Students Learn Number

Participation in the Advanced Numeracy Project was linked to an enhancement of teachers' understanding of how students learn number.

Three percent of the 13% of teachers who reported no change to their understanding of how students learn number were either beginning teachers or had participated in the Early Numeracy Project. Less than 1% of teachers completing the questionnaire did not answer this question. Eighty-six percent of those completing the questionnaire reported a greater understanding, particularly in the areas of mental strategies (28%). Several teachers explained how this had spin-offs for their feelings about teaching mathematics.

I enjoy teaching maths more as my children show greater enjoyment. I feel more confident about teaching maths as a result of greater knowledge of how children learn number knowledge and strategies.

My content knowledge of number has developed. I have become very interested in finding out how each pupil solves number problems – the strategies have become very important to me.

The need to understand how children are thinking as they are working out number problems – verbalising strategies – has given me a greater insight into how they do things.

Another area of greater understanding commented on by teachers was the developmental number framework area (24%). For some, using the framework clarified for them the next stage of the students' development, as is shown with one teacher's comment, "It consolidated my belief that teaching and building on a sequential set of strategies promotes a sound understanding of maths in children". Similar comments included, "I have always had a positive attitude towards maths, but I think this is a great programme, which highlights the next learning stage so easily" and "[I am] better [at] teaching to needs as [I] understand students' developmental mathematical stages [better]". Some teachers had acquired a broader view of problem solving (16%), reflected in the comment, "… all children, regardless of ability, need to be encouraged to think of number in a variety of ways and … hands-on, concrete materials in problem solving enhance thinking".

	Mental Strategies	Developmental Number Framework	Problem Solving	Group Work	Teaching Approaches
Teachers	28%	24%	16%	2%	0%
Facilitators	0%	50%	0%	44%	38%

Table 4.5: Percentages of Respondents Who Commented on Particular Factors Enhancing Teachers' Pedagogical Knowledge

All facilitators considered the Advanced Numeracy Project to have impacted on teachers' understanding of how students learn number. Specific areas commented on included teachers' knowledge of the developmental number framework (50%), as one facilitator put it, "… in thinking about their teaching through the lens of the developmental sequences presented in the framework" and another commented that "The framework [has] the most significant impact on teachers' pedagogical knowledge". Another commented that, "The assessment tool helped them pinpoint student needs, and the framework helped them plan programmes that focused the students' learning needs". Another area facilitators identified as having an impact on teachers' understanding was knowledge of group work (44%), as explained in the following comment that, "There appeared to be a tighter fit of the programme to the needs of students and a strong focus on group work". The third area identified by facilitators was teachers' knowledge of approaches to teaching number (38%). This is summed up in the following comment, "Teachers have rethought … how to teach number, how to record, how to develop solutions strategies".

Approaches to Teaching Mathematics

The majority of teachers (92%) felt that changes to the way in which they taught number had resulted from their participation in the project. The teachers regarded these changes positively. Most of the 8% who felt that they had not changed said that they had already changed from previous experiences with similar material such as ENP either as teachers or in their pre-service training. Very few teachers (6 out of the 254 who responded) commented that they had always taught this way.

Teachers identified the key theme of change as being an emphasis on strategies for solving number problems. Twenty-nine percent mentioned this. As one teacher put it, "It is almost an insight into their brains to see how they're thinking" or another, "It has been good to see the different levels of thinking that children use". Yet another teacher saw it as a more practical approach to teaching mathematics and said "I am more positive with the common-sense practical approach to numeracy that encourages thinking! I like strategies that focus rather than the 'learn this way' approach to solving equations, problems, etc". There were a number of related aspects that about one-fifth of the teachers mentioned, which included the use of equipment (17%), "hands-on" activities (18%) and group work (16%). Comments that illustrate these aspects include:

The breakdown of learning outcomes through knowledge, strategies and grouping, equipment, etc. was great. The variety of teaching tools, i.e., number lines, digit cards, hundreds boards, etc. all reinforced ideas.

I use concrete materials a lot more to teach concepts and do more head-work and strategic thinking.

... more hands on – games/activities, working with smaller groups, more intensive teaching.

[I] use far more equipment and materials – [I have] more group activities.

Thirteen percent of the teachers mentioned that they felt their teaching of number was more focused, as in the following typical comments, “I hated teaching maths, but now I love it. I know exactly where I am going and how to get there” and “I am now more precise in my thinking of what I want to teach” and “It’s more focused, interesting for the children. More about getting them to understand what they’re doing”. For one teacher it was a dramatic change:

[The way I teach] has changed completely. I am shocked when I look back to my teaching prior to ANP. My teaching is targeted more closely to individual needs ... the framework is at the centre of my teaching now. Teaching strategies as well as knowledge is new to me.

Principals were also asked to comment on whether they felt that the project had impacted positively or negatively on the mathematics programmes of participating teachers. Nearly all the principals (98%) commented that the project had had a positive impact.

	More Focused Teaching	**Inclusion of Strategies**	**Equipment**	**“Hands-on” Activities**	**Group Work**	**Diagnostic Interview**
Teachers	13%	29%	17%	18%	16%	0%
Principals	31%	26%	3%	5%	12%	23%
Facilitators	50%	25%	44%	5%	38%	0%

Table 4.6: Percentages of Respondents Who Commented on Common Factors That Enhance Their Approach to Teaching Mathematics

The key factors mentioned by principals included more focused teaching (31%). Typical comments included, “Generally teachers have felt more focused in their teaching and have enjoyed the improved understanding they have achieved of students’ stages” and “Teaching has become more focused. Teachers are more aware of what to teach and why. They have learnt strategies that have improved their teaching” and “Teachers know what the next step to teach is, how to teach it and what follows”. Another key factor that principals identified was the inclusion of strategies (26%). For instance, two principals commented that, “Teachers are able to teach more strategically and get better results from their students” and “I believe it has been very positive in terms of ‘measurable growth’ in the students’ competence with strategies. The knowledge of the strategies has allowed the teachers to be very specific over what the students can do”. The third main aspect that principals identified as impacting on teachers’ mathematics programmes was the use of the diagnostic interview (23%). Comments included:

Teachers found the interviews very significant as they highlighted learning points – on a par with running records ... having games to support specific learning outcomes ... Teacher understanding of children's learning processes improved.

Use of the ANP assessment has allowed teachers to identify specific learning needs of individuals. Teachers have grouped students more appropriately and selected teaching activities.

[Teachers are] extremely positive. Starting with the individual testing gave a focus on each student's understanding of concepts and led teachers to think about addressing needs and deficiencies.

While most principals (60% of the total) felt there had been no negative impact, some principals (38% of the total) felt there had been a negative impact on mathematics programmes. Reasons given by the 38% of principals were varied with the most common theme being the time taken on resource-making (9%).

All the facilitators felt that the project had had a positive impact on the participating teachers' approach to teaching mathematics. As with the principals and teachers, a common area of improvement mentioned was more focused teaching (50%). Comments from facilitators included:

Teachers have become more aware of the sequence of children's numeracy learning. They know the meaning/value of relevant warm-up/maintenance activities and the relationship different aspects of the lesson have in forming a good numeracy lesson. ... A lot of teachers have found [that] the project has given them a good "model" – helps them with organisation. [It] enables them to teach specifics after identifying these from testing.

Some saw this more as a change in focus, "The [focus has changed] from getting the right answer to questioning and probing to find out how [the students] worked questions out, so [the] focus [is] on the process rather than the product, using contexts that are meaningful to students". The facilitators also commented on the enhanced use of equipment (44%) and the better employment of group work (38%), as in the comment "[There is] more group teaching [and] more equipment in use. [Teachers are] teaching groups more specific focused content. Teachers more confident. [There is] more oral discussion". One facilitator associated the grouping of students with "children [being] engaged in a greater range of learning experiences [as in] a more stimulating integration of Figure It Out and games etc.". Some facilitators did comment on factors that had a negative influence on the impact of the project on teachers' practice. As one facilitator pointed out, "Any professional development needs commitment by teachers – no commitment, no change. This was noticed with about one teacher per school". Other facilitators stressed that "All teachers, regardless of ability to take on board new ideas, found positive aspects of the project to enhance their maths programmes and children's learning" and that "Teachers have been very prepared to embrace this development. They can see the gains for their children".

Impact on Teachers' Attitudes

Teachers were asked if they thought their attitude towards maths had changed as a result of their participation in the project. Sixty-seven percent responded that they did feel more positive about mathematics and teaching it as a result of their participation in the project. General comments included:

> *My attitude has changed greatly. I am more confident in teaching numeracy. I believe it is really important to spend increased time teaching and learning within the number strand of the curriculum.*

> *Maths has never been one of my best curriculum areas, however, participating in the project has been of great value personally and [that is] from a teaching perspective.*

> *My attitude towards maths is very positive and participation in the project has just reinforced it.*

Of the 285 who reported no change to their attitude, most commented that they had always been enthusiastic about mathematics. A small minority reported that their attitude had changed in the previous year because of their earlier involvement in the Count Me In Too pilot project and four respondents were beginning teachers. Only 5% reported a negative change to their attitude as a result of their involvement in the Advanced Numeracy Project.

Common themes emerging from the teachers' responses included some reference to changes in attitude to the use of mental strategies (68%). Comments included:

> *I feel the strategy progression is very reassuring for a teacher and keeps you from pushing children on too quickly.*

> *[I] feel more confident – able to teach strategies much better – break [things] down into simple steps, I used to be more concerned with the answer and not so much with the process.*

> *My attitude has changed. I'm more confident about letting children manipulate numbers in different ways.*

There were more general comments about increased interest and enthusiasm for mathematics (37%). Some of these comments indicated a more open disposition to teaching mathematics and a better sense of direction:

> *I think I am more prepared to think outside the square in terms of children's learning.*

> *The attitude of both the children and myself is more positive, and we have more direction.*

> *The project has provided me with new insights into the teaching of number. It has been a motivating factor in helping me review the way I teach maths and has provided me with many innovative ideas.*

	Mental Strategies	Interest and Enthusiasm for Mathematics	Teacher Confidence	More Flexible Approaches	Teacher Know-ledge	Student Achieve-ment
Teachers	68%	37%	24%	27%	25%	0%
Principals	0%	45%	55%	18%	17%	18%
Facilitators	0%	69%	81%	0%	0%	0%

Table 4.7: Percentages of Respondents Who Commented on Common Factors Impacting on Teachers' Attitudes

The next area of change in attitude related to different ways of and more flexible approaches to teaching mathematics (27%). Many teachers found the more focused approaches resulted in much more satisfying teaching.

> *It (my attitude) has definitely changed. When I see how well my pupils are progressing each day, I feel a sense of achievement as well.*
>
> *I feel much happier about the maths programme I provide in my classroom. Previously I understood the curriculum and was aware that in many aspects my programme was not achieving the "spirit" of the curriculum but didn't feel empowered to make significant changes.*

Twenty-five percent of teachers reported that increased teacher knowledge led to an improved attitude, and 24% cited increased confidence as improving their attitude. Comments included:

> *I feel empowered. I am better at maths myself so feel more confident teaching it.*
>
> *I've always enjoyed maths. The change in attitude was mainly towards my approach, e.g., teaching addition/subtraction together as opposed to two separate topics.*
>
> *I have greater confidence in the teaching of fractions and helping children with the relationships of number.*

Most principals (94%) responded that teacher attitude towards the project was generally positive. Typical comments were:

> *[Teachers are] generally more confident in teaching maths and have become more flexible in their approaches.*
>
> *Growth in understanding is evident. Confidence has developed.*
>
> *Teachers have shown more confidence and enthusiasm in teaching maths; particularly number.*
>
> *Teachers appear to have grown in confidence, having a clearer understanding of maths concepts and the variety of ways and flexibility in arriving at solutions.*
>
> *Several who were hesitant about maths have grown in confidence.*

The most common reason (18%) given by principals for teachers' improved attitude was the improvement in student achievement. Typical comments were:

> *[There have been some] very positive [impacts on teachers]. [Teachers] see children learning and progressing. [They] see children enthusiastic about maths. [Their] own understanding [is] broader [and] therefore [they] feel more confident.*
>
> *Great. They have all seen the worth, the development of children, and consider it a very worthwhile programme.*
>
> *Exceptionally positive. The teachers have been behind and involved 100%. They have seen the benefits and development of children's numeracy knowledge.*
>
> *Excellent. The programme challenged all staff members. The further they got into the programme the more committed they became – they could see the impact it was having on children's learning and their teaching.*

Seventy-five percent of facilitators felt that teachers' attitudes towards teaching mathematics had generally improved. They talked about interest and commitment particularly being reinforced by their students' achievement.

> *[Teachers have] gained in commitment and interest as [they] have got to know more about the structure and noticed the interest and achievement by students.*
>
> *Very different from ENP teachers – ANP teachers [are] more reserved, cautious, reluctant to change, sceptical, expressing frustration at resource-making, questioning. Over time – [they have become] appreciative, acknowledging its value.*
>
> *Teachers have been very enthusiastic toward the project. Some older teachers are saying initially "here we go again" or "how many changes are we going to make".*

Some facilitators (25%) noted a mixed response. They felt that this could be because of the stressful nature of change, as shown in comments such as:

> *Mixed. Most have been almost relieved because they have not felt very confident teaching maths. However, many found it a stressful process – often [having to come] to grips with very new methods of assessing and teaching number.*
>
> *Initially negative in one school, which improved over time – i.e., the school had a lot on; [they] were involved because Juniors [had been] involved the previous year – although some teachers [were] more keen. Not as keen to embrace [ANP] as ENP.*
>
> *A mixed reaction from extremely positive and enthusiastic to apprehensive and doubtful. For those that have been doubtful, I believe it is because the project challenged them to reflect on how they teach – the change is too difficult.*
>
> *Most teachers have been very positive about the project. They have implemented the project and are seeing very positive results in their students through increased levels of strategic thinking apparent in group discussions and increased enthusiasm for maths. However, there are teachers who do not really take on any of the ideas/philosophies behind the project and make no observable shift in their classroom practice.*

A related question asked of principals and facilitators was what impact the project had on the teachers "as a person". Sixty-eight percent of principals and 69% of facilitators generally reported a positive impact on teachers. Confirming the trends emerging from other questions, principals felt the biggest positive impact had been on teachers' confidence (55%) and on their enthusiasm for teaching number (45%), their teaching approach (18%) and on their knowledge of number (17%). Facilitators also saw the biggest impact as being on teachers' confidence (81%) and enthusiasm for teaching number (69%).

On commenting on any negative impact both principals (25%) and facilitators (44%) commented on teachers' tiredness and stress associated with implementing a new project. Both groups were also asked about any impact on other teachers in the school. Twenty-eight percent of principals and 44% of facilitators felt that the project had impacted positively on other teachers. Both groups noted that many of the other staff were also participating in the project.

Impact on Students

Teachers, principals and facilitators were asked what impact the project had had on students. The overwhelming response from the three groups of participants was that the impact on students had been positive.

Respondents	Positive Impact of Project on Students
Teachers	87%
Principals	98%
Facilitators	100%

Teachers' comments included:

More fun, less stress. Explaining 'how you got the answer' has opened up dialogue and enabled children to communicate more easily and confidently about maths. Children are more aware of their successes and seeing failure as learning steps.

[There has been a] marked development in understanding; even better attitude towards maths (which was really positive to start with); but the greatest impact [has been in] the strategies the children are now using.

Certainly mental processes are sharper, and children are more positive about maths. Strategies [are] seen to be applied in context in other areas of maths.

Very positive response to new stations and activities set up. Referred to as the 'Awesome Number Project' (ANP). Clearer understanding of number concepts and strategies.

Very enthusiastic about maths – [the children are] amazed at themselves, at what they can do – [ANP has] given them the ability to use what they know to get what they don't, for example, one boy who knows 10 times, 5 times tables did 16 x 6 = like this – (10 x 6) + (5 x 6) + 6 = 96 – [It's] rewarding for the teacher too!

Principals' comments included:

> *Children love maths – over 80% [were] very happy in [the] pupil survey. Advanced maths learners [have] now [been] identified and [are] being catered for.*
>
> *Children have shown more willingness to try out ideas, greater thinking, flexibility and increased number knowledge.*
>
> *Children have shown greater understanding and a willingness to use a variety of approaches – collaborative interchange in maths activities has been enhanced – the ability to articulate ideas has been most evident.*
>
> *Children have grown in confidence – their attitudes are good, and they enjoy the explaining ([This is] quite different from always being told!).*

Facilitators' comments included:

> *I have been impressed with the positive spin-offs ANP has had [on the] confidence and competence of most children.*
>
> *Overall, students are extremely positive about the project because it has meant more focused, individualised and "hands-on" teaching. In some instances, huge "gaps" in their understandings have been filled.*
>
> *Generally the children are more keen to do maths. They have enjoyed the discovery of finding new ways (strategies) to problem solve. Much more interaction has taken place between children during maths.*
>
> *Very positive – [children] enjoyed the activity, games and interactive approach. Children knew and could discuss new strategies they had mastered [and] (excited) families also reported new enthusiasm in their children.*

Sustainability of the Project

The Development of School Policies and the Use of Resources

Teachers and principals were asked about any changes to policies or resources that have taken place as a result of participating in the Advanced Numeracy Project. Seventy-five percent of teachers and 82% of principals reported that there had been policy/resource changes.

About 40% of the teachers' comments made reference to changes in policy and particularly noted changes to the long-term plans and the shift towards a numeracy focus and about 49% of the comments made reference to changes in resources and equipment. In the principals' responses, the pattern was similar, with 38% of the comments referring to extra resources and equipment and about 38% referring to the development of policy, again noting changes to the long-term plans to ensure more of an emphasis on numeracy.

Reasons for Participating in the Advanced Numeracy Project

Seventy-five percent of principals reported that their main reason for participating in the Advanced Numeracy Project was their school's previous involvement in the Early Numeracy Project. Some principals (34%) also mentioned that they saw their continued participation in the numeracy projects as a way of establishing consistency in the school's approach to mathematics teaching. Chapter 5 will discuss the benefits of this in ensuring the sustainability of new practices, such as those introduced through the Advanced Numeracy Project. The fostering of a community of practice in the school is seen as an important factor in this.

Parents

Parents form an important part of any school community. Both teachers (44%) and principals (58%) reported a positive reaction from their parent communities. Parents are by and large concerned that their children achieve in all curriculum areas. This was summed up by the principal who said that parents were "Very positive [and were] aware of their [child's] progress and sense of achievement". Parents feedback to teachers included comments that focused on students' ability to manipulate numbers such as, "[Parents were] quite astounded as to the knowledge of children", "Children are using strategies at home/supermarket to work out problems", "Good – [they were] amazed at the different ways children come up with an answer. [The children are] able to help older brothers and sisters with their maths!!" and "[Parents are] pleased with their children's 'dexterity' with numbers". Similar comments from principals included "Very positive. Excellent response and participation in our open night for parents and students. Parents wished they had 'this' when [they were] at school" and "Parents have been very supportive – visiting to view aspects in progress, supporting games sessions, supporting children in home-based tasks". Some of the comments included the impact of the project on reporting to parents, for instance, teachers commented that "They are pleased to have 'specific' number information about kids and where to next" and "They haven't really been formally informed, but I mentioned it to one or two at interviews and they were suitably impressed (about their children rather than the actual project)". Other comments referred to the need for parent education. For example, one teacher stated that "Data from assessments have been invaluable when reporting to parents. Further education of parents [is] needed to assist with their understanding of the number strategies approach". Principals commented that parents were "… positive, supportive, curious. We now consider it important that parent education become a goal for 2002" and "We have reported to parents and sent samples of children's work, but the new terminology and strategies often confuse them". Many of the mixed reactions from parents were to do with the issue of when to teach the algorithm and [the] extent to which primary schools should prepare students for "intermediate" maths. Parents have their own "markers" of what should be taught, as expressed in the following comment from a principal, "Fine – not really interested for the most part – provided children know their tables".

Least Helpful Aspects of the Project to Teachers

The least helpful aspect of the project, as commented on by 39% of the teachers, was the folder. Comments included reference to the organisation of the folder and omissions. Several of the respondents commented that they looked forward to these aspects being improved in the 2002 delivery. The next least helpful aspect of the project was the resource–making sessions (11%).

Most Helpful Aspects of the Project to Teachers

The project was very helpful to teachers in improving their practice. Comments such as "[The project] opened up my eyes to a better way of teaching maths" and "Through the project [I'm] being given the freedom to get away from 'book work' – I like the stages of development and the indicators of expectations at each level" are testimony to this. Twenty-one percent of teachers reported that the in-class support was the most helpful aspect of the project, and 22% felt that the provision of resources, including activities and games, was the most helpful aspect of the project. Comments that reflect the power of the in-class support are typified by the following examples:

… having the facilitator switched on and maths focused – also the turnaround and crunch time for me was when she came in to watch me teach and was so affirming.

… the professional development, which was people and classroom based, and the emphasis on the importance of number … the encouragement to force children to think mathematically to move children on.

… the resources and the facilitation… the fact that I could ask someone how to do something that involved an exploratory function. The more experienced teachers have tended to give me an old-fashioned method as advice.

… the practical demonstrations of activities by the co-ordinator in my own classroom with my kids – I got to learn how the equipment was used instantly and in context and could view the result!

The folder [was most helpful]. The day we spent with our facilitator going through the concepts, teaching techniques, resources, etc. and interacting with the facilitator generally – watching her model and [having] opportunities to clarify issues with her once I started teaching – was [great].

Fifteen percent of teachers mentioned the developmental number framework and the ANPA as being helpful, as shown in comments such as "being able to gauge the children's thinking processes has been the largest positive aspect for me – Next the clearer benchmarks of the learning progression".

Concluding Comments

The overwhelming sense coming from the comments in the questionnaires was that the Advanced Numeracy Project had impacted favourably on teachers and schools as a whole. The following comments from teachers and principals provide evidence of this.

It has been very positive for our school. Hopefully it will continue to move teachers away from prescriptive maths teaching with little pencil and paper pre and post tests (yuk!). Teachers are more [focused] on and teaching fractions better. Children are using better strategies to work problems out in the head. We're getting away from "this is the only way to work out/solve a problem/sum". … The project has been fabulous. (Teacher)

This has been the most satisfying professional development I have been involved in. I now really enjoy teaching maths and would really like to go on and further develop my knowledge of maths – content and (mostly) knowledge of children as learners in maths. … Thank you for the opportunity to give feedback. I am always so disappointed and frustrated when the opportunity is not given to teachers (and often it is not). (Teacher)

This has been the best professional development ever offered to schools and the MOE should use this as a model for other curriculum areas and note the cost, which I believe was quite high – but worth it. (Principal)

The delivery of this project has been the very best I have been involved in during my 28 years of teaching. The professional development provided has been absolutely outstanding. It is very, very effective model of curriculum professional development. (Principal)

Chapter 5: Effective Facilitation: Teachers' Knowledge Developing in Context

Contexualising Professional Development: Evidence from Practice

There is a critical factor that determines whether teachers will be able to modify their practice. This factor is the extent to which professional development is contextualised within the practice setting. School policies and structures, student backgrounds, the teacher's pedagogical style and associated classroom dynamics, and the teacher's knowledge of learners' cognitions in mathematics are features of such a context. Embedding these factors in the teacher's professional development ensures a more robust and long-term integration of the suggested new ideas and practices into the teacher's existing practice.

We need to build knowledge of what constitutes effective facilitation in teacher professional development in mathematics. A facilitator needs to carefully balance the teacher's existing practice with the suggested approaches of the project. The facilitator's effectiveness can be thought of as the management of the interrelationships between the teacher's content knowledge, their pedagogical style and their knowledge of how students learn mathematics. All of this constitutes the teacher's context of practice. The dynamics of the individual components and the way they relate to other components, together with the facilitator's knowledge of each situation, create the complexities of the facilitator's task.

Evidence of the effectiveness of facilitators' work with teachers therefore includes evidence of teachers critically examining their practice, as well as evidence of how the professional development has helped teachers to better understand the mathematical purpose of what they are teaching. Further evidence is teachers' improved ability to articulate the mathematical content of their lessons and to identify features of their practice that have led to students engaging with the mathematical concepts. Other aspects of the facilitator role include modelling teaching approaches; being a subject specialist and resource developer; being a confidant and an inspirer of teachers; and liaising between levels of the school, between the school and the community, and between the school and government policy. Effective implementation of all aspects of this complex role require facilitators to exercise sound professional judgement, to have a robust subject knowledge of the mathematical concepts underpinning the framework, to have the expertise to model a range of possible approaches to teachers and to be able to provide feedback to teachers on their classroom practice.

Establishing communities of practice is an important factor for the longer-term efficacy of professional development. In the case of the numeracy project, these can be thought of as nested communities of teachers and facilitators. A teacher's community of practice provides opportunities for teachers to talk with other teachers about their practice with and without the facilitator present. In the same way, similar opportunities exist for facilitators to talk with other facilitators about their practices in working with teachers. This chapter presents evidence from professional development modelled on a teacher-centred approach. The data were gathered from interviews with four facilitators and the eight teachers they were working with (see Chapter 2 for more details). This chapter examines the issues faced by teachers in modifying their classroom practice to meet the aims of the project and the implications of this for the role of the facilitator.

Explanatory Framework

The Number Framework provides teachers with an explanation of the key mathematical ideas associated with the learning of number, along with an account of the development progression in students' thinking as they move from simple counting strategies to more sophisticated part/whole strategies. The diagnostic interview encapsulates the essence of these ideas and acts as a pathway for teachers in learning about the developmental levels identified by research as key points in students' understanding of number. It provides a common language for discussing these ideas. The framework positions assessment alongside learning through the diagnostic interviews, which enable teachers to find out not only what their students know but also the strategies students use to solve number problems. This information is a useful benchmark against which to compare the effectiveness of classroom practices in raising student achievement.

The facilitator acts as a mediator between the ideas and associated practices of the explanatory framework and the teachers' existing practices. In the case of a more structured approach, such as the Advanced Numeracy Project, situating teacher development within a teacher's context of practice presents particular challenges to both the facilitator and the teachers. The structure is two-fold and incorporates the structure of the project as exemplified by The Number Framework, as well as the structure of the professional development model. As one facilitator commented, this complexity in structure presented a challenge in the balance between considering the context in which the teachers are working, and the expectation for teachers of raising student achievement as measured by the assessment element of the programme.

> *…There is a very specific role that I have this year that's not necessarily how I would see [the role of] a facilitator [to be] in general. Like, I see that this is a very structured project, and I think sometimes I feel it is difficult being a facilitator of this project because it is so structured, … I want to know what [teachers] needs are, but having said that, I recognise there are certain things we are asking them to do, and it's non-negotiable basically. … It's allowing them to see the content that they need to be planning and teaching, and it gives them a lot more understanding in terms of their own mathematics. So I see that there are a lot of different things that I am trying to achieve out of my role.* (Kay, Interview 1)

The Diagnostic Interview

The diagnostic interview as a catalyst for change has again been identified as having a profound impact on both teachers' content knowledge and classroom practice. This confirms the findings of the 2000 evaluations for both the Early Numeracy Project and Advanced Numeracy Project pilots (see Higgins, 2001a; Thomas and Ward, 2001). The facilitators in particular considered the diagnostic interview to be the key to enhancing teacher content and pedagogical content knowledge. This is in contrast to the strong theme of mental strategies that came through in the teachers' responses to the questionnaire. This might be explained as the differences between the teachers' and facilitators' contexts of practice. The teachers for their part are most concerned with "what they will teach children on Monday" whereas the facilitators' concern is with developing teachers' professional knowledge. They believe that the detailed knowledge of the diagnostic interview enables teachers to more closely match the task to the learner. The interview provided principals and their staff with hard data for setting school expectations of student achievement and for using in their self-review process. Student results derived from the interview enable more precise reporting of student progress to parents. What follows is an examination of the ways in which facilitators and some teachers and principals saw the diagnostic interview impacting on teachers' knowledge and practices.

Typical of facilitators' comments are those of two of the facilitators in the 2001 evaluation.

> *One of the most essential triggers is that initial diagnostic assessment. That came through loud and clear when I asked for feedback. ... The comment over and over and over again, overwhelmingly, was that doing that diagnostic assessment changed something for [teachers] because first of all they were given time, the time to actually sit with their students and for so many of them it was an eye-opener. ... So, in general, teachers felt confirmed that they had professionally made some pretty good judgements about the levels that their children were working at in number. But, having said that, there were always surprises and that was really revealing for those teachers and it was disquieting for them because it meant that they had to re-evaluate, sometimes, a student very seriously. ... Children who they had thought were the whizz-bangers of the class, they passed every paper test, and suddenly they were finding massive gaps in their strategies. Conversely, children who had never really demonstrated their knowledge very well on paper tests suddenly came through with some incredible knowledge of strategies in this oral interview, and so for many, many teachers that threw them into a little bit of a state of uncertainty. But it was alongside the fact that they felt they had confirmed some of their own professional judgement, so they felt good, but then they had this hook, they wanted to know why [it] was that they had missed out on this information with these kids. It made them want to have [facilitators] come into their classrooms, made them want to look at their teaching. ... We had experienced teachers who said "I really thought I was a good teacher in maths, you know, I have taught for twenty years. I was really happy with my maths programme, but you have thrown me into a state of feeling like there is something new to know here". And [the diagnostic assessment] more than anything, I think, hooked them and continued to [do so].* (Kay, Interview 2)

This facilitator's comments highlight the way in which the diagnostic interviews provide opportunities for teachers to spend time talking one-on-one with students. She emphasises that the information gained from any assessment situation is related to the nature of the tasks given. In contrast to the orientation of the more traditional pencil and paper test towards capturing the extent of a student's number knowledge, the diagnostic interview is more likely to reveal students' strategic thinking. As the facilitator pointed out, the sophistication of this thinking may be at odds with the teachers' perceptions of the performance of individuals based on a pencil and paper test. While in the case of some students, it confirms teachers' judgements of students' capabilities, in other cases it is in sharp contrast, giving rise to "a state of uncertainty". She suggested that this uncertainty was "the hook" into working with the facilitators on their teaching and highlights the important explanatory function of the framework. The diagnostic interview has an important function in revealing the ways in which the student's knowledge might be applied to building more robust and sophisticated strategies for solving number problems. In a later section of this chapter, the facilitators talk about how they presented this more complex interrelationship between knowledge and strategy to teachers.

In a similar vein, another facilitator also talked about the diagnostic interview identifying where students were up to in developing an understanding of ideas about number. A key factor for teachers was the way in which the information gained sometimes disrupted their assumptions about where students might be at as measured against the framework.

> *The diagnostic assessment is key because that identifies the [children's learning] need[s]. ... Getting teachers to see through the diagnostic assessment that there is a need and the children in their class will benefit from this and have something to gain. [For instance] it's the assessment when I say "Oh my goodness, I had these assumptions about this student and either this student is operating at a far higher level than I am or gosh I thought the student was way up here and they are actually here".* (Rachel, Interview 2)

Three teachers in particular talked about how the diagnostic interview challenged the assumptions teachers make about students' abilities. One of these teachers explained how evidence from the interview confirmed their beliefs that teaching made a difference.

> *The individual assessments in particular show "the gaps" in knowledge that we assume children know at a certain age/level. [It] has confirmed my long-time belief that [if one can] teach a student how [then] they can apply the skill/strategy to solve anything. Children also learn effectively through the constant challenges.*

The interview also enabled teachers to conduct more in-depth work with their students by providing them with greater knowledge of the students' problem-solving strategies.

> *[I] certainly gained a great deal of pleasure from being able to "dig deeper", particularly with my slower groups to find out how they thought and where they were along the maths processes learning line. It was also wonderfully revealing to see strategies employed in such a variety of ways by children who were apparently able in Number (on initial forecasts), but who were significantly less able in being able to define and state their thoughts – and see their own patterning processes and draw conclusions from them.*
>
> *During the initial one-to-one interview, some different paths to solving problems were taken that I had not realised children could follow!*

From the principal's perspective, the diagnostic interview provided the school with "hard data" about students' progress. One facilitator commented on how she thought principals used this information.

> *The principals who are switched on, who we've visited, have been able to use the hard data. For the first time, they have actually been able to say these kids have made good progress in that area and we need to work on these areas here because they have got hard data. And those schools will continue to grow and flourish without me now because they have got that done.* (Emma, Interview 2)

One principal explained how they might use this information to inform programme planning.

> *[We gain] confidence in mathematics using data to inform about programme needs. [We were] more focused on the differing needs of students and the strategies [required] to address those needs.*

As discussed in the previous chapter, principals also found the data useful in setting school expectations for achievement in numeracy and for reporting in more detail to parents.

One aspect of new practice emerging from the case studies and questionnaires was more focused teaching through linking the task more closely to the student's stage of learning. The diagnostic interview has enabled teachers to plan more focused classroom programmes based on the information about individual students' number knowledge and level of sophistication in problem solving strategies. Two facilitators talked about how the diagnostic interview can act as a guide for focusing teachers' planning more carefully without being a precise prescription of what to do next. Rather they thought that the explanatory nature of the framework is more likely to develop teachers' understanding of the sequence of key mathematical ideas.

> *From an assessment point of view, [the diagnostic interview] is absolutely ideal because it gives you the diagnosis and where to go. It's exactly what we want for good learning and teaching, to find out what they know and then give us some steps to where we can lead them on [to]. That's what we're doing, we're giving teachers a structure without giving them "You will do page this or that". ... I'm in favour of resources ... whatever you might be using, [but] it's which bit are you going to choose to meet the needs of a framework and the kids within that framework now, not just because you have got that resource to use. ... And that's the bit that's missing. Teachers need to know the framework. I've worked through a lot of [different] assessment [tools], and it's the same, take a piece of kid's work, find out where they are, what's the next step, what do we have to teach them, all of the bits along the way. And I think this follows that kind of thing that's been successful.* (Emma, Interview 1)

> *If they have really come to grips with the assessment, it means they have started to really probe at those different stages and what are the variances and what are the differences. And to do that, that's when they have to go back to the frameworks and start to understand what's in the frameworks. And use the frameworks not just as part of this assessment, but as a part of their planning and a way to get into the curriculum and a way to understand what are sometimes very broad statements in those achievement objectives. [For example] between stages 2 and 3, fractions somehow seem to almost disappear for teachers, and so it's a way for them to break down some of these achievement objectives that seem a bit confusing, into some very specific stages that they can look at.* (Kay, Interview 1)

Teachers' comments confirmed that the interviews empowered them. As one teacher put it, "The ability to now recognise benchmarks in students' thinking has been empowering". Other teachers explained how it also helped them to be more focused in their teaching through a better match between the learner and the task.

> *The one-on-one interviews at the beginning really highlighted each child's strengths and weaknesses and how they worked things through. [This] enabled much more focused teaching.*

> *Yes, I really like using the developmental stages mapped out in the project for goal setting with the children. The project has drawn attention to the finer details/strategies children use.*

An essential part of the facilitator's role is to highlight the links between current policy documents and the new development. One facilitator explained how she made a link between the cycle of assessment and planning and the diagnostic interview that she saw as an enactment of such policy.

There is a visual in one of the Ministry books that I often refer to that shows this cycle of assessment and planning for teachers, and I think for a lot of teachers that was an abstract thing. But now it actually means something because they have seen that when they have done a diagnostic assessment on a child, it informs them about their planning and their teaching and then once they begin teaching, they are continually assessing as they teach. (Kay, Interview 2)

Another facilitator also commented on the way in which such information might impact on group teaching and teachers' knowledge of the developmental sequence. She suggested that the information gained from the diagnostic interview might act as a catalyst in shifting teachers from old practices to more focused teaching. A loose match between the task and the learner typifies these old practices. In her comment, she sees these old practices as exercises from books not closely matched to the stage at which the learner is at in their number development.

Teacher pedagogy – I think there is more effective group teaching because they have had some good hard data to work with, you can't go back and give them all the same page. I think that struck quite well. I think teacher knowledge of the developmental sequence [is] far, far better. I don't know whether they have ever had anything like it to hang their hats on in terms of being able to say "Where are we going? ... We can try these things with these kids. There is no point in doing these with those kids because they are nowhere near". So I think their knowledge about how kids learn maths is much better overall. (Emma, Interview 2)

She summed up:

... Success for most of [the teachers] has been undeniable when they see the results, and they can tell you about their kids far more than they ever could. It has been about building teacher knowledge, their own personal knowledge in maths. (Emma, Interview 2)

Teacher comments confirmed that teacher knowledge was indeed built up in this way.

Yes, I really like using the developmental stages mapped out in the project for goal setting with the children. The project has drawn attention to the finer details/strategies children use.

In-class Modelling

The second aspect identified as being a key factor in enhancing teacher content and pedagogical content knowledge was the in-class and in-school support of the facilitators. It was by this means that the facilitators were able to build a link between the diagnostic interview and the practice setting of the teachers, enabling the teachers' learning to be situated in practice. Having challenged teachers' assumptions about students' learning and about practices that emphasised number knowledge at the expense of developing students' strategies, the facilitator has an important role to play in mediating between teachers' old practices and their adoption of the new practices. One facilitator talked about the importance of "teacher buy-in".

I think first of all the teacher has got to see the need for what we are doing, the project we are working on, see a need for their children and themselves. So I think that is really important so they have got to, I suppose, buy in. So seeing the need and *then being able to apply it to their classrooms. And I think that is when the working one-on-one with teachers in the classrooms is kind of valuable because it's actually only when you walk into the classroom that you really get a feel for what's happening. The key part [after the diagnostic interview] is the working one-on-one with teachers. Yes, I think so. The cluster [meetings] are great at giving an overview but in terms of responding to teachers' individual needs, I think in the classroom one-on-one and at the syndicate level as well.* (Rachel, Interview 2)

Another facilitator talked about how she established her role as facilitator.

To say to the teachers I am here to teach you, I am using your children to teach you, I'm not here to teach your kids … that's one of the messages of the first session that we gave. The first term [it's] all about you learning and after that we'll talk about whether the kids are learning anything. Teachers ask [me] a lot of basic questions. … [I] said to the teachers "I am here to teach you, I am using your children to teach you, I'm not here to teach your kids". … There is a very strong message for teachers that they are the ones who are supposed to be learning out of this. (Emma, Interview 2)

Yet another facilitator talked about the importance of working in the classroom with the teacher, which she found challenging.

What I find most effective is the actual in-the-classroom working alongside the teacher. …The skills of working in a supportive way, I suppose, but challenging the teacher – that's the area that has been a bit of a learning curve for me. (Rachel, Interview 2)

This in-class support complements the diagnostic interview. It helps teachers to use the information gained about students' achievements to enhance their classroom programmes so that they became more student-focused. By working alongside teachers in their classrooms, facilitators can enact the explanatory framework through, for instance, modelling the teaching of different strategies for solving number problems. This is illustrated by one facilitator's comments:

It takes time, and a most effective thing that this contract has done is to be in [the teacher's] classroom with their children modelling what it looks like and what's possible. We model with their kids, and they say "I never knew that kid knew that" or "this is where this is coming from". I think the effective thing has been being in there facing their classroom and the fact that you keep on coming back. … You are able to talk with the teacher … not just show what the activity is but talk about underlying concepts because we still have a lot of teachers teaching an activity without any concept and to model the pedagogy of good questioning and good classroom management, so a teacher gives me a group or something – "Just give me one of your groups, and I will now show you this activity, and I will raise it up or down depending on where the children are at." … and modelling things as basic in pedagogy as giving the kids three or four minutes to orient themselves with the equipment. If they are using the beans for the first time, let them play with them - let them stack, how do they fit together before you do that? … modelling the good questioning, lots of effective questioning. … We are modelling the mathematical questions, "So how do you know that?" and "What were you thinking?" and "I can hear Jo that you know those doubles but how did you know that double became that bit of the fraction?" (Emma, Interview 2)

Another facilitator talked about the modelling in terms of exposing the dynamic of teaching and assessment and building on what teachers already know.

> *Assessment is not just a thing we put on paper with questions 1–10. As a teacher and facilitator of learning, you are always making mental notes as you go in your teaching, and based on that, it might be within one session, you make drastic changes to what you had planned to teach based on what you see. And that's been exactly what we have been trying to hit at in our modelling, and that's where the teachers are really making comments. ... I continually have teachers saying, "This is so useful to me because I can actually sit back first of all and step outside and watch from the outside". This is hard to do sometimes when you are with your own students, but more importantly, they can suddenly see the knowledge and strategy gaps opening right in front of them with a group of eight children [who] they thought were at the same strategy level. Suddenly it's like someone comes with a knife and cleaves them in half, and they see, "Oh, my gosh, those are advanced counters on one side, and those are early part/whole thinkers on the other side." ... That's been a very powerful message coming through that in their own teaching, they need to be trying to push at that edge, push at the edge of where their students are to try and make those divisions so that they can see where those gaps are. And [having] the frameworks there for them has been such a powerful aid. And for me, what I have been really emphasising as I go throughout this year, ... "This is not something you start from A and go to Z and work your way through. This is not telling you to throw out everything you know about teaching. You already did good things in your teaching. This is adding to your tool-box of information and adding to your knowledge as a professional. And with that, you now can look at your learners and identify in very specific ways what each of those kids' needs. And that if you can keep your sights on the framework [you should have] a clear sense of those progressions embedded in your head. It doesn't matter what resources you particularly have in front of you or don't [have]. You can make do with a few things and you have got then the sense of where your children are and where you will be too", and so that's really what I have been trying to get teachers to see as the aims of this project.* (Kay, Interview 2)

This facilitator's comments touch on one of the most important issues in professional development; that of mediating between old and new practices. As she pointed out, it is not a matter of abandoning everything from the old practice but rather of building on previous teaching practices or, as she put it, "adding to your tool-box of information and adding to your knowledge as a professional". The key point in this is having guidance about what to build on. She suggested that the framework could be this guide, "If you can keep your sights on the framework, [you should have] a clear sense of those progressions embedded in your head". In her role as facilitator in enacting the framework for teachers in their practice setting, she demonstrated how teachers can use the framework to "push at that edge, push at the edge of where their students are to try and make those divisions so that they can see where those gaps are".

Another facilitator explained why she saw the facilitator's work alongside the teacher as being so important:

> *... When you are removed from the classroom and talking with teachers about maths ... it's almost what they know you want to hear, what they know is best practice kind of knowledge, their knowledge of it rather than what they are actually doing*

necessarily. I think that's key in their classroom. Otherwise … a common comment is "Oh, that's all very well, you know, for you to say that, but in my classroom I've got special needs, 32 children, …" all of that. So for the teacher, not only to see and be exposed to the theory and the new ideas but also to show them how to put that in place in their classroom. And then you can identify any, and work with the teacher on any key issues. Like if it's a management issue they are struggling with, that is perhaps getting in the way of their maths learning, or if they have got all the management structures in place but perhaps it's … strategy identification and working on those. Whether it's more content or the wider classroom teaching and management. … Well I can just see the importance of actually working inside the classroom with the teacher rather than the after-school syndicate meeting. Even though you might do in-depth planning along with the syndicate about what their needs are and all the rest, that's not going to be anywhere near as effective as the in-classroom support. (Rachel, Interview 1)

This facilitator's comment highlights the importance of seeing what is really happening so that the teacher's professional development can be situated at the heart of the practice context. Comments from teachers detail the types of changes that teachers made and illustrate the fundamental aspects of pedagogy that teachers modified, such as providing students with the opportunity to explain their solution strategies, incorporating equipment into mathematical tasks, and modifying their questioning techniques. Through their in-class work with teachers facilitators illuminated students' explanations for the teachers.

This is the first time I have really appreciated the variety of strategies that children can use and use effectively. Many children are very good at explaining how they know something.

The many different ways that children came to the answer was a surprise – and provided lots of discussion – now several different ways are incorporated into problem-solving activities.

Teachers also became more selective and focused about how they used equipment. The Number Framework provides teachers with an explanatory structure that informs their rationale for equipment use.

I have always emphasised questioning, discussion, looking for patterns and using their learning in different contexts, but ANP has given more structure to all this with things like 10s boards, 100s boards, number line activities, etc. I ask children, "How would/did you get your answer?" all the time now.

The way I teach has definitely changed. Using equipment and resources has become important; asking questions to find out how students solve problems is really helping them to think about strategies.

Teachers also became more focused in the way they questioned students about how they solve a problem.

My questioning for various strategies being employed has become more directed.
I have better questioning skills and know exactly what to do next.

I question slightly differently, for example, what's a different way of working this out or how did you work it out? I would look closer at time allocation for number at the beginning of the year – more time on number!

I now encourage children to share their strategies with the class – the focus in teaching maths has changed to "How did you find out the answer?" [rather than] "What was the answer?"

Mediating "Sticking Points"

One important aspect of effective facilitation is that of mediating between existing policies and practices that inhibit change and new policies and practices that are associated with implementing those changes. This section explores three types of "sticking points"; those concerning pedagogy, those concerning policy, and those concerning professional matters.

Pedagogical Concerns

Two principals reflected on the way in which the facilitators were able to support the teachers in developing their practice.

The facilitator has encouraged and informed teachers. The soundly based principles of ANP convince teachers quickly that it works, is practical and logical.

It has been as if our facilitator waved a magic wand over our school – and now teachers and children all look upon maths as being fun and an area where all can succeed. It has blown fresh air into the school's maths programme. The programme was good – but now it is even better.

One principal likened refinement of practice to "blowing apart" some long-held beliefs that had been part of teachers' "teaching armoury". One facilitator referred to teachers' "very strong views" in describing mediation between existing and new practices as the most challenging aspect of her facilitation.

Teachers have very strong views about how they [teach] their maths lessons [you have] to work alongside them to effect some change. ... When you walk in and you see there is something in the teacher's practice that is basically in the way maybe of effective maths learning, being able to address that effectively, I think, is the most challenging. And it might be the teacher's own personal knowledge or it might be some management issue. (Rachel, Interview 1)

One principal described the facilitator's role in supporting the students through the change process.

[Teachers] had a willingness and openness to learn to increase their teaching effectiveness. At times, they felt confused with the programme. However, follow-through by the facilitator assisted them forward.

Questioning Skills

One of the teachers described in detail how she saw the facilitator shift teachers into new ways of working with students.

I've been watching Emma take lessons and actually take kids that step further. Like they give her an answer but she'll always come back and ask them that extra step, and I think that's what made teachers realise that we never get to extend the kids to where they're actually at. I think without watching Emma in action like that, that would have been hopeless. I think people would have carried on the same way and hoped that new concepts would have gone in. ... you then start realising what your own kids are capable of. If I was watching in another school or another class I think people would automatically say "my kids can't do that or my kids are actually a step further so that doesn't really mean much. ... Once again, just questioning – and questioning of us as well. I mean through the kids "But how did you get there?" I mean every time she asked a kid a question, we did it ourselves. I mean, we were answering those same sorts of questions. But what we learnt from her was the questioning techniques of how to get kids onto that next bit of how to actually understand. So I think what we've learnt is how to question to get kids' mathematical learning out in the open. So, therefore we were stopping children because I think we were afraid that our own knowledge wouldn't go that far. And we're now listening ... I think that's what's probably been a teaching issue, is that we're now prepared to listen to a variety of ways and in our own mathematical knowledge we've had to learn this variety of ways as well.

The key to the shift in practice is to situate the professional development in the context of practice. The setting in which the facilitator modelled the questioning appeared to be crucial in determining how seriously teachers took the suggested new practice. Teachers became unable to dismiss the new practice as unsuitable on contextual grounds because the demonstration was situated in the teacher's own classroom. The other "sticking point" is the fear of teachers in exposing inadequate content knowledge to themselves and the students with whom they are working. Restricting their questioning of students' explanations has been one way teachers have been able to prevent this potential exposure. The challenge for the facilitator is in carefully managing the development of the pedagogy and building up the teacher's content knowledge in unison.

The Relationship between Knowledge and Strategy

As one teacher commented, "Teaching knowledge and strategy together is rewarding as the children show their understanding and progress seems more rapid". However, the division of the framework into knowledge and strategy components was something that some teachers found confusing. Each facilitator had their own way of easing teachers through this potential confusion. One facilitator saw the framework as essentially being about leading students to use more sophisticated strategies. She described the main message that she wanted teachers to get as:

... looking at the strategies the children are using to work things out mentally rather than being product-focused – rather than who can work out this answer. So they are fluent with the strategies. [Things] that they will recognise through experience, for example if they are doing 8 + 7, if they say something like "7 and 7 is 14, so 1 more is going to be 15", ... the teacher recognises that that is an example of part/whole thinking. The teacher is becoming fluent with the different strategies and being able to recognise them in a variety of different ways. I would say [ANP] is a focus on strategies. Looking at the mental strategies that the children are using but also with The Number Framework having a hierarchy of strategies – strategy sophistication, I suppose. And giving guidance, if the student is working at this sophistication level of strategies, how to extend them, also how to broaden them, and also how to move them up into more sophisticated strategies. I think teachers find it really confusing

having them in two separate key bits [of] the knowledge and the strategy. How I approach it is that the knowledge is an essential component of the children being able to successfully use a strategy. The example I used before of 8 + 7, it's going to … if the student is not fluent in doubles it's actually going to make no sense for them to say "7 + 7 and 1 more" because it's not going to be any easier for them, or if [they are] building up to 10 if they said, for example, "8 + 2 + 5" if they don't know 10 + 5 is 15 and have that base-10 basic knowledge, it's actually not going to make any sense to them. … So I see knowledge as a key part of being able to build strategies successfully, and a previous strategy almost contributes to the knowledge base that the student will use to move up to a more advanced strategy. (Rachel, Interview 1)

This facilitator's message about strategies encapsulates the complexities of the interrelationship between knowledge and strategy that is potentially so confusing for teachers. The confusion arises from the interrelationship of knowledge and strategy. The facilitator described this as being about "a previous strategy almost contribut[ing] to the knowledge base that the student will use to move up to a more advanced strategy".

Informal versus Formal Mathematics

Learning algorithms are an important marker in a student's schooling. The Advanced Numeracy Project challenges the way in which algorithms have typically been taught as reflected in the curriculum over the last 30 years or so. Alongside the theoretical debates on the optimum timing for introducing the formal algorithm there are also contextual factors that a teacher needs to consider when deciding what emphasis should be given to algorithms in a classroom programme.

One facilitator outlined the sticking points around the formality of mathematics that faced teachers in implementing the numeracy project. She saw these as being the assessment practices related to the premature teaching of written algorithms, the uncritical use of textbooks, and pressure to prepare students for more traditional mathematics programmes at the intermediate level. She explained:

There seem to be certain sticking points where it makes it difficult for teachers to take on board and implement a lot of the aims of the project. … At the Year 3/4 level, [teachers] are the meat in the sandwich between the junior school and the senior school of standardised tests and written speed tests and competency tests … with vertical written algorithms. … And then … for the Year 6 teachers, … knowing their students are going off into intermediate. … And it really seems to be cutting as well at the heart of the school-wide policies and procedures for assessment that are in place in terms of their maths schemes or maths implementation plans. There are often guidelines about the kinds of basic-skills tests that will be administered at each grade level, at each year level. … There is a real conflict then that comes because the teachers on the one hand, have this new fledgling philosophy that's building based on these frameworks. And based on the fact that they really do see [the framework] is offering them powerful information about their students, and they see the reality of their students [achievement], but then on the other hand they have their systems and the administration telling them there are certain "must dos". And it creates a real conflict, I think, for them personally and professionally in terms then of how they can implement the things in the classroom with their kids. (Kay, Interview 2)

The facilitator needs to mediate between teachers' new priorities of a greater emphasis on informal mathematics and the emphasis on more formal mathematics traditionally represented in school policy. Another facilitator highlighted the role assessment can play in helping teachers to see the problems with the premature teaching of the algorithms.

> *In the assessment, the teachers actually recognise the value of what we are talking about. I think particularly at the advanced numeracy stage because most of the children will be familiar with working form and the different algorithms. And maybe it tends to inhibit the mental strategies. And it becomes apparent to teachers and you can sort of see the penny drop and teachers say ... "Oh gosh, you know, these children who I thought were really advanced, were actually ... just carrying out a procedure without an understanding to go behind it". It is definitely like seeing the light. I had that experience yesterday working with a teacher and she just said to me, "Gosh, we're teaching the addition and subtraction algorithm, we are teaching this too early. As soon as there is no understanding behind what the student has done here, the student has just carried out a procedure and all understanding has gone".* (Rachel, Interview 2)

Comments from teachers concur with this facilitator's observation of teachers' knowledge of strategies, illuminating the way in which algorithms encourage procedural rather than conceptual learning.

> *[It is] fascinating to see how children who are "hooked" into algorithms at a young age are stifled in their ability to play with numbers (i.e., part/whole thinking).*

> *[I've] greater "trust" that children will learn though verbalising ideas, [I allow] children to share strategies and work through "imaging". [I'm] aware that much of my practice previously had centred round my strategies of algorithms.*

> *I no longer "panic" if my year 3 and 4 children are not doing algorithms – I get a real buzz when I see number sense developing.*

The use of the words "trust" and "panic" underscore the seriousness for teachers in moving away from the algorithm. This was also evident in comments made in relation to parental involvement in the project. Typical comments such as, "Parents want their children to know how to do algorithms before strategies", "Parents continue to demonstrate 'that' way when supporting homework" or "[Parents are] insistent that the 'old way' is the best way". Teachers also commented that "It's teaching maths about the four operations that they are interested in not the pedagogy" and "It would be fair to say that parents like to see evidence of learning in books, i.e., algorithms". One teacher expressed frustration that parents' worry about algorithms led to some "independently teaching their children in my class at home, which does seem to block what I am trying to achieve at school as the children are just beginning early part/whole so [do] not really understand".

Part of informal mathematics includes students working collaboratively in groups. For a lot of teachers' group teaching was a new challenge as one teacher explained, "The effective management of groups is a skill I am still refining, but it is hugely beneficial and means very focused teaching". One principal put it, "Some [teachers] found group teaching not easy and have had a steep learning curve. Some want to include more formal methods and book work". One issue that arose was the existence of a common understanding of what group work means. Many of those commenting made reference to developing students' autonomy as a necessary component of effective independent group work.

Assessment Policies and Practices as Sticking Points

In many of the case-study schools, the facilitators led school policy development that integrated the principles of the Advanced Numeracy Project (and the Early Numeracy Project) into the school's mathematics policy. This appeared to be an important aspect of supporting changes that individual teachers made to their practice as well as informing school-wide policy development in all curriculum areas. One facilitator commented:

> *… I have been able to help schools write policy and help schools... connect together the pieces by asking questions, by not going and giving them ... and seeing the things in my folder that are not all beautiful and I don't give them as handouts but we look at, "Here's a dozen different ways, now what suits you?" and "What's your way?", and [I help] them to help themselves, but [I'm] coming from that knowledge background of what makes good assessment.* (Emma, Interview 2)

Such comments about how to best support schools in writing policies reflect the concerns of other facilitators – the dilemma of how to present possible examples yet not provide a model assessment policy. The important role of the facilitator is to guide the school by asking them questions that lead them to formulate policy that best fits their context. Another facilitator highlighted some of the specific considerations for school-wide assessment policies.

> *One of the things I want the teachers to reflect on is some of the implications for the school in terms of their mathematics policies. Things like the school-wide assessment policy, being a very algorithmic [one]. … You know, at year 4, the children will be able to do vertical algorithms for addition and subtraction, that sort of thing, the large focus in terms of assessment really, I suppose, on knowledge, you know, your basic facts tests twice a term and all of that. In a couple of the schools the long-term plans … five weeks on this, five weeks on that – five weeks number, five weeks measurement, for example – so five weeks number every term in a block, and then five weeks of say algebra and statistics. … And we talked about if you are doing measurement, for example, in one block in term one, and they are not seeing it for the rest of the year, they are not going to have any continuity, any flow through with the children. … I would say most are really eager to take on some new ideas at the planning and policy level and want some guidance in that really. I think one of the difficulties we are now a bit wary of is showing a school or giving an example of what we feel is a successful way of implementing things like the project into their school policy and long-term plan, [and] the school just picking it up and saying, "Okay, scrub out the name, put our name on top and that's it". Ideally what we would like is to get them to start critically thinking about it themselves.* (Rachel, Interview 2)

A key consideration that arises in conjunction with the Advanced Numeracy Project is the level at which algorithms feature in assessment policies. As the above quote pointed out, algorithms, through their suitability for pencil and paper testing, often feature strongly in assessment practices in the middle school years. Another consideration is the way in which the long-term plan evenly distributes the topics across five-week blocks. Facilitators were hesitant about providing a model policy or long-term plan to schools. They felt that the provision of such models might inhibit schools' critical reflection on policy and practice.

Facilitators also commented on the importance of coherence between school policies for mathematics and the aims of the Advanced Numeracy Project.

… [Teachers] are looking at redrafting some of their assessment procedures, redrafting some of their maths implementation plans and so really getting at the heart of what we have currently got in terms of what we are mandating for teachers in terms of assessment and planning consistent with the aims of this contract. So I am really interested to get feedback. I've had feedback coming from teachers who keep in touch with me through email and phone calls, and there are some really great things happening because they are starting to actually make significant changes to their policies to ensure that they are not at cross-purposes, especially things like skills tests and basic facts tests with what the aims of the project are. (Kay, Interview 2)

Aligning school policy, implementation/long-term plans and new assessment practices is of vital importance to the sustainability of changes. Facilitators can give further support to teachers and principals by being the recipients of feedback, at which point they have the opportunity to make further suggestions to ensure the tight alignment between the aims of ANP and the evolving school policies.

The Uncritical Use of Textbooks as a Sticking Point

Textbook sets in schools can be a sticking point in implementing different practices for teaching mathematics. The use of textbooks is often mandated in policy as one facilitator explained.

It's particularly a year 4–6 issue, I found … the types of resources [the teachers] already had available to them and by that I mean book resources, textbooks or maths books. Some schools have bought in large scale to maths class … or whatever their personal preference is, and in some cases, that has been a huge outlay of money that has come from their maths budgets. It may have been just last year, it may have been this year, it may have been five years ago, but there is certainly that [concern over] where do these resources sit, which they felt were an integral part of their maths programmes? And again, what we have been trying to do rather than, say, throw out the baby with the bath water is [ask] "How can we look at these resources as resources?" (Kay, Interview 2)

This facilitator explained the questions she posed to help schools though this dilemma at the same time as developing an ongoing critique amongst staff of such issues of practice.

Rather than as a maths programme that I start on page one and work through, … What can we use to complement what we are doing? What can we use for our other strand coverage? [They critique] what doesn't sit well with what we are talking about in the numeracy project, and teachers … are actually identifying that now. Like they will ... say, "Take a look at this page …. I am worried because it says it's for level three. … My kids aren't doing renaming and they are not part/whole thinkers yet but yet they are supposed to be doing these algorithms with two-digit addition." And so we'll talk about that and [question whether it is] an appropriate thing for them to go away and do as independent work or maintenance work. So they are trying to critique that as well, and in some instances, it has meant they are not continuing with that series. Things like that are really heartening because it's telling me that as a staff they are making decisions together about what is working, what's not working and what's consistent in terms of the philosophy that they now have. Because they actually now have a philosophy of maths teaching and learning that in some cases they can really articulate. In some cases, they can't put it in perhaps sophisticated wording as someone in our position might, but they have a sense now *of what they are trying to achieve, and they now have a feel for what's clashing with that, and I think that is really positive.* (Kay, Interview 2)

The opportunities arising from this dilemma underscore the importance of developing a community of practice within school settings. The contextual nature of such teaching and learning decisions are better mediated through the introduction of new practices within the community of collaborative decision-making rather than the provision of guidelines for the purchase of textbooks.

The Preparation of Students for "Intermediate Maths" as a Sticking Point

The transition between stages of schooling sometimes highlights different emphases in practice. The transition between contributing primary schools and intermediates is no exception. The Advanced Numeracy Project's emphasis on strategic problem solving is at odds with the more traditional emphasis on the written algorithm in the intermediate setting. Reflecting on this, one teacher commented that:

> *All we ask the intermediates to do is don't tell [them] they are wrong, just listen to the way they are getting the answers. ... I think there is a really big issue with the intermediates not understanding what the kids are doing. I mean one school turned around and said to us "But the high schools want us to be able to send kids on with this ... so we need your kids coming through being able to solve". I said "They'd be able to solve [problems], but they just won't do it in the same way that you [are] teaching them".*

There appears to be a classic case of top-down pressure often observed in our education system. Teachers also commented on this in relation to parents' concerns. They noted that while parents had a positive reaction to the project "some were worried about Y6s moving to intermediate and no continuity", and some felt that "their children may be at a disadvantage going to intermediate".

One facilitator elaborated on problematic practices in intermediate schools in relation to the numeracy project:

> *For the purposes of the kids going on to intermediate now, we have another issue because if they are going on to a school where they will be presented with [algorithms] in textbooks and they are going to be streamed on the basis of a test on vertical algorithms for number, then I think we have got a responsibility to those kids to teach them how to do it that way as well. ... If they do it like that, number line and picture for an answer to something, [then] that's acceptable as the vertical algorithm, and we have got two intermediates prepared to change so that instead of doing a 50-question or 25-question algorithm test, they are going to ask half the number of questions and let the kids do each one in two different ways. So if we can get that as their assessment, if we can make that little change in those kids out of the project, we'll have a chance to show some of their thinking, and you'll find the kids who are basically facile can do the process and then the ones who actually know two different ways of being able to do it or a different way but not the algorithm. [A possible way ahead] is to actually talk with intermediates. We ran a meeting, and we invited the six ANP schools plus all the intermediates that they contribute to, just to an afternoon of information sharing. It wasn't a course, it was full of questions, it was some of the schools that had been in talking about what their kids are likely to come out with. For some of the intermediate principals, they say now, "What is this*

numeracy project about? What is it all about?" There was complete ignorance on some of their parts as to the numeracy development happening. ... I think it has to be face to face. I don't think the paper is getting there in terms of informing on the breadth of the whole project. I don't think all the curriculum updates ... and all the stuff that is in there, I don't think it has been read. Now that's not a complete ignorance, but there was major ignorance by people ... And that was amongst intermediates in general. (Emma, Interview 2)

The facilitator suggested that a solution to this problem is to educate intermediate school teachers about the numeracy projects generally and more specifically about the use of strategies to solve problems.

Professional Concerns: The Dilemma of the Facilitator as a "Critical Friend"

Several of the facilitators interviewed talked about the dilemmas associated with exposing teachers' weaknesses as part of the teacher development process. One of these dilemmas is about confidentiality.

> *In the role of the facilitator there are often some very interesting and challenging positions you are put [in] in terms of the judgement. ... We are very much there for the teachers. ... Our role is [as] someone who can come in as a supportive and critical friend who they can confide in, and we don't want to break that trust, and suddenly they find out that we have gone and told everything to their principal. So ... that's a really tricky aspect of this role because we are stuck between a lot of different areas and people in the school, and they all want pieces from us and so that can be [tricky] in terms of the professional judgements you make. I have found you have really got to think carefully. We have tried to talk about that as a team [for instance] when teachers have asked us to watch them and give them feedback, some of us are in the habit of jotting down some notes, if the teacher would like some feedback, on paper. Again, is that something just for the teacher? ... We have had principals and associate principals and senior teachers who have asked for copies of those things. ... We have had to talk about that as a team [and decide] what is appropriate and what is ethically right in terms of the dealings we have with teachers and the feedback we are giving them, so in terms of those judgements that's been interesting.* (Kay, Interview 2)

This comment underscores the importance of professional judgement in the role of the facilitator. Such judgements are context-bound, and therefore it is important to have a sound set of ethical principles in place on which to base judgements.

Sustainability of the Project

Developing Autonomous Teachers and Schools

An important consideration for policy-makers is the on-going sustainability of the new knowledge and practices of teachers and the supporting school structures. Aside from the goal of raising student achievement, the project aimed to enhance teachers' content and pedagogical content knowledge. This enhancement of teachers' professional knowledge is an important factor in developing autonomous professionals. The quality of that knowledge is evident in teachers' ability to make connections.

> *And I guess, in the end, it's about teachers being able to make connections. The good ones are actually able to go back and start using texts again within that time*

and are able to actually see all the connections – "I can use that bit there, but I wouldn't want to do that, and I can use that one" and so on. The best teachers are the ones who make connections between past and present and can see why they are doing those things. The others are still looking for handouts. And we've got teachers from a much broader spectrum in [the ANP] than we had with ENP. (Emma, Interview 2)

This facilitator's comments encapsulate the essence of this autonomy with her statement about teachers who are able "to make connections between past and present and … see[ing] why they are doing those things". Her observation continued:

If we want the schools to stand on their own two feet afterwards, which we want them to do, you have got to be more than just doing the maths. ... Different schools have done it [in] completely different ways ... and it comes back to that school culture thing. Whether they have had it within their culture and their school to be able to shake everything as a result of the maths and see what's working and build on that. ... But they can now go and do that in language or PE or art or anything else that they want to do because they have now got a framework. (Emma, Interview 2)

These comments are a reminder that each school has their own set of contextual factors within which a project such as ANP is situated. Part of this context is a school's way of doing things, or as the facilitator quoted above termed it, "the school culture" or a school's way of doing things.

Communities of Practice

The development of professional autonomy is fostered through communities of practice both within and across schools that advance the ideas of the project through discussion and debate. One of the strengths of the project noted by facilitators and teachers is the existence of a common language in the explanatory framework. In this way, the intent of school policies and practices can become aligned. One facilitator talked about the advantages of across-school discussion in helping teachers to change practices within their own schools.

I would like to try and encourage the schools I am working in [to provide] some kind of ongoing support or sharing system happening at the syndicate level. I think it is starting to at some schools. In some schools, the culture of the school doesn't lend itself so much to that.

I like [it] when the teachers from different schools come [to cluster meetings] because if it's just at one school, you can be a bit limited by the school culture, whereas if it is a couple of different schools you're getting ideas from a wider cross section. It depends on the school, but sometimes there might be a culture of doing things a certain way. There might have an assessment practice that was set in concrete almost that wouldn't be necessarily challenged by the teachers themselves in the workshop, but if they are with a wider group, and you're all talking about assessment, there is more likely to be a wider range [of ideas]. If there is a culture of sitting in your desk and working from a textbook, then that might be more easily challenged when there are teachers with different experiences willing to share. (Rachel, Interview 1)

It is through professional debate that ideas are refined and become part of a school's culture.

Concluding Comments

The comments of one of the facilitators hit at the essence of effective mediation:

> *The comment that has come back from teachers is that this contract has been so different and so successful because we can walk the walk not just talk the talk. And so, as facilitators, we are the bridge between these Ministry policies and these Ministry initiatives and what the broad things are in this curriculum and the actual "how-to" in a very down-to-earth, practical [way], in the classroom [level]. How do I actually plan my week? What do I actually put on paper? What resources do I use? What have you tried that's been effective? ... Talking to teachers at a really practical level about how to implement it has been really critical.* (Kay, Interview 2)

> *It's about confidence and a change in their articulating, from "What do I do next with this bit?" to the kinds of things that they say about why they had done the things. The kinds of things they say about why they have changed, why they have chosen particular activities, about why they abandoned particular activities and said "I won't do that because that, that, that, that and that didn't work" and why I have gone back to doing this or that.* (Emma, Interview 2)

The facilitators' interpretation of the dialectical relationship between a national focus on numeracy programmes in schools and the contextual nature of teaching practice informs their approach and has implications for their role. A key element of effective facilitation appears to be the extent to which a facilitator is able to make the professional development relevant to the context in which a teacher is working. This context is broad and encompasses not only the immediate classroom environment but also the wider context of the school and community. The facilitator's role in helping teachers to understand the mathematical purpose of their teaching is a complex one.

Chapter 6: Summary

Number is an important part of primary mathematics. Student performance at years 4 to 6 improved across six aspects of number monitored over the duration of the Advanced Numeracy Project in 2001. This improvement was irrespective of students' age, gender, ethnicity, school region or decile ranking. By the final assessment, most students had advanced to part/whole thinking. The more advanced levels were characterised by a key transition from counting-based problem-solving strategies to more sophisticated and efficient strategies based on part/whole thinking or the partitioning of numbers. The gains made by students were variable, with students of Asian and European descent making greater gains at the more advanced levels in all six aspects of number.

Teachers, principals and facilitators who participated in this evaluation reported that teachers' professional knowledge and practices were enhanced through participation in the Advanced Numeracy Project. Teachers' practice became more tightly focused on number, with a particular emphasis on the teaching of mental strategies to solve number problems. In particular, changes to teachers' questioning and instructional grouping provided greater opportunities for students to explain their problem-solving strategies. This enhancement of teachers' practice was informed by a greater knowledge of how students develop an understanding of number. Both teacher and student attitudes towards the teaching and learning of number in particular, and mathematics generally, improved over the duration of the project.

Communication about teaching and learning was enhanced through participation in the project. Teachers, principals and facilitators were able to communicate using a common language to describe in a detailed manner student achievement in mathematics. This common language was particularly helpful when communicating across the transition points of schooling, such as between the junior and middle and senior parts of a primary school as well as between primary and intermediate schools. Teachers and principals reported that they were able to report more specifically to parents by using student performance in the diagnostic interviews and by referring to The Number Framework.

Teachers and principals reported the development of communities of practice arising out of a joint focus through participation in the project. Such communities of practice were helpful to teachers in providing a context for discussing their practice and developing their knowledge of number concepts.

In-class support by facilitators provided a focal point for discussing existing practice alongside the approaches suggested in the Advanced Numeracy Project. Facilitators were able to act as mediators between teachers' existing and new practices within individual teachers' contexts of practice. Facilitators also worked with schools to align school policies and procedures with the intent of the Advanced Numeracy Project. An important factor in facilitators' work was to ensure the sustainability of the changes made as a result of participation in the project.

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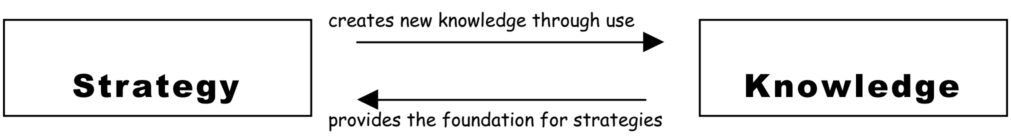
Appendix A: The Number Framework

Introduction

The Number Framework has been established to help teachers and children to understand the requirements of the Number strand from *Mathematics in the New Zealand Curriculum*. The framework relates to most of the achievement aims and objectives in levels 1 to 4.

There are two main sections to the framework; Strategy and Knowledge. The Strategy section is about how children solve number problems, in particular the mental processes they use. The Knowledge section considers the key items of knowledge that children need to acquire.

It is essential that children make progress in both sections of the framework as these parts are dependent on one another. Clearly it is impossible for a child to apply a clever strategy to solve a number problem if they have weak knowledge. For example, a child will not work out 8 + 6 as 10 + 4 if they do not know that 10 + 4 = 14. Similarly, using clever strategies helps to develop knowledge. For example, a child who uses doubling of the three times tables to work out the six times tables will soon learn their tables through appropriate repetition.



Underpinning the framework is a sequence of global strategy stages. The stacked layout of the stages represents increasingly sophisticated concepts about numbers.

Part-whole

Advanced Proportional
Advanced Multiplicative
Advanced Additive
Early Additive

Counting

Advanced Counting
One-to-one Counting
Emergent

We have defined these stages in order to make it easier to identify and describe the kinds of mental strategies that children are using so that they can be lead to develop more sophisticated ones. Children appear to be very consistent in their view of numbers, and this consistency helps us to anticipate what kinds of strategies they are likely to use and to plan appropriate learning activities and questions.

An Overview of the Strategy Section of the Framework

Below, we have described each strategy stage and linked it to the child's view of numbers as being units. Each stage assumes that the child has the ability to call up these unit structures and manipulate them mentally. Teachers need to have experience with using the *Early Numeracy Project Assessment (ENPA)* technique for interviewing children in order to understand these stages.

It is important to remember that children are often between stages. That is, they display characteristics of one stage given a certain problem but may use more or less advanced strategies given different problems. Consider this to be metaphorically like a children having one foot in both stages and "shifting weight" until they are confident in stepping to the more advanced stage.

Each stage contains the operational domains of addition and subtraction, place value, multiplication, and fractions. In the table below, information in the cells, reading across each stage, relates to the strategies that are consistent with the unit structure used by the child at that stage.

		Operational Domains			
Stages		Addition Subtraction	Place value	Multiplication	Fractions
Counting	Pre-Counting				
	Count All from One				
	Advanced Counting				
Part-Whole	Early Additive				
	Advanced Additive				
	Advanced Multiplicative				
	Advanced Proportional				

The most advanced stage demonstrated by children on any of the operational domains should be seen as their "zone of proximal development" and is indicative of the most advanced unit structure that they understand.

Strategy Stages

This section contains descriptions of the unit structures that children at each stage are using along with some examples of specific unit structures.

Stage One: Pre-Counting

Children at this stage have no counting unit. They are unable to count a given number of objects because they lack an understanding of counting sequences, one-to-one correspondence, or both.

Stage Two: Count All from One

Children at this stage have a counting unit of one. Given a joining or separating problem they represent all the objects in both sets, either with materials or later in their mind as an image. They count all the objects in both sets to find an answer.

Stage Three: Advanced Counting

This stage marks the point where the child realises that a number can represent a completed count that can be built on. For example, instead of counting all objects to solve 8 + 4, the child recognises that “8” represents the act of having counted 8 objects and counts on from there (8, 9, 10, 11, 12). Children at this stage also develop the ability to put together completed counts as in 10, 20, 30, 40, 50 to get $50 in $10 notes and increments in tens as in 14, 24, 34, 44, 54.

Stage Four: Early Additive Part-whole

At this stage, the child has begun to recognise that numbers are abstract ideas (units) that can be treated as a whole or can be partitioned and recombined to solve addition and subtraction problems. A particular characteristic of this stage is the deriving of results from closely related known facts, such as finding addition answers by using doubles.

The unit structures that the children are using can be represented as:

(i) Compensation
Example: 7 + 8: 7 + 7 is 14 so 7 + 8 is 15

7	7	→	7	7	1	→	7	8

(ii) Standard place value partitioning,
Example: 23 + 12 is (20 + 10) + (3 + 2) = 30 + 5

23	12	→	20	3	10	2	→	20	10	3	2

The material covered in depth in this folder finishes at this stage.

Stage Five: Advanced Additive/Early Multiplicative Stage

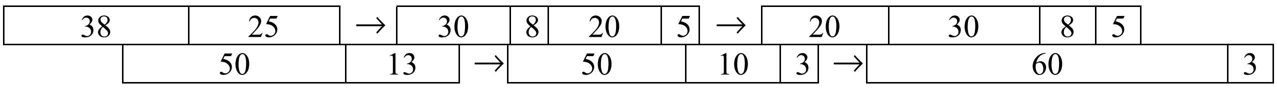
(Stages Five to Seven are covered in more detail in the Advanced Numeracy Project (ANP) folder.)

Children who are at the advanced additive part-whole stage are able to interpret a wide range of contextual problems that involve addition and subtraction, and they can choose appropriately from a rich repertoire of part-whole strategies to solve such problems. This view sees numbers as whole units in themselves, but also "nested" within these units are multiple possibilities for subdivision. Critically this stage includes the use of inverse operations, as in using addition to solve subtraction problems.

The key strategies used by children at this stage may be represented as:

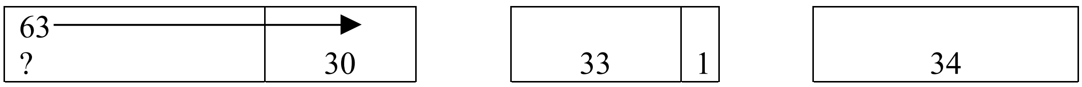
(i) Standard place value partitioning

Example: 38 + 25 as 30 + 20 + 8 + 5 = 50 + 13



(ii) Compensation

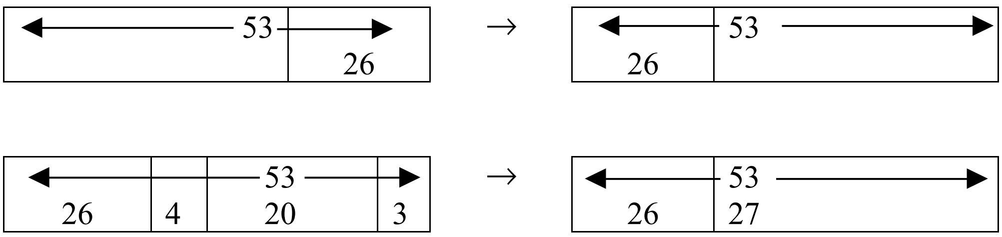
Example: 63 – 29 as 63 – 30 + 1



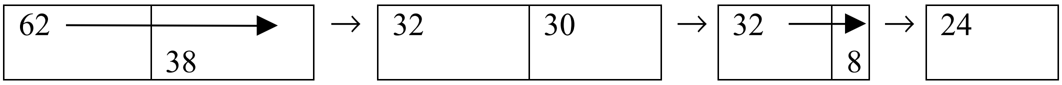
(iii) Inverse operation

Example: 53 – 26: 26 + ? = 53 and 26 + 4 + 20 + 3 = 53

so 53 – 26 = 27

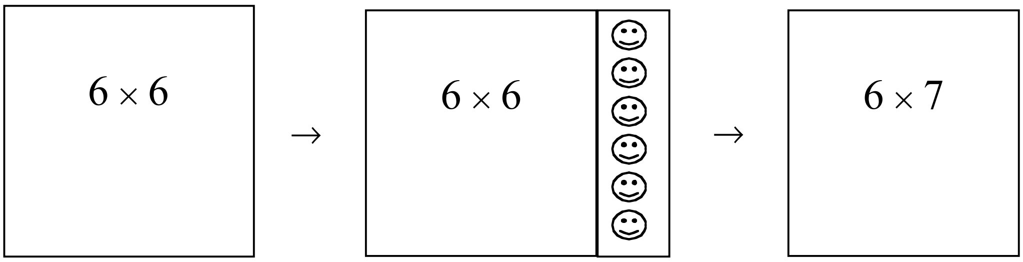


(iv) Decomposition with place value
Example: 62 – 38: 62 – 30 = 32, 32 – 8 = 24



Children at the advanced additive stage treat multiplicative units simultaneously as whole structures and units that can be partitioned and re-combined. A significant characteristic of children at this stage is their ability to derive multiplication and division facts from closely related known results. This is a direct result of their fluency with additive unit structures.

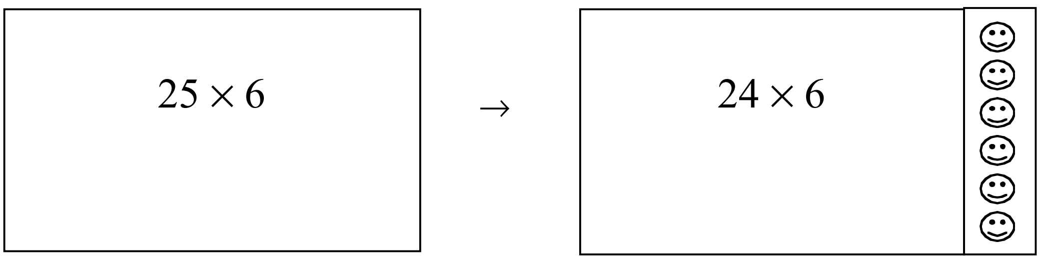
Example: $6 \times 6 = 36$ so 7×6 must be 6 more which is 42.



Stage Six: Advanced Multiplicative/Early Proportional Part-whole
Children at the advanced multiplicative stage can solve contextual problems by choosing appropriately from a wide range of part-whole strategies that involve multiplication and division. This entails viewing multiplicative units as both complete and sub-dividable. A critical development at this stage is the use of inverse operations, for example, solving a division problem using multiplication.

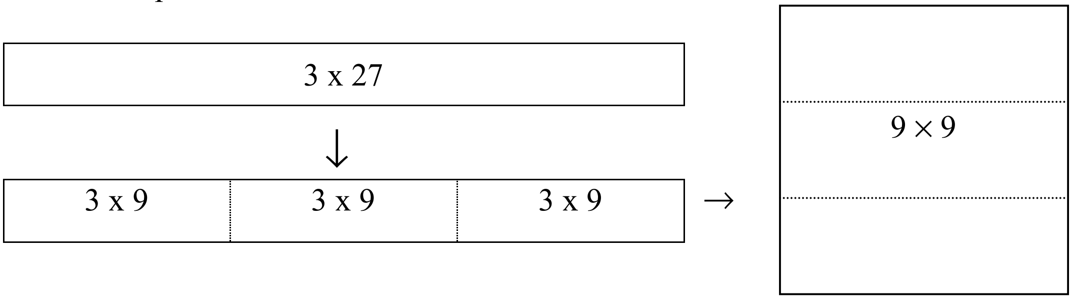
Key strategies for advanced multiplicative children are summarised diagrammatically below:

(i) Compensation
Example: 24×6 as $25 \times 6 - 6$



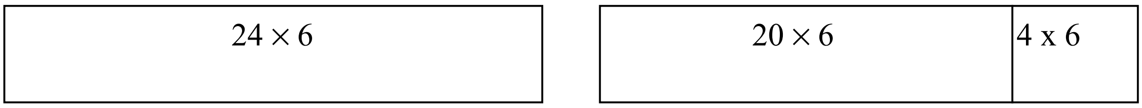
(ii) Associative property

Example: 3×27 as $3 \times 3 \times 3 \times 3 = 9 \times 9$



(iii) Distributive property

Example: 24×6 as $(20 \times 6) + (4 \times 6)$

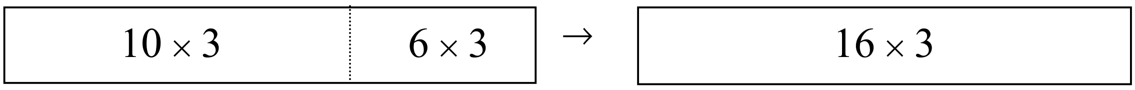


(iv) Proportion

Example: $80 \div 5$ as $(80 \div 10) \times 2$

(v) Inverse operation

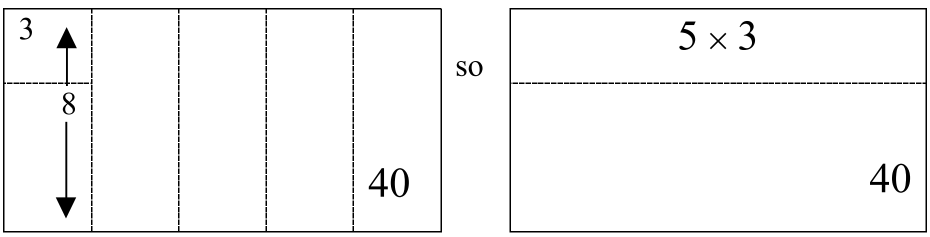
Example: $48 \div 3$ as $? \times 3 = 48$, $10 \times 3 = 30$ and $6 \times 3 = 18$ so $48 \div 3 = 16$



Advanced multiplicative children develop the ability to apply multiplication and division unit structures to solve problems that involves fractions and proportions. This allows them to understand fractional and proportional equivalence, which is a foundation for the decimal system.

Example: Out of every 8 lollies in the jar 3 of them are raspberry. There are 40 lollies in the jar. How many of them are raspberry?

(3:8 as ?:40, $5 \times 8 = 40$ so $5 \times 3 = ?$)



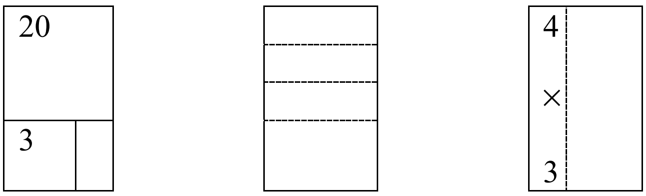
Stage Seven: Advanced Proportional Part-whole

Children who are at the advanced proportional stage are able to select from a wide range of part-whole strategies to solve problems that involve fractions and proportions. Critically this entails applying inverse operations, for example, solving a division problem by multiplication.

Key strategies for Advanced Multiplicative children are summarised diagrammatically below:

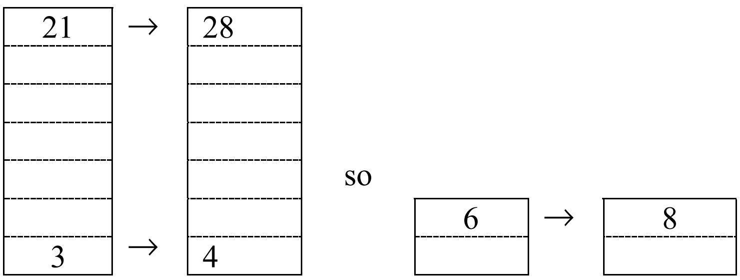
(i) Within strategies

Example: Of every 5 children in the class, 3 are boys. There are 20 children in the class. How many of them are boys?
(3:5 as ?:20: $4 \times 5 = 20$ so $4 \times 3 = 12$)



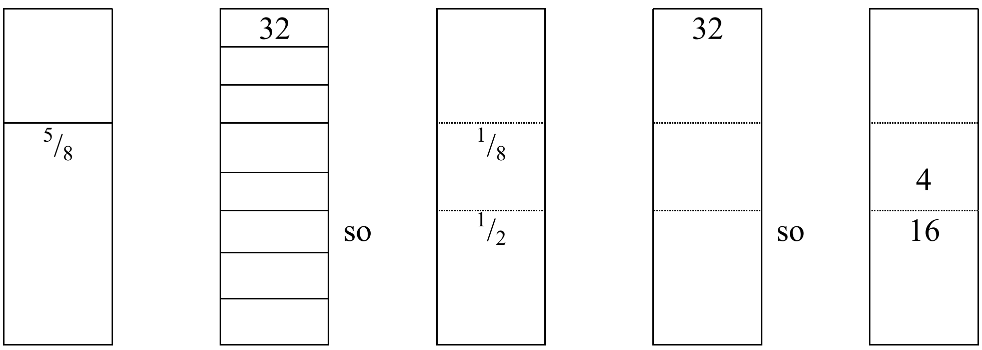
Between strategies

Example: You can make 21 glasses of lemonade from 28 lemons. How many glasses can you make using 8 lemons? (21:28 as ?:8, 21 is ${}^3/_4$ of 28 so ${}^3/_4$ of 8 is 6)



(iii) Distributive property

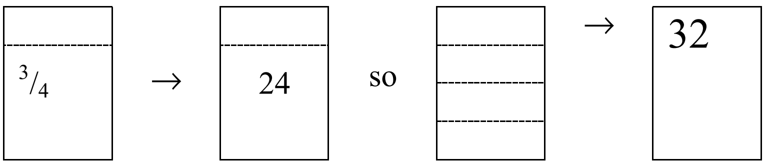
Example: Albert has 32 matchbox toys. Five-eighths of them are sports cars. How many of the matchbox toys are sports cars?
(${}^5/_8$ of 32 as ${}^1/_2$ of 32 is 16 and ${}^1/_8$ of 32 is 4, so ${}^5/_8$ of 32 is 16 + 4 = 20)



(iv) Inverse operation

Example: Marina has 24 metres of fabric to use to make T-shirts for the Kapahaka group. Each T-shirt takes three-quarters of a metre of fabric. How many T-shirts can Marina make?

($24 \div {}^3/_4 = ?$ by ${}^3/_4 \times ? = 24$)



An Overview of the Knowledge Section of the Framework

The intention of the Knowledge section of the framework is to outline the important items of knowledge that children should learn as they progress through the strategy stages. This knowledge plays a critical role in children applying their available strategies with proficiency and fluency across all the numbers and problem types that they may encounter.

The framework has knowledge categorised under four content domains: Numeral Identification, Number Sequence and Order, Grouping/Place Value and Basic Facts, and Written Recording. Items of knowledge can be taught to children across a range of strategy stages. This avoids limiting the children's exposure to important mathematical ideas. For example, children can come to read and write decimals long before their strategies are sufficiently developed for them to use decimals in operational problems.

It is important to recognise that the knowledge framework reflects some significant shifts in emphasis from previous interpretations as well as a restatement of the importance of some knowledge items. These shifts are in keeping with the trends observed across most overseas countries.

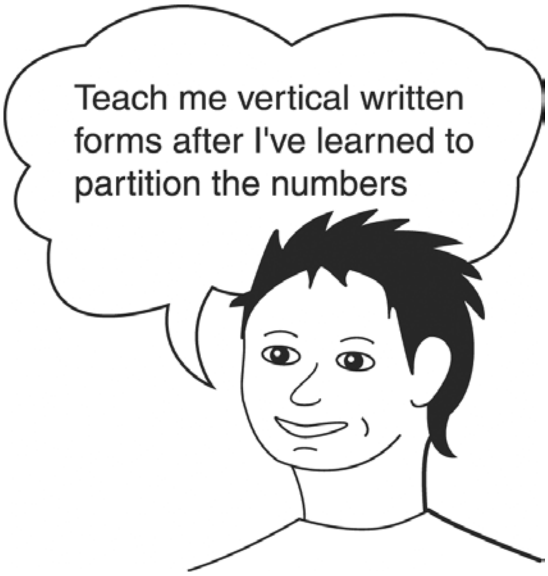
Written recording is seen primarily as a means to "think through" calculations **so that children are not exposed to standard vertical algorithms until mental strategies are sufficiently advanced**.

Counting needs to be developed with all primary-aged children. This counting should extend past counting by ones, twos, and other simple multiples into counting in powers of ten, in decimals, and in fractions. The framework also acknowledges the importance of backwards counting sequences, particularly in the early years of primary schooling.

The framework encourages greater use of children's natural inclination to use groupings of fives. There is significant emphasis on using finger patterns in the early years and in using multiples of five in later work, for example, 35 + 35 = 70, 500 + 500 = 1000.

The most significant model of fractions that needs to be developed is the partitioning of sets. Continuous models such as regions and lengths are useful spatial contexts for fractions, but sets models offer the more direct link to understanding the decimal system.

Basic fact knowledge is critical. *The Number Framework* emphasises that the process of coming to know and derive number facts is as important as knowing them. It also demands that children come to know a broader range of facts than previously, including groupings of "benchmark" numbers, knowledge of decimal, and fraction conversions. It is vital to the development of fluency and flexibility that children be able to automatically recall key number facts.



Appendix B: Advanced Numeracy Project Assessment

Things that the interviewer says to the child appear in **plain bold** type. Comments for the interviewer appear in *italic* type.

Knowledge Questions

Whole Number Identification/Order and Sequence

(1) **Tell me what number am I showing, then tell me which number is one more and which number is one less than that number?**

Use the yellow numeral cards provided. The question may need to be repeated a few times, that is, **What number is one more?** *(child responds)*

What number is one less?

Stage and Behavioural Indicator
4 FNWS and BNWS to 100 The child identifies numbers in the range 1 to 100 and can give the number one after and one before.
5 FNWS and BNWS to 1000 The child identifies numbers in the range 1 to 1000 and can give the number one after and one before.
6 FNWS and BNWS whole numbers The child identifies numbers in the range 1 to 1 000 000 and can give the number one after and one before.

Fraction, Decimal, Percentage Identification, Order and Sequence

(2) **What number I am showing?**
Use all of the blue cards provided.

Put these numbers in order with the smallest over here *(left)* **and the largest over here** *(right)***.**

Give the child the cards $^1/_2$, $^1/_3$, and $^1/_4$, first, to find out whether they can order these unit fractions before using the other fractions.

(3a) **Put these numbers in order with the smallest over here** *(left)* **and the largest over here** *(right)*. *Use the green cards provided that show 0.8, 0.47, 0.598.*

(3b) *Add the green cards marked 60% and $^1/_3$ and mix with the other cards.* **Put all of these numbers in order.**

Stage and Behavioural Indicator
5 Unit Fraction ID The child identifies unit fractions (for example, $^1/_4$, $^1/_3$).
6 Unit Fraction Order, Decimal ID The child identifies decimals to two places and orders unit fractions.
7 Fraction Order, FNWS and BNWS Decimal The child gives the number one more and one less than decimals to two places and orders fractions with different numerators and denominators.
8 Fraction, Decimal, Percentage Order The child orders fractions, decimals, and percentages.

Grouping/Place Value

(4) **Suppose you had to make this much money using only $10 notes. How many notes would you get?** *Show the child the envelopes one at a time. Allow them to make the amounts for $60 and $230 with money, if necessary. If children cannot complete all parts of this question correctly, miss out questions 5 and 6.*

(5) **Suppose you had to make this much money using only $100 notes. How many notes would you get?**
Show the child the envelopes one at a time.

(6) **Here are some decimal numbers. How many tenths are in each number?**
Show the child the envelopes one at a time. (3.2 and 1506.9)

4	**Building up Tens** The child finds the tens in numbers to 100 by repeated counting, that is, 10, 20, 30, 40,....
5	**Tens in 100** The child finds how many tens are in numbers to 1000 using knowledge that 10 tens are 100, for example, for 230: 10, 20, 23.
6	**Tens in Whole Numbers** The child knows how many tens are in whole numbers.
7	**Tens and Hundreds in Whole Numbers, Tenths in 1** The child knows how many tens and hundreds are in whole numbers. They find the number of tenths in a decimal number using knowledge that 10 tenths make 1. For example, for 2.40: 10, 20, 24.
8	**Tenths in Decimals** The child knows how many tenths are in decimal numbers.

Strategy Questions

For all questions, if the child's strategy is not obvious ask "Tell me how you worked that out?" This is an assessment of mental strategies. The child's first attempt on each question must be to solve it mentally. Pencil and paper may be provided after reasonable thinking time has elapsed and only if the interviewer believes that access to it will support the child in finding a solution. Successful completion of a question using vertical written forms shows nothing about the child's part/whole understanding.

Addition and Subtraction/Place Value

(7) Place a pile of loose beans in front of the child. **Please count out 8 beans for me.** Allow the child to count out 8 beans. Cover the beans with a masking card. Place another 6 beans under a separate masking card. **I have another 6 beans under here** (indicating with hands). **That's 8 beans and another 6 beans. How many beans is that altogether?**
Gather up the beans used in Question 7 away and start again.

(8) Place 9 beans under one card and 7 beans under the other card. **I have 9 beans under this card and another 7 beans under here** (indicating with hands). **How many is that altogether?**

(9) Add 2 bags of 10 beans to the collection of 9 and mask it. **Now I have 29 beans under here and there are still 7 under here. How many is that altogether?**

If the child counts from one or counts on to solve questions 7, 8, and 9, miss out questions 10 and 11.

(10) *Read the birds and cats problem.* **On this page there are 15 birds and 27 cats. How many animals is that altogether?** *Once the child has responded correctly and you*

have questioned them about their strategy, ask: **You are correct. Can you think of another way to work that out?**

If the child is unable to fluently answer question 10, move on to question 11.

(11) *Display the card with the bus problem and read it to the child.* **There are 53 people on the bus, and 26 people get off. How many people are left on the bus?**

Allow the child sufficient time to solve the problem mentally, but offer them a pencil and paper if you consider it will assist them.

Stage and Behavioural Indicator	
2-3	**Count from One** The child solves addition and subtraction problems by counting from one, for example, 8 + 7 as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
4	**Advanced Counting** The child solves addition and subtraction problems by counting on or counting back, for example, 8 + 7 as 8, 9, 10, 11 … or 25 + 17 as 25, 26, 27, 28, … They also use tens counts to solve addition problems, for example, 25 + 17 as 25, 35, 36, 37, 38, 39 …
5	**Early Additive Part/Whole** The child solves addition and subtraction problems using a limited range of part/whole strategies like doubles or standard place value partitioning, for example, 8 + 7 as 7 + 7 + 1 or 25 + 17 as (20 + 10) + (5 + 7).
6	**Advanced Additive Part/Whole** The child solves multidigit addition and subtraction problems using a range of part/whole strategies. Note that correctly solving question 11 (53–26) is a strong indicator.

Multiplication and Division

(12) **Here is a field of cows** *(6 × 5* array*)*. **There are 5 cows in each row and there are 6 rows** *(using horizontal and vertical sweeps with index finger). Mask all but one horizontal and one vertical edge of the array.* **How many cows are there in the field altogether?** *If the child is unable to give an answer, uncover the rest of the sheet.*

Show 15 more cows on individual cards. **Here are 15 more cows. If they join the others in the rows of 5, how many rows will there be then? How many cows will that be altogether***?* *If the child is unable to solve the problem mentally, allow them to manipulate the cards.*

If the child solves question 12 using one-to-one counting or skip counting with finger tracking, then miss out questions 13, 14, and 15.

(13) **17 × 6 = 102 so what does 18 × 6 equal?**
27 × 2 = 54 so what does 27 × 4 equal?

If the child cannot solve question 13 by connecting the multiplication facts, that is, 102 add 6 is 108, then miss out questions 14 and 15.

(14) **There are 24 muffins in each basket. How many muffins are there altogether?**

(15) **At the car factory, they need 4 wheels to make each car. How many cars can they make with 72 wheels?**

Stage and Behavioural Indicator	
2-3	**Count from One** The child solves multiplication problems by counting all of the objects.
4	**Advanced Counting** The child solves multiplication problems by skip counting or a combination of skip counting and counting in ones, for example, 5, 10, 15, 20 …
5	**Early Additive Part-Whole** The child solves multiplication problems using repeated addition, eg. for 6 x 5: 5 + 5 = 10, 10 + 10 + 10 = 30
6	**Advanced Additive Part/Whole** The child solves multiplication and division problems by deriving from known multiplication facts, for example, $(17 \times 6) + 6 = 18 \times 6$.
7	**Advanced Multiplicative Part/Whole** The child uses a range of part/whole strategies to solve multiplication and division problems involving multidigit numbers. These strategies include the distributive property, eg. 24×6 as $(20 \times 6) + (4 \times 6)$, and compensation, for example, $72 \div 4$ as $(80 \div 4) - (8 \div 4)$.

Fractions

(16) *Make a set of 12 beans and mask them.* **Here are 12 beans. You have to get one half of them. How many should you take?** *If the child cannot answer the question, allow them to manipulate the beans to find an answer.*

If the child needs to manipulate the materials to solve question 16, conclude the interview.

(17) *Put another 16 beans under the masking card.* **There are now 28 beans under the card. You need to take $^3/_4$ of them.** *(Show $^3/_4$ of 28 written down.)* **How many should you take?** *If the child cannot answer the question, allow them to manipulate the beans to find an answer.*

If the child is not able to solve question 17 using mental strategies, then conclude the interview.

(18) **Of every 8 apples in the box, 3 are bad. There are 40 apples in the box. How many are bad?**

If the child does not solve question 18 using multiplication, conclude the interview.

(19) **It takes 10 balls of wool to make 15 mittens. How many balls of wool does it take to make 6 mittens?**

(20) **There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class are boys?**

Stage and Behavioural Indicator	
2-3	**Count from One** The child finds a fraction of a number by equal sharing of objects.
4	**Advanced Counting** The child finds a fraction of a number by equal sharing of objects.
5	**Early Additive Part/Whole** The child finds a fraction of a number using addition facts, for example, $^1/_2$ of 12 as 6 + 6.
6	**Advanced Additive Part/Whole** The child finds a fraction of a number using a combination of addition facts and multiplication, for example, $^3/_4$ of 28 as 14 + 14 = 28, 7 × 2 = 14 so $^1/_4$ of 28 is 7, 14 + 7 = 21.
7	**Advanced Multiplicative Part/Whole** The child finds a fraction of a number using multiplication and division, for example, $^3/_4$ of 28 as 28 ÷ 4 = 7, 3 × 7 = 21. They also solve proportions problems using multiplication, for example, 3:8 as ?:40, 8 × 5 = 40 so 5 × 3 = 15.
8	**Advanced Proportional Part/Whole** The child finds a fraction of a number using a range of part/whole strategies based on multiplication and division and solves proportion problems using ratio. Note that a successful solution to either question 19 or question 20 is a strong indicator.

Appendix C: Summary of Questionnaires

Facilitator Questionnaire

1. From your perspective, what has been the **attitude** toward the project of your participating teachers? Please specify?
2. In regard to the teachers that you worked with, do you think that ANP had an impact on their **maths content knowledge**? If so, please give specific examples in relation to the teachers you were working with.
3. In regard to the teachers that you worked with, do you think that ANP had an impact on their **pedagogical knowledge**? If so, how?
4. In your view, has the project had any **positive** impact on the mathematics programme of the participating teachers? Please elaborate on your response.
5. In your view, has the project had any **negative** impact on the mathematics programme of the participating teachers? Please elaborate on your response.
6. In your view, has the project had any impact on the teachers "**as a person**" either in or outside the classroom? (eg confidence, enthusiasm, tiredness)
7. From your perspective, has the project had any impact on the **other teachers** in your school who were not participating in the project?
8. In your view, what impact has the project had on the **children** participating in the project? (consider attitudes, skills, understandings etc)
9. What aspects of this project could be **improved** ? Why?
10. Are there any other comments you would like to make about the project?

Background Information

Your age:
Gender:
Years of teaching experience (including this year):
Years of advisory experience:

Principal Questionnaire

1. What factors most influenced your decision to apply to participate in the Advanced Numeracy Project?
2. From your perspective, what has been the attitude toward the project of your participating teachers? Please specify.
3. In your view, has the project had any positive impact on the mathematics programme of the participating teachers? Please elaborate on your response.
4. In your view, has the project had any **negative** impact on the mathematics programme of the participating teachers? Please elaborate on your response.
5. In your view has the project had any impact on the teachers "**as a person**" either in or outside the classroom? (eg confidence, enthusiasm, tiredness)
6. From your perspective, has the project had any impact on the **other teachers** in your school who were not participating in the project?
7. In your view, what impact has the project had on the **children** participating in the project? (Consider attitudes, skills, understandings etc)

8. What has been the general reaction of **parents** to the Advanced Numeracy Project?
9. What aspects of this project could be **improved**? Why?
10. Are there any other comments you would like to make about the project?

Background Information

Region:
Size of School:
Decile:

Teacher Questionnaire

1. Do you think that your attitude towards maths has changed as a result of your participation in the project? Please explain your response.
2. Has your content knowledge of maths been developed in any way as a result of your participation in the project? Please elaborate on your response.
3. Has your understanding of how children learn number changed as a result of your participation in the project? Please elaborate on your response.
4. Has the way you teach number changed as a result of your participation in the project? Please elaborate on your response.
5. In your opinion, what aspects of the project helped you most? How and why?
6. What aspects of the project were least helpful or confusing? Why?
7. What has been the general reaction of parents to ANP?
8. What suggestions can you offer for the improvement of the project?
9. What aspects of this project could be improved? Why?
10. Are there any other comments you would like to make about the project?

Background Information

Region:
Size of school:
Decile:
Your age:
Gender:
Years of teaching experience (including this year):
What year level are you currently teaching? (tick all that apply)
How many years experience have you been teaching in years 4-6?
How long have you taught at this school?
Highest level of teaching qualifications completed:
Are you currently engaged in furthering your qualifications?

Appendix D: Mean Gains for Each Aspect

AGE	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole
7-year-olds	0.84	0.77	0.34	0.13
8-year-olds	0.97	0.72	0.51	0.34
9-year-olds	0.96	0.74	0.56	0.39
10-year-olds	1.11	0.81	0.71	0.46
11-year-olds	1.09	0.91	0.60	0.32
Total	0.99	0.76	0.61	0.42

Table D1: Mean Gains in Multiplication and Division: Stages by Age

ETHNICITY	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole
Asian	1.18	0.86	0.61	0.41
European	1.03	0.8	0.63	0.43
Māori	0.92	0.69	0.54	0.34
Pacific Islands	0.89	0.55	0.51	0.37
Other	1.0	0.82	0.55	0.35
Total	1.0	0.76	0.61	0.41

Table D2: Mean Gains in Multiplication and Division: Stages by Ethnicity

DECILE	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole
Low (1–3)	0.97	0.72	0.58	0.42
Middle (4–7)	0.99	0.74	0.58	0.38
High (8–10)	1.06	0.88	0.66	0.45
Total	0.99	0.76	0.61	0.41

Table D3: Mean Gains in Multiplication and Division: Stages by Decile Rating

AGE	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole	Advanced Multiplicative Part/Whole
7-year-olds	1.11	0.55	0.46	0.36	0.00
8-year-olds	1.01	0.65	0.36	0.52	0.10
9-year-olds	0.98	0.70	0.46	0.37	0.27
10-year-olds	0.93	0.93	0.62	0.59	0.23
11-year-olds	0.71	0.74	0.52	0.37	0.11
Total	0.98	0.74	0.50	0.50	0.22

Table D4: Mean Gains in Fractions: Stages by Age

ETHNICITY	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole	Advanced Multiplicative Part/Whole
Asian	1.27	0.73	0.61	0.56	0.25
European	1.04	0.78	0.53	0.51	0.20
Māori	0.99	0.70	0.36	0.45	0.32
Pacific Islands	0.76	0.54	0.35	0.37	0.13
Other	1.00	0.75	0.59	0.39	0.27
Total	0.98	0.74	0.50	0.50	0.22

Table D5: Mean Gains in Fractions: Stages by Ethnicity

DECILE	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole	Advanced Multiplicative Part/Whole
Low (1–3)	0.94	0.76	0.41	0.43	0.37
Middle (4–7)	0.96	0.66	0.54	0.52	0.19
High (8–10)	1.32	0.89	0.55	0.54	0.18
Total	0.98	0.74	0.50	0.50	0.22

Table D6: Mean Gains in Fractions: Stages by Decile Rating

Appendix E: Knowledge Aspects Pre- and Post-Instruction

Stage	Pre-Instruction	Post-Instruction
Stage 3	2% (180)	0% (43)
Stage 4	23% (1838)	9% (717)
Stage 5	43% (3492)	34% (2742)
Stage 6	32% (2580)	57% (4588)
Total	100% (8090)	100% (8090)

Table E1: Whole Number Pre- and Post-Instruction

Stage	Pre-Instruction	Post-Instruction
Stage 4	56% (4497)	13% (1078)
Stage 5	34% (2775)	48% (3892)
Stage 6	7% (614)	26% (2055)
Stage 7	2% (143)	8% (682)
Stage 8	1% (61)	5% (383)
Total	100% (8090)	100% (8090)

Table E2: Fraction Knowledge Pre- and Post-Instruction

Stage	Pre-Instruction	Post-Instruction
Stage 3	8% (644)	1% (112)
Stage 4	51% (4160)	26% (2097)
Stage 5	25% (2039)	34% (2762)
Stage 6	12% (930)	24% (1907)
Stage 7	2% (195)	8% (631)
Stage 8	2% (122)	7% (581)
Total	100% (8090)	100% (8090)

Table E3: Grouping Pre- and Post-Instruction

Appendix F: Strategy Aspects Pre- and Post-Instruction Adjusted by Age

Stage	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	23% (45)	25% (597)	14% (357)	7% (198)	11% (23)	15% (1220)
Advanced Counting	53% (105)	48%(1159)	38% (995)	28% (734)	37% (75)	38% (3068)
Early Additive Part/Whole	19% (38)	20% (480)	31% (791)	36% (953)	31% (62)	29% (2324)
Advanced Additive Part/Whole	4% (8)	7% (169)	14% (366)	22% (573)	17% (34)	14% (1151)
Advanced Multiplicative Part/Whole	1% (2)	1% (32)	3% (83)	8% (202)	4% (8)	4% (327)
Total	100% (198)	100% (2437)	100% (2592)	100% (2660)	100% (202)	100% (8090)*

* Note: Percentages have been rounded to the nearest whole number where necessary.
* One 12-year-old not included

Table F1: Multiplication, Adjusted by Age (pre-instruction)

Stage	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	10% (19)	6% (145)	4% (97)	2% (52)	4% (8)	4% (321)
Advanced Counting	30% (60)	34% (835)	24% (615)	14% (373)	16% (32)	24% (1915)
Early Additive Part/Whole	38% (76)	36% (876)	34% (881)	29% (781)	37% (75)	32% (2689)
Advanced Additive Part/Whole	19% (38)	18% (437)	27% (699)	32% (837)	28% (56)	26% (2067)
Advanced Multiplicative Part/Whole	3% (5)	6% (144)	12% (300)	23% (617)	15% (31)	14% (1098)
Total	100% (198)	100%(2437)	100% (2592)	100% (2660)	100% (202)	100%(8090)*

* Note: Percentages have been rounded to the nearest whole number where necessary.
* One 12-year-old not included

Table F2: Multiplication and Division Adjusted by Age (post-instruction)

Stage	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	29% (57)	32% (781)	19% (483)	13% (355)	15% (31)	21% (1707)
Advanced Counting	39% (77)	39% (948)	34% (871)	27% (705)	26% (53)	33% (2654)
Early Additive Part/Whole	26% (52)	25% (606)	35% (918)	37% (986)	44% (88)	33% (2650)
Advanced Additive Part/Whole	6% (11)	3% (71)	9% (235)	13% (356)	9% (19)	9% (692)
Advanced Multiplicative Part/Whole	1% (1)	1% (30)	3% (78)	9% (237)	5% (9)	4% (356)
Advanced Proportional Part/Whole	0% (0)	0% (1)	0% (7)	1% (21)	1% (2)	0% (31)
Total	100% (198)	100% (2437)	100% (2592)	100% (2660)	100% (202)	100% (8090)*

* Note: Percentages have been rounded to the nearest whole number where necessary.
* One 12-year-old not included

Table F3: Fractions Adjusted by Age (pre-instruction)

Stage	7-year-olds	8-year-olds	9-year-olds	10-year-olds	11-year-olds	Total
Count All	8% (15)	10% (234)	6% (145)	5% (129)	7% (15)	7% (538)
Advanced Counting	31% (62)	32% (780)	25% (636)	16% (418)	17% (34)	24% (1930)
Early Additive Part/Whole	43% (85)	42% (1023)	39% (1014)	33% (870)	39% (79)	38% (3071)
Advanced Additive Part/Whole	14% (27)	11% (277)	20% (516)	22% (588)	23% (46)	18% (1454)
Advanced Multiplicative Part/Whole	4% (7)	5% (113)	9% (238)	20% (539)	11% (23)	11% (921)
Advanced Proportional Part/Whole	1% (2)	0% (10)	2% (43)	4% (116)	3% (5)	2% (176)
Total	100% (198)	100% (2437)	100% (2592)	100% (2660)	100% (202)	100% (8090)*

* Note: Percentages have been rounded to the nearest whole number where necessary.
* One 12-year-old not included

Table F4: Fractions Adjusted by Age (post-instruction)

Appendix G: Final Status of Students Who Were Advanced Counters

AGE	No Change	Became Early Additive	Became Advanced Additive	Became Advanced Multiplicative	Total
7-year-olds	42% (44)	40% (42)	17% (18)	1% (1)	100% (105)
8-year-olds	43% (495)	44% (508)	12% (137)	1% (19)	100% (1159)
9-year-olds	42% (417)	43% (431)	13% (131)	2% (16)	100% (995)
10-year-olds	39% (286)	44% (324)	14% (104)	3% (20)	100% (734)
11-year-olds	33% (25)	45% (34)	19% (14)	3% (2)	100% (75)

Table G1: Final Status of Advanced Counters by Age for Multiplication and Division

ETHNICITY	No Change	Became Early Additive	Became Advanced Additive	Became Advanced Multplicative	Total
Asian	38% (47)	45% (56)	12% (15)	5% (7)	100% (125)
European	39% (735)	44% (842)	15% (285)	2% (39)	100% (1901)
Māori	44% (284)	44% (281)	11% (71)	1% (7)	100% (643)
Other	33% (24)	54% (39)	10% (7)	3% (2)	100% (72)
Pacific Islands	54% (178)	37% (121)	8% (26)	1% (3)	100% (328)

Table G2: Final Status of Advanced Counters by Ethnicity for Multiplication and Division

DECILE	No Change	Became Early Additive	Became Advanced Additive	Became Advanced Multiplicative	Total
Low (1–3)	44% (563)	43% (553)	11% (149)	2% (25)	100% (1290)
Middle (4–7)	43% (537)	42% (527)	13% (172)	2% (21)	100% (1257)
High (8–10)	32% (167)	50% (259)	16% (83)	2% (12)	100% (521)

Table G3: Final Status of Advanced Counters by Decile for Multiplication and Division

DECILE	No Change		Became Early Additive		Became Advanced Additive		Became Advanced Multiplicative		Total	
1	63%	(114)	33%	(59)	4%	(8)	0%	(1)	100%	(182)
2	39%	(197)	45%	(229)	14%	(69)	2%	(8)	100%	(503)
3	42%	(253)	44%	(265)	12%	(72)	2%	(16)	100%	(606)
4	41%	(265)	43%	(284)	14%	(93)	2%	(13)	100%	(655)
5	41%	(33)	48%	(38)	11%	(9)	0%	(0)	100%	(80)
6	43%	(117)	40%	(109)	15%	(41)	1%	(4)	100%	(271)
7	49%	(122)	38%	(96)	12%	(29)	1%	(4)	100%	(251)
8	32%	(68)	46%	(99)	19%	(40)	3%	(7)	100%	(214)
9	38%	(9)	54%	(13)	8%	(2)	0%	(0)	100%	(24)
10	32%	(90)	52%	(147)	14%	(41)	2%	(5)	100%	(283)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table G4: Final Status of Advanced Counters by Decile for Multiplication and Division

AGE	No Change		Became Early Additive		Became Advanced Additive		Became Advanced Multiplicative		Became Advanced Proportional		Total	
7-year-olds	50%	(39)	44%	(34)	6%	(4)	0%	(0)	0%	(0)	100%	(77)
8-year-olds	47%	(442)	44%	(413)	8%	(80)	1%	(12)	0%	(1)	100%	(948)
9-year-olds	47%	(410)	39%	(336)	11%	(100)	3%	(22)	0%	(3)	100%	(871)
10-year-olds	39%	(274)	38%	(268)	16%	(110)	6%	(47)	1%	(6)	100%	(705)
11-year-olds	43%	(23)	42%	(22)	13%	(7)	2%	(1)	0%	(0)	100%	(53)

Table G5: Final Status of Advanced Counters by Age for Fractions

ETHNICITY	No Change		Became Early Additive		Became Advanced Additive		Became Advanced Multiplicative		Became Advanced Proportional		Total	
Asian	46%	(56)	39%	(48)	11%	(13)	2%	(2)	2%	(2)	100%	(121)
European	43%	(724)	41%	(689)	12%	(210)	4%	(64)	0%	(4)	100%	(1691)
Māori	45%	(249)	43%	(237)	9%	(52)	2%	(10)	1%	(3)	100%	(551)
Other	45%	(34)	37%	(28)	15%	(11)	3%	(2)	0%	(0)	100%	(75)
Pacific Islands	58%	(126)	33%	(72)	7%	(15)	2%	(4)	0%	(1)	100%	(218)

Table G6: Final Status of Advanced Counters by Ethnicity for Fractions

DECILE	No Change	Became Early Additive	Became Advanced Additive	Became Advanced Multiplicative	Became Advanced Proportional	Total
1	65% (62)	28% (27)	5% (5)	1% (1)	0% (0)	100% (95)
2	36% (184)	46% (232)	14% (72)	4% (18)	0% (0)	100% (506)
3	43% (194)	45% (205)	10% (46)	1% (4)	1% (5)	100% (454)
4	44% (266)	40% (241)	12% (75)	3% (15)	1% (4)	100% (601)
5	32% (27)	53% (44)	11% (9)	4% (3)	0% (0)	100% (83)
6	64% (150)	23% (54)	8% (19)	5% (12)	0% (0)	100% (235)
7	64% (145)	26% (59)	9% (19)	1% (3)	0% (0)	100% (226)
8	37% (79)	40% (84)	15% (32)	8% (16)	0% (1)	100% (212)
9	54% (14)	42% (11)	4% (1)	0% (0)	0% (0)	100% (26)
10	31% (68)	53% (117)	11% (23)	5% (10)	0% (0)	100% (218)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table G7: Final Status of Advanced Counters by Decile for Fractions

DECILE	No Change	Became Early Additive	Became Advanced Additive	Became Advanced Multiplicative	Became Advanced Proportional	Total
Low (1–3)	42% (440)	44% (464)	12% (123)	2% (23)	0% (5)	100% (1055)
Middle (4–7)	51% (588)	35% (398)	11% (122)	3% (33)	0% (4)	100% (1145)
High (8–10)	35% (161)	47% (212)	12% (56)	6% (26)	0% (1)	100% (456)

Table G8: Final Status of Advanced Counters by Decile for Fractions

Appendix H: Strategy Aspects – Patterns of Improvements

Age/Period of Instruction	Count All		Advanced Counting		Early Additive Part/Whole		Advanced Additive Part/Whole		Total	
7 Initial	10%	(20)	49%	(98)	30%	(59)	11%	(21)	100%	(198)
7 Final	3%	(6)	22%	(44)	46%	(90)	29%	(58)	100%	(198)
8 Initial	11%	(275)	50%	(1216)	31%	(757)	8%	(189)	100%	(2437)
8 Final	2%	(59)	27%	(645)	46%	(1120)	25%	(613)	100%	(2437)
9 Initial	7%	(188)	40%	(1033)	38%	(980)	15%	(391)	100%	(2592)
9 Final	2%	(44)	19%	(493)	43%	(1111)	36%	(944)	100%	(2592)
10 Initial	4%	(97)	33%	(880)	41%	(1079)	23%	(604)	100%	(2660)
10 Final	1%	(18)	13%	(336)	38%	(1008)	49%	(1298)	100%	(2660)
11 Initial	4%	(8)	40%	(81)	37%	(75)	19%	(38)	100%	(202)
11 Final	2%	(4)	15%	(30)	40%	(80)	44%	(88)	100%	(202)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table H1: Patterns of Improvement for Addition and Subtraction by Age

Age/Period of Instruction	Count All		Advanced Counting		Early Additive Part/Whole		Advanced Additive Part/Whole		Advanced Multiplicati ve Part/Whole		Total	
7 Initial	23%	(45)	53%	(105)	19%	(38)	4%	(8)	1%	(2)	100%	(198)
7 Final	10%	(19)	30%	(60)	38%	(76)	19%	(38)	3%	(5)	100%	(198)
8 Initial	24%	(597)	48%	(1159)	20%	(480)	7%	(169)	1%	(32)	100%	(2437)
8 Final	6%	(145)	34%	(835)	36%	(876)	18%	(437)	6%	(144)	100%	(2437)
9 Initial	14%	(357)	38%	(995)	31%	(791)	14%	(366)	3%	(83)	100%	(2592)
9 Final	4%	(97)	24%	(615)	34%	(881)	27%	(699)	11%	(300)	100%	(2592)
10 Initial	7%	(198)	28%	(734)	36%	(953)	21%	(573)	8%	(202)	100%	(2660)
10 Final	2%	(52)	14%	(373)	29%	(781)	32%	(837)	23%	(617)	100%	(2660)
11 Initial	11%	(23)	37%	(75)	31%	(62)	17%	(34)	4%	(8)	100%	(202)
11 Final	4%	(8)	16%	(32)	37%	(75)	28%	(56)	15%	(31)	100%	(202)

* Note: Percentages have been rounded to the nearest whole number where necessary.

Table H2: Patterns of Improvement for Multiplication and Division by Age

Age/Period of Instruction	Count All	Advanced Counting	Early Additive Part/Whole	Advanced Additive Part/Whole	Advanced Multiplicative Part/Whole	Advanced Proportional Part/Whole	Total
7 Initial	29% (57)	39% (77)	26% (52)	5% (11)	1% (1)	0% (0)	100% (198)
7 Final	8% (15)	31% (62)	43% (85)	14% (27)	3% (7)	1% (2)	100% (198)
8 Initial	32% (781)	39% (948)	25% (606)	3% (71)	1% (30)	0% (1)	100% (2437)
8 Final	10% (234)	32% (780)	42% (1023)	11% (277)	4% (113)	1% (10)	100% (2437)
9 Initial	19% (483)	34% (871)	35% (918)	9% (235)	3% (78)	0% (7)	100% (2592)
9 Final	6% (145)	24% (636)	39% (1014)	20% (516)	9% (238)	2% (43)	100% (2592)
10 Initial	13% (355)	27% (705)	37% (986)	13% (356)	9% (237)	1% (21)	100% (2660)
10 Final	5% (129)	16% (418)	33% (870)	22% (588)	20% (539)	4% (116)	100% (2660)
11 Initial	15% (31)	26% (53)	44% (88)	9% (19)	5% (9)	1% (2)	100% (202)
11 Final	7% (15)	17% (34)	39% (79)	23% (46)	11% (23)	3% (5)	100% (202)

Table H3: Patterns of Improvement for Fractions by Age